On gauge-dependence of gravitational waves from 1\textsuperscript{st}-order phase transitions

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April 27, 2019@CCNU

2019 CCNU - cfa@USTC Junior Cosmology Symposium

based on

Cheng-Wei Chiang (Natl Taiwan U), E.S., arXiv: 1707.06765 (PLB)
Outline

• Introduction
  • Gauge-dependence ($\xi$) of the effective potential
• Impact of $\xi$ on 1st-order phase transition in classical scale-inv. U(1) models: $T_N$, GW
• Summary
Introduction

- 1st-order phase transition (PT) has interesting physical implications:
  Electroweak Baryogenesis, Gravitational Waves (GW), etc.

- Mostly, effective potential is used for such calculations.

problem

- Effective potential inherently depends on gauge-fixing parameter (ξ).

- Nucleation temperature (TN), GW can be ξ dependent.

Q. How (numerically) serious?
Thorny problem

Effective potential is gauge dependent!!

Because

\[ V_{\text{eff}} \ni \ni \]

1PI diagrams only

Leg corrections are needed to remove the \( \xi \) dependence.

Jackiw, PRD9,1686 (1974)
Gauge dependence of $V_{\text{eff}}$

- Generally, VEV depends on a gauge parameter $\xi$

- Energies at stationary points do not depend on $\xi$

$$\frac{\partial V_{\text{eff}}}{\partial \xi} = C(\phi, \xi) \frac{\partial V_{\text{eff}}}{\partial \phi}$$

(Nielsen-Fukuda-Kugo (NFK) identity)
**Abelian-Higgs model**  

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi|^2 - V(|\Phi|^2), \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu \Phi = (\partial_\mu - ie A_\mu) \Phi, \]

\[ V(|\Phi|^2) = -\nu^2 |\Phi|^2 + \frac{\lambda}{4} |\Phi|^4, \quad \Phi(x) = \frac{1}{\sqrt{2}} \left( v + h(x) + iG(x) \right). \]

**gauge boson:** \[ D^{-1}_{\mu\nu}(k) = (-k^2 + \bar{m}_A^2) \Pi^T_{\mu\nu}(k) + \frac{1}{\xi} (-k^2 + \xi \bar{m}_A^2) \Pi^L_{\mu\nu}(k), \]

\[ \Pi^T_{\mu\nu}(k) = \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right), \quad \Pi^L_{\mu\nu}(k) = \frac{k_\mu k_\nu}{k^2}, \]

**NG boson:** \[ \Delta^{-1}_G(k) = k^2 - \bar{m}_G^2 - \xi \bar{m}_A^2 \]

**ghost:** \[ \Delta^{-1}_c(k) = i(k^2 - \xi \bar{m}_A^2) \]
Abelian-Higgs model ~ an example ~

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi|^2 - V(|\Phi|^2),
\]

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad D_\mu \Phi = (\partial_\mu - ieA_\mu)\Phi,
\]

\[
V(|\Phi|^2) = -\nu^2|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4, \quad \Phi(x) = \frac{1}{\sqrt{2}}(v + h(x) + iG(x)).
\]

gauge boson: \[
D_{\mu\nu}^{-1}(k) = (-k^2 + \bar{m}_A^2)\Pi_{\mu\nu}^T(k) + \frac{1}{\xi}(-k^2 + \xi\bar{m}_A^2)\Pi_{\mu\nu}^L(k),
\]

\[
\Pi_{\mu\nu}^T(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right), \quad \Pi_{\mu\nu}^L(k) = \frac{k_\mu k_\nu}{k^2},
\]

NG boson: \[
\Delta_G^{-1}(k) = k^2 - \bar{m}_G^2 - \xi\bar{m}_A^2
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ghost: \[
\Delta_c^{-1}(k) = i(k^2 - \xi\bar{m}_A^2)
\]
1-loop effective potential

e.g., Abelian-Higgs model

\[
\mu^e V_1^{A+G+c}(\varphi, \xi) = -\frac{i}{2} \mu^e \int \frac{d^D k}{(2\pi)^D} \left[ (D - 1) \ln(-k^2 + \bar{m}_A^2) + \ln(-k^2 + \xi \bar{m}_A^2) \right. \\
+ \ln(-k^2 + \xi \bar{m}_A^2) + \ln \left( 1 + \frac{\bar{m}_G^2}{-k^2 + \xi \bar{m}_A^2} \right) \\
\left. - 2 \ln(-k^2 + \xi \bar{m}_A^2) \right]
\]

\[
= -\frac{i}{2} \mu^e \int \frac{d^D k}{(2\pi)^D} \left[ (D - 1) \ln(-k^2 + \bar{m}_A^2) + \ln \left( 1 + \frac{\bar{m}_G^2}{-k^2 + \xi \bar{m}_A^2} \right) \right].
\]

\(\xi\)-dependence disappears at \(\bar{m}_G^2(\varphi = v) = 0, \quad \frac{\partial V_0}{\partial \varphi} \bigg|_{\varphi = v} = 0\)

NFK identity at 1-loop level:

\[
\frac{\partial V_1(\varphi, \xi)}{\partial \xi} = C(\varphi, \xi) \frac{\partial V_0(\varphi)}{\partial \varphi}
\]
1-loop effective potential

e.g., Abelian-Higgs model

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\mu^e V_1^{A+G+c}(\varphi, \xi) = -\frac{i}{2} \mu^e \int \frac{d^D k}{(2\pi)^D} \left[ (D - 1) \ln(-k^2 + \tilde{m}_A^2) + \ln(-k^2 + \xi \tilde{m}_A^2) + \ln(-k^2 + \xi \tilde{m}_A^2) \right.
\]
\[
+ \ln(-k^2 + \xi \tilde{m}_A^2) + \ln \left( 1 + \frac{\tilde{m}_G^2}{-k^2 + \xi \tilde{m}_A^2} \right)
\]
\[
- 2 \ln(-k^2 + \xi \tilde{m}_A^2) \right]
\]

= -\frac{i}{2} \mu^e \int \frac{d^D k}{(2\pi)^D} \left[ (D - 1) \ln(-k^2 + \tilde{m}_A^2) + \ln \left( 1 + \frac{\tilde{m}_G^2}{-k^2 + \xi \tilde{m}_A^2} \right) \right].
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\xi -dependence disappears at \( \tilde{m}_G^2(\varphi = v) = 0, \quad \frac{\partial V_0}{\partial \varphi} \bigg|_{\varphi = v} = 0 \)

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\[ + \ln(-k^2 + \xi \tilde{m}_A^2) + \ln \left( 1 + \frac{\tilde{m}_G^2}{-k^2 + \xi \tilde{m}_A^2} \right) \]

\[ - 2 \ln(-k^2 + \xi \tilde{m}_A^2) \]

\[ \left. \right] \]

\[ = -\frac{i}{2} \mu^e \int \frac{d^D k}{(2\pi)^D} \left[ (D - 1) \ln(-k^2 + \tilde{m}_A^2) + \ln \left( 1 + \frac{\tilde{m}_G^2}{-k^2 + \xi \tilde{m}_A^2} \right) \right]. \]

\[ \xi \text{-dependence disappears at} \quad \tilde{m}_G^2(\varphi = v) = 0, \quad \frac{\partial V_0}{\partial \varphi} \bigg|_{\varphi=v} = 0 \]

NFK identity at 1-loop level:

\[ \frac{\partial V_1(\varphi, \xi)}{\partial \xi} = C(\varphi, \xi) \frac{\partial V_0(\varphi)}{\partial \varphi} \]
1-loop effective potential

e.g., Abelian-Higgs model

\[
\mu^\epsilon V_1^{A+G+c}(\varphi, \xi) = -\frac{i}{2} \mu^\epsilon \int \frac{d^D k}{(2\pi)^D} \left[ (D - 1) \ln(-k^2 + \bar{m}_A^2) + \ln(-k^2 + \xi \bar{m}_A^2) \right.
\]
\[
\quad + \ln(-k^2 + \xi \bar{m}_A^2) + \ln \left( 1 + \frac{\bar{m}_G^2}{-k^2 + \xi \bar{m}_A^2} \right)
\]
\[
\left. + 2 \ln(-k^2 + \xi \bar{m}_A^2) \right]
\]
\[
= -\frac{i}{2} \mu^\epsilon \int \frac{d^D k}{(2\pi)^D} \left[ (D - 1) \ln(-k^2 + \bar{m}_A^2) + \ln \left( 1 + \frac{\bar{m}_G^2}{-k^2 + \xi \bar{m}_A^2} \right) \right].
\]

\(\xi\)-dependence disappears at \(\bar{m}_G(\varphi = v) = 0\), \(\varphi = v\)

NFK identity at 1-loop level:

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\frac{\partial V_1(\varphi, \xi)}{\partial \xi} = C(\varphi, \xi) \frac{\partial V_0(\varphi)}{\partial \varphi}
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1-loop effective potential

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\[ \xi \text{-dependence disappears at } \tilde{m}_G^2(\varphi = v) = 0, \quad \frac{\partial V_0}{\partial \varphi} \bigg|_{\varphi=v} = 0 \]

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\]

\[
= -\frac{i}{2} \mu^e \int \frac{d^D k}{(2\pi)^D} \left[ (D - 1) \ln(-k^2 + \tilde{m}_A^2) + \ln\left(1 + \frac{\tilde{m}_G^2}{-k^2 + \xi \tilde{m}_A^2}\right) \right].
\]

\(\xi\)-dependence disappears at \(\tilde{m}_G(\varphi = v) = 0\), \(\frac{\partial V_0}{\partial \varphi} \bigg|_{\varphi=v} = 0\)

NFK identity at 1-loop level:

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\mu^e V_1^{A+G+c}(\varphi, \xi) = -\frac{i}{2} \mu^e \int \frac{d^D k}{(2\pi)^D} \left[ (D - 1) \ln(-k^2 + m^2_A) + \ln(-k^2 + \xi m^2_A) \right. \\
\left. + \ln(-k^2 + \xi m^2_A) + \ln \left( 1 + \frac{m^2_G}{-k^2 + \xi m^2_A} \right) \right] \\
= -\frac{i}{2} \mu^e \int \frac{d^D k}{(2\pi)^D} \left[ (D - 1) \ln(-k^2 + \bar{m}^2_A) + \ln \left( 1 + \frac{m^2_G}{-k^2 + \xi \bar{m}^2_A} \right) \right].
\]

\[\xi\text{-dependence disappears at } \bar{m}^2_G(\varphi = v) = 0, \quad \frac{\partial V_0}{\partial \varphi}\bigg|_{\varphi=v} = 0\]

NFK identity at 1-loop level:

\[
\frac{\partial V_1(\varphi, \xi)}{\partial \xi} = \frac{\partial V_0(\varphi)}{\partial \varphi} C(\varphi, \xi)
\]
Plotting $V_{\text{eff}} = V_0 + V_1$, 

No $\xi$-dependence at $\frac{\partial V_0}{\partial \varphi} = 0$ but it is no longer a minimum at 1-loop level.
When 1-loop minimization condition is imposed, \[ \frac{\partial (V_0 + V_1)}{\partial \varphi} = 0 \]

Energy at \( \phi = 246 \text{ GeV} \) depends on \( \xi \).!!
1-loop effective potential at $T \neq 0$

$$V_1(\varphi, \xi; T) = \sum_i \frac{T^4}{2\pi^2} I_B(a_i^2),$$

where

$$I_B(a^2) = \int_0^\infty dx \ x^2 \ln \left[ 1 - e^{-\sqrt{x^2 + a^2}} \right].$$

Using a high-$T$ expansion of $I_B(a^2)$, one gets

$$V_1(\varphi, \xi) + V_1(\varphi, \xi; T)$$

$$= \frac{T^2}{24} (\bar{m}_h^2 + \bar{m}_G^2 + 3\bar{m}_A^2) - \frac{T}{12\pi} \left[ (\bar{m}_h^2)^{3/2} + (\bar{m}_G^2 + \xi \bar{m}_A^2)^{3/2} + (3 - \xi^{3/2})(\bar{m}_A^2)^{3/2} \right]$$

$$+ \frac{1}{64\pi^2} \left[ \bar{m}_h^4 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + (\bar{m}_G^2 + \xi \bar{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + 3\bar{m}_A^4 \left( \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + \frac{2}{3} \right) - (\xi \bar{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} \right].$$
1-loop effective potential at T≠0

\[ V_1(\varphi, \xi; T) = \sum_i \frac{T^4}{2\pi^2} I_B(\alpha_i^2), \]

where

\[ I_B(a^2) = \int_0^\infty dx \, x^2 \ln \left[ 1 - e^{-\sqrt{x^2+a^2}} \right]. \]

Using a high-T expansion of \( I_B(a^2) \), one gets

\[
\begin{align*}
V_1(\varphi, \xi) + V_1(\varphi, \xi; T) &= \frac{T^2}{24}(\bar{m}_h^2 + \bar{m}_G^2 + 3\bar{m}_A^2) - \frac{T}{12\pi} \left[ (\bar{m}_h^2)^{3/2} + (\bar{m}_G^2 + \xi \bar{m}_A^2)^{3/2} + (3 - \xi^{3/2})(\bar{m}_A^2)^{3/2} \right] \\
&\quad + \frac{1}{64\pi^2} \left[ \bar{m}_h^4 \ln \left( \frac{\alpha_B T^2}{\bar{\mu}^2} \right) + (\bar{m}_G^2 + \xi \bar{m}_A^2)^2 \ln \left( \frac{\alpha_B T^2}{\bar{\mu}^2} \right) + 3\bar{m}_A^4 \left( \frac{\ln \left( \frac{\alpha_B T^2}{\bar{\mu}^2} \right)}{\bar{\mu}^2} + \frac{2}{3} \right) - (\xi \bar{m}_A^2)^2 \ln \left( \frac{\alpha_B T^2}{\bar{\mu}^2} \right) \right].
\end{align*}
\]

\( V_{\text{eff}} \) at \( T\neq 0 \) also depends on \( \xi \) except "\( T^2 \)-terms".
Figure 5. A comparison of the critical temperatures as computed using the following methods: the standard method (solid line); the gauge-independent methods described in the text, derived from the full theory (dashed lines); and by performing lattice simulations (arrow). Note that the lattice result is much higher than the perturbative estimations and is displayed on a separate scale.

Also included are lattice results, that yield a critical temperature of 126.8 GeV, independent of $\xi$ by construction. Our estimate of the higher-order contributions included in (5.15) leads to a substantially larger value of $T_C$, suggesting that the difference between the non-perturbative and $O(\epsilon)$ perturbative results arises in part from the omission of higher-order contributions. In addition, we note that the precise definition of $T_C$ as obtained from the lattice studies differs from the one we have employed here as well as in other perturbative analyses (For a discussion of the lattice determinations, see, e.g., refs. [4, 6, 7]). We speculate that part of the difference between the lattice and perturbative results may also be due to this difference in definition.

While the gauge-independent perturbative estimation of $T_C$ falls below the lattice value, it is interesting that the dependence on the relevant couplings follows the trend observed in non-perturbative studies. To illustrate, we plot the one-loop $T_C$ as a function of the Higgs quartic self-coupling $\lambda$ in figure 6. We observe that increasing $\lambda$ increases $T_C$ in agreement with our qualitative expectations in (3.11). As we discuss shortly, this trend implies that the efficiency of sphaleron-induced baryon number washout increases with $\lambda$ and, thus, with the value of the Higgs boson mass. This trend is also observed in non-perturbative studies as well as in earlier gauge-dependent perturbative analyses.

We now turn our attention to the sphaleron scale, $\bar{\nu}(T)$, which we plot in figure 7. We observe that in the vicinity of the $T_C$ obtained at $O(\epsilon)$ in the full theory, $\bar{\nu}(T)$ drops rapidly to zero. This behavior makes the perturbative estimate of the sphaleron rate at the critical temperature highly sensitive to small changes in $T_C$. Therefore, statements about the efficiency of baryon number preservation are susceptible to large uncertainties. To illustrate, we first consider the value of this scale at the one-loop $T_C$ in the full theory,
Figure 5. A comparison of the critical temperatures as computed using the following methods: the standard method (solid line); the gauge-independent methods described in the text, derived from the full theory (dashed lines); and by performing lattice simulations (arrow). Note that the lattice result is much higher than the perturbative estimations and is displayed on a separate scale. Also included are lattice results, that yield a critical temperature of 126.8 GeV, independent of $\xi$ by construction. Our estimate of the higher-order contributions included in (5.15) leads to a substantially larger value of $T_C$, suggesting that the difference between the non-perturbative and $O(\hbar)$ perturbative results arises in part from the omission of higher-order contributions. In addition, we note that the precise definition of $T_C$ as obtained from the lattice studies differs from the one we have employed here as well as in other perturbative analyses (For a discussion of the lattice determinations, see, e.g., refs. [4, 6, 7]). We speculate that part of the difference between the lattice and perturbative results may also be due to this difference in definition.

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We now turn our attention to the sphaleron scale, $\bar{v}(T_C)$, which we plot in figure 7. We observe that in the vicinity of the $T_C$ obtained at $O(\hbar)$ in the full theory, $\bar{v}(T_C)$ drops rapidly to zero. This behavior makes the perturbative estimate of the sphaleron rate at the critical temperature highly sensitive to small changes in $T_C$. Therefore, statements about the efficiency of baryon number preservation are susceptible to large uncertainties. To illustrate, we first consider the value of this scale at the one-loop $T_C$ in the full theory, $\bar{v}(T_C)$. We observe that in the vicinity of the $T_C$ obtained at $O(\hbar)$ in the full theory, $\bar{v}(T_C)$ drops rapidly to zero. This behavior makes the perturbative estimate of the sphaleron rate at the critical temperature highly sensitive to small changes in $T_C$. Therefore, statements about the efficiency of baryon number preservation are susceptible to large uncertainties. To illustrate, we first consider the value of this scale at the one-loop $T_C$ in the full theory, $\bar{v}(T_C)$.

$\lambda=0.035 \ (m_H \approx 65 \text{ GeV})$

- $\text{latt: } 126.8 \text{ GeV}$
- $\mathcal{O}(\hbar^2) + \Delta V_{\text{ring}}^{G.I.}(T) : 104.2 \text{ GeV}$
- $\text{Landau: } 78.0 \text{ GeV}$
- $\mathcal{O}(\hbar) : 70.6 \text{ GeV}$

SM case

Figure 5. A comparison of the critical temperatures as computed using the following methods: the standard method (solid line); the gauge-independent methods described in the text, derived from the full theory (dashed lines); and by performing lattice simulations (arrow). Note that the lattice result is much higher than the perturbative estimations and is displayed on a separate scale. Also included are lattice results, that yield a critical temperature of 126.8 GeV, independent of \( \xi \) by construction. Our estimate of the higher-order contributions included in (5.15) leads to a substantially larger value of \( T_C \), suggesting that the difference between the non-perturbative and \( O(\hbar) \) perturbative results arises in part from the omission of higher-order contributions. In addition, we note that the precise definition of \( T_C \) as obtained from the lattice studies differs from the one we have employed here as well as in other perturbative analyses (For a discussion of the lattice determinations, see, e.g., refs. [4, 6, 7]).

We speculate that part of the difference between the lattice and perturbative results may also be due to this difference in definition. While the gauge-independent perturbative estimation of \( T_C \) falls below the lattice value, it is interesting that the dependence on the relevant couplings follows the trend observed in non-perturbative studies. To illustrate, we plot the one-loop \( T_C \) as a function of the Higgs quartic self-coupling \( \lambda \) in figure 6. We observe that increasing \( \lambda \) increases \( T_C \) in agreement with our qualitative expectations in (3.11). As we discuss shortly, this trend implies that the efficiency of sphaleron-induced baryon number washout increases with \( \lambda \) and, thus, with the value of the Higgs boson mass. This trend is also observed in non-perturbative studies as well as in earlier gauge-dependent perturbative analyses.

We now turn our attention to the sphaleron scale, \( \bar{v}(T) \), which we plot in figure 7. We observe that in the vicinity of the \( T_C \) obtained at \( O(\hbar) \) in the full theory, \( \bar{v}(T) \) drops rapidly to zero. This behavior makes the perturbative estimate of the sphaleron rate at the critical temperature highly sensitive to small changes in \( T_C \). Therefore, statements about the efficiency of baryon number preservation are susceptible to large uncertainties. To illustrate, we first consider the value of this scale at the one-loop \( T_C \) in the full theory.
Classical scale-inv. U(1) model

SM + U(1)' w/ scale symmetry

\[
\mathcal{L} = \mathcal{L}_{SM'} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} + |D_\mu S|^2 - V(H, S)
\]

\[
Z'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu, \quad D_\mu S = (\partial_\mu + ig'Q'_S Z'_\mu)S,
\]

scalar potential

\[
V(H, S) = \lambda_H (H^\dagger H)^2 + \lambda_{HS} H^\dagger H |S|^2 + \lambda_S |S|^4
\]

singlet scalar field: \[ S(x) = \frac{1}{\sqrt{2}} (v_S + h_S(x) + iG(x)) \]

After U(1) is radiatively broken (<S>≠0), EW symmetry is broken if \( \lambda_{HS} < 0 \).

\[ m_h^2 = -\lambda_{HS} v_S^2 \quad \rightarrow \quad -\lambda_{HS} = m_h^2/v_S^2 = \mathcal{O}(10^{-3}) \]
Classical scale-inv. $U(1)$ model

SM + $U(1)'$ w/ scale symmetry

$$\mathcal{L} = \mathcal{L}_{SM'} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} + |D_\mu S|^2 - V(H, S)$$

$$Z'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu, \quad D_\mu S = (\partial_\mu + ig'Q'_S Z'_\mu)S,$$

scalar potential

$$V(H, S) = \lambda_H (H^\dagger H)^2 + \lambda_{HS} H^\dagger H |S|^2 + \lambda_S |S|^4$$

singlet scalar field: $S(x) = \frac{1}{\sqrt{2}}(v_S + h_S(x) + iG(x))$

After $U(1)$ is radiatively broken ($<S>\neq 0$), EW symmetry is broken if $\lambda_{HS} < 0$. $m_h^2 = -\lambda_{HS} v_S^2 \quad \rightarrow \quad -\lambda_{HS} = m_h^2/v_S^2 = \mathcal{O}(10^{-3})$
Classical scale-inv. U(1) model

ξ dependence is different from the massive U(1) model case.

\[
V_{\text{eff}}(\varphi_S) = \frac{\lambda_S}{4} \varphi_S^4 + 3 \frac{\bar{m}_{Z'}^4}{64\pi^2} \left( \ln \frac{\bar{m}_{Z'}^2}{\bar{\mu}^2} - \frac{5}{6} \right) + \frac{\bar{m}_{G,\xi}^4}{64\pi^2} \left( \ln \frac{\bar{m}_{G,\xi}^2}{\bar{\mu}^2} - \frac{3}{2} \right) - \frac{(\xi \bar{m}_{Z'}^2)^2}{64\pi^2} \left( \ln \frac{\xi \bar{m}_{Z'}^2}{\bar{\mu}^2} - \frac{3}{2} \right),
\]

where \( \bar{m}_{Z'}^2 = (g' Q'_S \varphi_S)^2, \bar{m}_{G,\xi}^2 = \lambda_S \varphi_S^2 + \xi \bar{m}_{Z'}^2. \)

Minimization condition \( \rightarrow \lambda_S = O(g'^4/16\pi^2) \)

One gets

\[
V_{\text{eff}}(\varphi_S) \simeq 3 \frac{\bar{m}_{Z'}^4}{64\pi^2} \left( \ln \frac{\varphi_S^2}{v_S^2} - \frac{1}{2} \right), \quad \xi \text{ independent!!}
\]

- Finite-T 1-loop effective potential is also \( \xi \) independent.
- \( \xi \) dependence will appear from 2-loop order.
Classical scale-inv. U(1) model

dependence is different from the massive U(1) model case.

\[
V_{\text{eff}}(\varphi_S) = \frac{\lambda_S}{4} \varphi_S^4 + 3 \frac{m_{Z'}^4}{64\pi^2} \left( \ln \frac{m_{Z'}^2}{\bar{\mu}^2} - \frac{5}{6} \right) + \frac{m_{G,\xi}^4}{64\pi^2} \left( \ln \frac{m_{G,\xi}^2}{\bar{\mu}^2} - \frac{3}{2} \right) - \frac{(\xi m_{Z'}^2)^2}{64\pi^2} \left( \ln \frac{\xi m_{Z'}^2}{\bar{\mu}^2} - \frac{3}{2} \right),
\]

where \( m_{Z'}^2 = (g'Q'_S \varphi_S)^2 \), \( m_{G,\xi}^2 = \lambda_S \varphi_S^2 + \xi m_{Z'}^2 \).

Minimization condition \( \rightarrow \) \( \lambda_S = O(g'^4/16\pi^2) \)

One gets

\[
V_{\text{eff}}(\varphi_S) \simeq \frac{3m_{Z'}^4}{64\pi^2} \left( \ln \frac{\varphi_S^2}{v_S^2} - \frac{1}{2} \right), \quad \xi \text{ independent!!}
\]

- Finite-T 1-loop effective potential is also \( \xi \) independent.
- \( \xi \) dependence will appear from 2-loop order.
Classical scale-inv. U(1) model

\(\xi\) dependence is different from the massive U(1) model case.

\[
V_{\text{eff}}(\varphi_S) = \frac{\lambda_S}{4} \varphi_S^4 + 3 \frac{\bar{m}_{Z'}^4}{64\pi^2} \left( \ln \frac{\bar{m}_{Z'}^2}{\bar{\mu}^2} - \frac{5}{6} \right)
+ \frac{\bar{m}_{G,\xi}^4}{64\pi^2} \left( \ln \frac{\bar{m}_{G,\xi}^2}{\bar{\mu}^2} - \frac{3}{2} \right) - \frac{(\xi \bar{m}_{Z'}^2)^2}{64\pi^2} \left( \ln \frac{\xi \bar{m}_{Z'}^2}{\bar{\mu}^2} - \frac{3}{2} \right),
\]

where \(\bar{m}_{Z'}^2 = (g'Q'_S \varphi_S)^2\), \(\bar{m}_{G,\xi}^2 = \lambda_S \varphi_S^2 + \xi \bar{m}_{Z'}^2\).

Minimization condition \(\rightarrow \lambda_S = O(g'^4/16\pi^2)\)

One gets

\[
V_{\text{eff}}(\varphi_S) \simeq \frac{3\bar{m}_{Z'}^4}{64\pi^2} \left( \ln \frac{\varphi_S^2}{v_S^2} - \frac{1}{2} \right), \quad \xi \text{ independent!!}
\]

- Finite-T 1-loop effective potential is also \(\xi\) independent.
- \(\xi\) dependence will appear from 2-loop order.
Thermal resummation

At high $T$

$$g^2 \sim g^2 T^2$$

$V_{\text{eff}}$ is no longer $\xi$ independent due to $\Delta m_S^2 \neq 0$

\[
\bar{m}_S^2, \xi \rightarrow \bar{m}_S^2, \xi + \Delta m_S^2
\]

to leading order:

\[
\Delta m_S^2 = \frac{(g' Q'_S)^2}{4} T^2
\]

\[
V_{\text{eff}}(\varphi_S; T) = \frac{(\xi \bar{m}_{Z'}^2 + \Delta m_S^2)^2}{64 \pi^2} \left( \ln \frac{\xi \bar{m}_{Z'}^2 + \Delta m_S^2}{\bar{\mu}^2} - \frac{3}{2} \right) - \frac{(\xi \bar{m}_{Z'}^2)^2}{64 \pi^2} \left( \ln \frac{\xi \bar{m}_{Z'}^2}{\bar{\mu}^2} - \frac{3}{2} \right)
\]

\[
+ \frac{T^4}{2 \pi^2} \left[ I_B \left( \frac{\xi \bar{m}_{Z'}^2 + \Delta m_S^2}{T^2} \right) - I_B \left( \frac{\xi \bar{m}_{Z'}^2}{T^2} \right) \right]
\]

where $I_B$ is a 1-loop thermal function.

$V_{\text{eff}}$ is no longer $\xi$ independent due to $\Delta m_S^2 \neq 0$
Thermal resummation

At high $T$

\[ g^2 \sim g^2 T^2 \]

Prescription

e.g. NG boson

\[ \bar{m}_{G,\xi}^2 \rightarrow \bar{m}_{G,\xi}^2 + \Delta m_S^2 \]

to leading order:

\[ \Delta m_S^2 = \frac{(g' Q_S')^2}{4} T^2 \]

where $I_B$ is a 1-loop thermal function.

\[ V_{\text{eff}}(\varphi_S; T) = \frac{(\xi \bar{m}_Z' + \Delta m_S^2)^2}{64\pi^2} \left( \ln \frac{\xi \bar{m}_Z'}{\bar{\mu}^2} + \Delta m_S^2 \right) - \frac{(\xi \bar{m}_Z')^2}{64\pi^2} \left( \ln \frac{\xi \bar{m}_Z'}{\bar{\mu}^2} - \frac{3}{2} \right) \]

\[ + \frac{T^4}{2\pi^2} \left[ I_B \left( \frac{\xi \bar{m}_Z' + \Delta m_S^2}{T^2} \right) - I_B \left( \frac{\xi \bar{m}_Z'}{T^2} \right) \right] . \]

$V_{\text{eff}}$ is no longer $\xi$ independent due to $\Delta m_S^2 \neq 0$
Gravitational Waves from 1\textsuperscript{st}-order EWPT

GWs are induced by the 1\textsuperscript{st}-order EWPT.

Sources of GW

(1) Bubble collisions,
(2) Sound waves,
(3) Turbulence

See Ref. [C.Caprini et al, 1512.06239(JCAP)]

2 important parameters: [Grojean, Servant, hep-ph/0607107(PRD)]

latent heat ($\alpha$), duration of PT ($\beta$)

$$\alpha \equiv \frac{\epsilon(T_*)}{\rho_{\text{rad}}(T_*)} \quad \text{and} \quad \beta \equiv H_* T_* \left. \left( \frac{S_3(T)}{T} \right) \right|_{T=T_*}, \quad \epsilon(T) = \Delta V_{\text{eff}} - T \frac{\partial \Delta V_{\text{eff}}}{\partial T} \quad \text{and} \quad \rho_{\text{rad}}(T) = \frac{\pi^2}{30} g_*(T) T^4,$$
Gravitational Waves from 1\textsuperscript{st}-order EWPT

\[ \Omega_{GW} h^2 = \Omega_{\text{col}} h^2 + \Omega_{\text{sw}} h^2 + \Omega_{\text{turb}} h^2 \]

Dominant source is sound waves:

\[ \Omega_{\text{sw}} h^2(f) = 2.65 \times 10^{-6} \tilde{\beta}^{-1} \left( \frac{\kappa_v \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{1/3} v_w \left( \frac{f}{f_{sw}} \right)^3 \left( \frac{7}{4 + 3(f/f_{sw})^2} \right)^{7/2} \]

\[ f_{sw} = 1.9 \times 10^{-2} \text{ mHz} \quad \tilde{\beta} = \frac{\beta}{H_*} \]

\[ \kappa_v \simeq \frac{\alpha}{(0.73 + 0.083 \sqrt{\alpha + \alpha})} \text{ for } v_w \simeq 1. \]

- Most calculations of \( \alpha \& \beta \) in the literature depends on \( \xi \).
- How much \( \xi \) dependence can affect GW?
From Eqs. (17) and (18), one obtains

\[ Q'_S = 2, \quad \alpha' = g'^2 / 4\pi = 0.015, \quad m_{Z'} = 4.5 \text{ TeV} \quad \text{and} \quad m_{\nu_{R1,2,3}} = 1.0 \text{ TeV}. \]

\[
S_3 = 4\pi \int_0^\infty dr \, r^2 \left[ \frac{1}{2} \left( \frac{d\phi_S}{dr} \right)^2 + V_{\text{eff}}(\phi_S; T) \right]
\]

\[
\frac{d^2 \phi_S}{dr^2} + \frac{2}{r} \frac{d\phi_S}{dr} - \frac{\partial V_{\text{eff}}}{\partial \phi_S} = 0
\]

\[ \lim_{r \to \infty} \phi_S(r) = 0 \quad \text{and} \quad \frac{d\phi_S(r)}{dr} \bigg|_{r=0} = 0. \]

\begin{table}
\begin{tabular}{|c|c|c|c|c|}
\hline
& \text{no resum} & \text{\(\xi = 0\)} & \text{\(\xi = 1\)} & \text{\(\xi = 5\)} \\
\hline
\(v_S(T_*)/T_*\) & 5.181/0.328 = 15.8 & 5.181/0.368 = 14.1 & 5.180/0.405 = 12.8 & 5.163/0.490 = 10.5 \\
\hline
\(\alpha\) & 2.27 & 1.44 & 0.99 & 0.48 \\
\(\tilde{\beta}\) & 89.4 & 97.5 & 105.4 & 135.0 \\
\hline
\end{tabular}
\end{table}
From Eqs. (17) and (18), one obtains

\[ Q'_S = 2, \quad \alpha' = g'^2/4\pi = 0.015, \quad m_{Z'} = 4.5 \text{ TeV and } m_{\nu_{R1,2,3}} = 1.0 \text{ TeV.} \]

\[ \xi \text{ dep.} \]

\[ S_3 = 4\pi \int_0^\infty dr \ r^2 \left[ \frac{1}{2} \left( \frac{d\phi_S}{dr} \right)^2 + V_{\text{eff}}(\phi_S; T) \right] \]

\[ \frac{d^2 \phi_S}{dr^2} + \frac{2}{r} \frac{d\phi_S}{dr} - \frac{\partial V_{\text{eff}}}{\partial \phi_S} = 0 \]

\[ \lim_{r \to \infty} \phi_S(r) = 0 \text{ and } d\phi_S(r)/dr|_{r=0} = 0. \]

\[
\begin{array}{|c|c|c|c|c|}
\hline
v_S(T_*)/T_* & 5.181/0.328 = 15.8 & 5.181/0.368 = 14.1 & 5.180/0.405 = 12.8 & 5.163/0.490 = 10.5 \\
\hline
\alpha & 2.27 & 1.44 & 0.99 & 0.48 \\
\beta & 89.4 & 97.5 & 105.4 & 135.0 \\
\hline
\end{array}
\]
From Eqs. (17) and (18), one obtains at least one bubble must nucleate within the Hubble volume. We thus define with the boundary conditions: \( \lim_{r \to \infty} \phi_S(r) = 0 \) and \( d\phi_S(r)/dr|_{r=0} = 0 \).

\[
S_3 = 4\pi \int_0^\infty dr \, r^2 \left[ \frac{1}{2} \left( \frac{d\phi_S}{dr} \right)^2 + V_{\text{eff}}(\phi_S; T) \right]
\]

\[
\frac{d^2\phi_S}{dr^2} + \frac{2}{r} \frac{d\phi_S}{dr} - \frac{\partial V_{\text{eff}}}{\partial \phi_S} = 0
\]

\[
\xi \text{ dep.}
\]

\[
Q'_S = 2, \alpha' = g'^2/4\pi = 0.015, m_{Z'} = 4.5 \text{ TeV and } m_{\nu_{R1,2,3}} = 1.0 \text{ TeV}.
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\xi & \xi = 0 & \xi = 1 & \xi = 5 \\
\hline
t_S(T_*)/T_* & 5.181/0.328 = 15.8 & 5.181/0.368 = 14.1 & 5.180/0.405 = 12.8 & 5.163/0.490 = 10.5 \\
\alpha & 2.27 & 1.44 & 0.99 & 0.48 \\
\tilde{\beta} & 89.4 & 97.5 & 105.4 & 135.0 \\
\hline
\end{array}
\]
Impact of $\xi$ on $T_N$

$Q'_S = 2$, $\alpha' = g'^2/4\pi = 0.015$, $m_{Z'} = 4.5$ TeV and $m_{\nu_{R1,2,3}} = 1.0$ TeV.

\[
S_3 = 4\pi \int_0^\infty dr \, r^2 \left[ \frac{1}{2} \left( \frac{d\phi_S}{dr} \right)^2 + V_{\text{eff}}(\phi_S; T) \right]
\]

\[
\frac{d^2\phi_S}{dr^2} + \frac{2}{r} \frac{d\phi_S}{dr} - \frac{\partial V_{\text{eff}}}{\partial \phi_S} = 0
\]

\[
\lim_{r \to \infty} \phi_S(r) = 0 \text{ and } d\phi_S(r)/dr|_{r=0} = 0.
\]

<table>
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<tr>
<th>$v_S(T_<em>)/T_</em>$</th>
<th>$\xi = 0$</th>
<th>$\xi = 1$</th>
<th>$\xi = 5$</th>
</tr>
</thead>
<tbody>
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<td>2.27</td>
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</tr>
</tbody>
</table>
Impact of $\xi$ on gravitational wave
Impact of $\xi$ on gravitational wave

$$\Omega_{\text{GW}} h^2$$

$\xi = 0$

$\xi = 1$

$\xi = 5$

no resum
Impact of $\xi$ on gravitational wave

$$\xi = 0$$

$\Omega_{GWh^2}$ vs $f$ [Hz]
Impact of $\xi$ on gravitational wave

\[ \Omega_{GWh^2} \]

\[ f \text{ [Hz]} \]

\[ \xi = 0 \]
\[ \xi = 1 \]
\[ \xi = 5 \]
Impact of $\xi$ on gravitational wave

$\sim 1$ order change!!
Impact of $\xi$ on gravitational wave

~ 1 order change!!

$\xi = 0$

$\xi = 5$

$\xi$ dependence of $V_{\text{eff}}$ propagates to GW spectrum significantly!
Summary

- We have evaluated the gauge fixing parameter \( \xi \) dependence on GW from the 1st-order phase transitions.

- Effective potential is \( \xi \) dependent.

- Such \( \xi \) dependence can propagate to nucleation temperature and eventually gravitational waves.
  \( \Omega_{GW} \) can change \( O(1) \) in magnitude varying \( \xi = 0 - 5 \).

- Gauge-inv. method with consistent thermal resummation is necessary to get reliable results.
Backup
1-loop effective potential $T \neq 0$

Using a high-$T$ expansion, one gets

$$V_1(\varphi, \xi) + V_1(\varphi, \xi; T) = \frac{T^2}{24}(\mathring{m}_h^2 + \mathring{m}_G^2 + 3\mathring{m}_A^2) - \frac{T}{12\pi}\left[ (\mathring{m}_h^2)^{3/2} + (\mathring{m}_G^2 + \xi\mathring{m}_A^2)^{3/2} + (3 - \xi^{3/2})(\mathring{m}_A^2)^{3/2} \right]$$

$$+ \frac{1}{64\pi^2} \left[ \mathring{m}_h^4 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + (\mathring{m}_G^2 + \xi\mathring{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + 3\mathring{m}_A^4 \left( \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + \frac{2}{3} \right) - (\xi\mathring{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} \right].$$

$$V_{\text{eff}}(\varphi, \xi; T) = V_0(\varphi) + V_1(\varphi, \xi) + V_1(\varphi, \xi; T)$$

$$= D(T^2 - T_0^2)\varphi^2 - ET(\varphi^2)^{3/2} + \frac{\lambda_T}{4}\varphi^4$$
1-loop effective potential $T \neq 0$

Using a high-$T$ expansion, one gets

$$V_1(\varphi, \xi) + V_1(\varphi, \xi; T) = \frac{T^2}{24} (\tilde{m}_h^2 + \tilde{m}_G^2 + 3\tilde{m}_A^2) - \frac{T}{12\pi} \left[ (\tilde{m}_h^2)^{3/2} + (\tilde{m}_G^2 + \xi \tilde{m}_A^2)^{3/2} + \frac{1}{2} (\xi^2)^{3/2} (\tilde{m}_A^2)^{3/2} \right]$$

$$+ \frac{1}{64\pi^2} \left[ \tilde{m}_h^4 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + (\tilde{m}_G^2 + \xi \tilde{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + 3\tilde{m}_A^4 \left( \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + \frac{2}{3} \right) - (\xi \tilde{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} \right].$$

“$T^2$-terms” are gauge-independent.

$$V_{\text{eff}}(\varphi, \xi; T) = V_0(\varphi) + V_1(\varphi, \xi) + V_1(\varphi, \xi; T)$$

$$= D(T^2 - T_0^2) \varphi^2 - ET(\varphi^2)^{3/2} + \frac{\lambda_T}{4} \varphi^4$$
1-loop effective potential $T \neq 0$

Using a high-$T$ expansion, one gets

$$V_1(\varphi, \xi) + V_1(\varphi, \xi; T) = \frac{T^2}{24}(\bar{m}^2_n + \bar{m}^2_G + 3\bar{m}^2_A) - \frac{T}{12\pi} \left[ (\bar{m}^2_n)^{3/2} + (\bar{m}^2_G + \xi \bar{m}^2_A)^{3/2} + (3 - \xi^{3/2})(\bar{m}^2_A)^{3/2} \right]$$

$$+ \frac{1}{64\pi^2} \left[ \bar{m}^4_n \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + (\bar{m}^2_G + \xi \bar{m}^2_A)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + 3\bar{m}^4_A \left( \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + \frac{2}{3} \right) - (\xi \bar{m}^2_A)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} \right].$$

“$T^2$-terms” are gauge-independent.

$\xi$ terms disappear if $\bar{m}^2_G = 0$

$$V_{\text{eff}}(\varphi, \xi; T) = V_0(\varphi) + V_1(\varphi, \xi) + V_1(\varphi, \xi; T)$$

$$= D(T^2 - T_0^2)\varphi^2 - ET(\varphi^2)^{3/2} + \frac{\lambda_T}{4} \varphi^4$$
1-loop effective potential $T \neq 0$

Using a high-$T$ expansion, one gets

\[
V_1(\varphi, \xi) + V_1(\varphi, \xi; T) = \frac{T^2}{24} (\bar{m}_h^2 + \bar{m}_G^2 + 3\bar{m}_A^2) - \frac{T}{12\pi} \left[ (\bar{m}_h^2)^{3/2} + (\bar{m}_G^2 + \xi \bar{m}_A^2)^{3/2} + (3 - \xi^{3/2})(\bar{m}_A^2)^{3/2} \right] \\
+ \frac{1}{64\pi^2} \left[ \bar{m}_h^4 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + (\bar{m}_G^2 + \xi \bar{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + 3\bar{m}_A^4 \left( \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + \frac{2}{3} \right) - (\xi \bar{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} \right].
\]

“$T^2$-terms” are gauge-independent.

$\xi$ terms disappear if $\bar{m}_G^2 = 0$

\[
V_{\text{eff}}(\varphi, \xi; T) = V_0(\varphi) + V_1(\varphi, \xi) + V_1(\varphi, \xi; T) \\
= D(T^2 - T_0^2)\varphi^2 - ET(\varphi^2)^{3/2} + \frac{\lambda_T}{4} \varphi^4
\]
1-loop effective potential $T \neq 0$

Using a high-$T$ expansion, one gets

$$V_1(\varphi, \xi) + V_1(\varphi, \xi; T)$$

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$$+ \frac{1}{64\pi^2} \left[ \bar{m}_h^4 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + (\bar{m}_G^2 + \xi \bar{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + 3\bar{m}_A^4 \left( \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + \frac{2}{3} \right) - (\xi \bar{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} \right].$$

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$$+ \frac{1}{64\pi^2} \left[ \bar{m}_h^4 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + (\bar{m}_G^2 + \xi\bar{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + 3\bar{m}_A^4 \left( \ln \frac{\alpha_B T^2}{\bar{\mu}^2} + \frac{2}{3} \right) - (\xi\bar{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\bar{\mu}^2} \right].$$

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$$V_{\text{eff}}(\varphi, \xi; T) = V_0(\varphi) + V_1(\varphi, \xi) + V_1(\varphi, \xi; T)$$

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1-loop effective potential $T \neq 0$

Using a high-$T$ expansion, one gets

$$V_1(\varphi, \xi) + V_1(\varphi, \xi; T) = \frac{T^2}{24} (\bar{m}_h^2 + \bar{m}_G^2 + 3\bar{m}_A^2) - \frac{T}{12\pi} \left[ (\bar{m}_h^2)^{3/2} + (\bar{m}_G^2 + \xi \bar{m}_A^2)^{3/2} + (3 - \xi^{3/2})(\bar{m}_A^2)^{3/2} \right]$$
$$\quad + \frac{1}{64\pi^2} \left[ \bar{m}_h^4 \ln \frac{\alpha_B T^2}{\mu^2} + (\bar{m}_G^2 + \xi \bar{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\mu^2} + 3\bar{m}_A^4 \left( \ln \frac{\alpha_B T^2}{\mu^2} + \frac{2}{3} \right) - (\xi \bar{m}_A^2)^2 \ln \frac{\alpha_B T^2}{\mu^2} \right].$$

"$T^2$-terms" are gauge-independent.

$\xi$ terms disappear if $\bar{m}_G^2 = 0$

$$V_{\text{eff}}(\varphi, \xi; T) = V_0(\varphi) + V_1(\varphi, \xi) + V_1(\varphi, \xi; T) = D(T^2 - T_0^2) \varphi^2 - E T (\varphi^2)^{3/2} + \frac{\lambda_T}{4} \varphi^4$$

$$\frac{v_C}{T_C} = \frac{2E}{\lambda_T} \quad \text{gauche dependent!!}$$
**Thermal resummation**

[Many refs: see, e.g., Parwani (92), Buchmüller et al (93), Chiku, Hatsuda (98), etc.]

Perturbative expansion gets worse at high $T$.

\[
\begin{align*}
\mathcal{L}_B &= \mathcal{L}_R + \mathcal{L}_{CT} \\
&\rightarrow \left[ \mathcal{L}_R + \Delta m_S^2 |S|^2 + \frac{1}{2} \Delta m_L^2 Z'^\mu L_{\mu\nu}(i\partial) Z'^\nu + \frac{1}{2} \Delta m_T^2 Z'^\mu T_{\mu\nu}(i\partial) Z'^\nu \right] \\
&+ \left[ \mathcal{L}_{CT} - \Delta m_S^2 |S|^2 - \frac{1}{2} \Delta m_L^2 Z'^\mu L_{\mu\nu}(i\partial) Z'^\nu - \frac{1}{2} \Delta m_T^2 Z'^\mu T_{\mu\nu}(i\partial) Z'^\nu \right]
\end{align*}
\]

where

\[
T_{00} = T_{0i} = T_{i0} = 0 , \quad T_{ij} = g_{ij} - \frac{k_i k_j}{-k^2} ,
\]

\[
L_{\mu\nu} = P_{\mu\nu} - T_{\mu\nu} , \quad P_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} ,
\]

[N.B.] Resummed Lagrangian preserves the gauge invariance.
Thermal resummation

[Many refs: see, e.g., Parwani (92), Buchmüller et al (93), Chiku, Hatsuda (98), etc.]

Perturbative expansion gets worse at high $T$.

\[
\begin{align*}
g^2 & \sim g^2 T^2 \\
\text{\#n sub-bubbles} & \sim \frac{g^4 T^3}{m} \left( \frac{g^2 T^2}{m^2} \right)^{n-1}
\end{align*}
\]

Dominant thermal terms are added and subtracted in the Lagrangian:

\[
L_B = L_R + L_{CT} \rightarrow \left[ L_R + \Delta m_S^2 |S|^2 + \frac{1}{2} \Delta m_L^2 Z''(i\partial) Z' + \frac{1}{2} \Delta m_T^2 Z''(i\partial) Z' + \frac{1}{2} \Delta m_\mu^2 Z''(i\partial) Z' \right]
\]

where

\[
T_{00} = T_{0i} = T_{i0} = 0 \; , \; \; \; T_{ij} = g_{ij} - \frac{k_i k_j}{-k^2} \; ,
\]

\[
L_{\mu\nu} = P_{\mu\nu} - T_{\mu\nu} \; , \; \; \; P_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \; ,
\]

[N.B.] Resummed Lagrangian preserves the gauge invariance.
**Thermal resummation**

[Many refs: see, e.g., Parwani (92), Buchmüller et al (93), Chiku, Hatsuda (98), etc.]

Perturbative expansion gets worse at high $T$. Dominant thermal terms are added and subtracted in the Lagrangian:

$$L_B = L_R + L_{CT} \rightarrow \left[ L_R + \Delta m_S^2 |S|^2 + \frac{1}{2} \Delta m_L^2 Z^{\prime \mu} L_{\mu \nu} (i \partial) Z^{\prime \nu} + \frac{1}{2} \Delta m_T^2 Z^{\prime \mu} T_{\mu \nu} (i \partial) Z^{\prime \nu} \right]$$

$$+ \left[ L_{CT} - \Delta m_S^2 |S|^2 - \frac{1}{2} \Delta m_L^2 Z^{\prime \mu} L_{\mu \nu} (i \partial) Z^{\prime \nu} - \frac{1}{2} \Delta m_T^2 Z^{\prime \mu} T_{\mu \nu} (i \partial) Z^{\prime \nu} \right]$$

where $T_{00} = T_{0i} = T_{i0} = 0$, $T_{ij} = g_{ij} - \frac{k_i k_j}{-k^2}$, $L_{\mu \nu} = P_{\mu \nu} - T_{\mu \nu}$, $P_{\mu \nu} = g_{\mu \nu} - \frac{k_{\mu} k_{\nu}}{k^2}$.

[N.B.] Resummed Lagrangian preserves the gauge invariance.
**Impact of $\xi$ on $v/T$**

e.g., scale-inv. $U(1)_{B-L}$ model  

$Q'_S = 2$, $\alpha' = g'^2/4\pi = 0.015$, $m_{Z'} = 4.5$ TeV and $m_{\nu_{R1,2,3}} = 1.0$ TeV.

\[
\frac{v_S(T_C)}{T_C} = 3.67, \quad \frac{4.851}{1.321} = 3.67, \quad \frac{4.833}{1.346} = 3.59, \quad \frac{4.816}{1.368} = 3.52, \quad \frac{4.695}{1.348} = 3.48
\]
Impact of $\xi$ on $v/T$

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$Q'_S = 2$, $\alpha' = g'^2/4\pi = 0.015$, $m_{Z'} = 4.5$ TeV and $m_{\nu_{R1,2,3}} = 1.0$ TeV.

$$Q'_S = 2, \alpha' = g'^2/4\pi = 0.015, m_{Z'} = 4.5 \text{ TeV and } m_{\nu_{R1,2,3}} = 1.0 \text{ TeV.}$$

### Table

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$\frac{v_S(T_C)}{T_C}$</th>
<th>$\frac{\xi = 0}{1.321} = 3.67$</th>
<th>$\frac{\xi = 1}{1.346} = 3.59$</th>
<th>$\frac{\xi = 5}{1.348} = 3.48$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no resum</td>
<td>$\frac{4.851}{1.321} = 3.67$</td>
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$Q'_S = 2$, $\alpha' = g'^2/4\pi = 0.015$, $m_{Z'} = 4.5$ TeV and $m_{\nu_{R1,2,3}} = 1.0$ TeV.

$\frac{v_S(T_C)}{T_C} = \begin{array}{c|c|c|c|c}
\text{no resum} & \xi = 0 & \xi = 1 & \xi = 5 \\
\hline
\frac{4.851}{1.321} = 3.67 & \frac{4.833}{1.346} = 3.59 & \frac{4.816}{1.368} = 3.52 & \frac{4.695}{1.348} = 3.48
\end{array}$
Onset of PT

- $T_c$ is not onset of the PT.
- Nucleation starts somewhat below $T_c$.

“Not all bubbles can grow”

expand? or shrink?

volume energy vs. surface energy

$\propto (\text{radius})^3$  \,  $\propto (\text{radius})^2$

There is a critical value of radius $\rightarrow$ critical bubble
Nucleation temperature

- Nucleation rate per unit time per unit volume

\[ \Gamma_N(T) \simeq T^4 \left( \frac{S_3(T)}{2\pi T} \right)^{3/2} e^{-S_3(T)/T} \]

[A.D. Linde, NPB216 ('82) 421]

- \( S_3(T) \): energy of the critical bubble at \( T \)

- Definition of nucleation temperature \( (T_N) \)

horizon scale \( \simeq H(T)^{-1} \)

\[ \Gamma_N(T_N)H(T_N)^{-3} = H(T_N) \]

\[ \frac{S_3(T_N)}{T_N} - \frac{3}{2} \ln \left( \frac{S_3(T_N)}{T_N} \right) = 152.59 - 2 \ln g_*(T_N) - 4 \ln \left( \frac{T_N}{100 \text{ GeV}} \right) \]

Roughly, \( S_3(T)/T \approx 150 \) is needed for the PT.