PBHs from cosmic domain walls

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Amplification of scalar perturbation comes from

1. ultra slow-roll inflation  
2. resonance due to oscillatory sound speed  
3. early dust stage

Regions where the average energy density in a Hubble horizon exceeds a threshold $\delta_c$ will collapse into primordial black hole (PBH)

Energy stored in topological defects shrink into PBHs
Content

1. Introduction to topological defects: cosmic domain walls
2. Quantum creation of topological defects during inflation
3. Dynamics of domain walls after inflation
4. Inflation model and the mass function of PBHs
5. Conclusion and discussion
Topological defects: domain wall

Domain walls are sheet-like objects which are formed in the early universe when a discrete symmetry is spontaneously broken.

\[ V(\phi) = \frac{m^2 v^2}{N^2} \left[ 1 - \cos \left( \frac{N \phi}{v} \right) \right] \]

2-dimensional domain walls

\[ V(\phi) = \frac{\lambda}{4} (\phi^2 - v^2)^2 \]

3-dimensional domain walls
Topological defects: domain wall

The initial value of scalar field in on the potential barrier, then fall in different vacuum.

On each side of the wall the scalar field settles in different vacuum. On the wall the field should clime up the barrier and energy is stored in the walls.
Static domain wall

1. Lagrangian and effective potential

\[ \mathcal{L} = -\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \]

\[ V(\phi) = \frac{\lambda}{4} (\phi^2 - v^2)^2 \]

2. Static solution perpendicular to the z-axis

\[ \phi(z) = v \tanh\left[ \sqrt{\frac{\lambda}{2v}} z \right] \]

3. Typical length scale of the spatial variation

\[ \delta \approx \left( \frac{\lambda}{2v} \right)^{-1} \]

4. The surface energy density (also the tension)

\[ \sigma = \frac{4}{3} \sqrt{\frac{\lambda}{2v^3}} \]
Quantum creation of topological defects during inflation


1. De Sitter space can be described as

\[-\tau^2 + \omega^2 + \sum_{i=1}^{n-2} \zeta_i^2 = H^{-2}\]

2. The nucleation can be described using the instanton language. The domain wall described above is a three-sphere of maximal radius and the tunneling probability is proportional to \(\exp(-S_E)\).

\[\zeta_1^2 + \zeta_2^2 + \zeta_3^2 + \tau^2 = H^{-2}, \quad \omega = 0\]

\[dN = \lambda e^{3Ht_0} d^3x dt_0 \quad \lambda \approx H^4 e^{-S_E} \quad S_E = 2\pi^2 \sigma H^{-3}\]
The domain wall is subcritical or supercritical depends on whether the energy of the domain wall is dominant in the local universe.

2. If the domain wall is subcritical, its gravitational field can be safely neglected before horizon crossing.

The initial mass can be expressed by

\[ M_{BH} = 4\pi\sigma C R_H^2 \]

\[ C_{dust} \approx 0.15 \text{ and } C_{radiation} \approx 0.62 \]

Where \( R_H \) is the radius of the domain wall when the Hubble horizon exceeds the domain wall.
3. In radiation dominant era, due to accretion the final mass is 2 times the initial mass according to numerical simulation. In matter dominant era, accretion effect is negligible.

\[ M_{f, RD} = 2M_{i, RD} \]
\[ M_{f, MD} = M_{i, MD} \]

Figure 4: $M_{BH}$ as a function of time for six subcritical walls in the background of radiation with the same surface tension ($\ell_s \approx 300$) but different radii. $r_i = 5, 6, 7, 8, 9$ and $10$ from the bottom. Blue curves are from simulations, and dashed red curves are from Eq. (39). For $r_i = 5$, $\frac{M_{BH}}{M_{BH,i}} \approx 1.5$; for $r_i = 6$, $\frac{M_{BH}}{M_{BH,i}} \approx 1.6$; for $r_i = 7$, $\frac{M_{BH}}{M_{BH,i}} \approx 1.8$; for $r_i = 8$, $\frac{M_{BH}}{M_{BH,i}} \approx 1.9$; for $r_i = 9$, $\frac{M_{BH}}{M_{BH,i}} \approx 2.0$; for $r_i = 10$, $\frac{M_{BH}}{M_{BH,i}} \approx 2.0$. The ratio increases to $\sim 2$ as we approach the critical regime.
Dynamics of domain walls after inflation

1. As for the supercritical case, the initial black hole mass has an upper bound which is the total mass enclosed by the horizon crossing radius of the unperturbed FRW region.

\[
M_{i, RD} = \frac{1}{2} \times 5.6 (RH_i)^2 \quad M_{i, MD} = \frac{1}{2} \times (RH_i)^3
\]

Where R is the radius of the domain wall at the end of inflation

2. The subsequent process is similar as the subcritical case.

\[
M_{f, RD} = 5.6 (RH_i)^2 \quad M_{f, MD} = \frac{1}{2} \times (RH_i)^3
\]
The model of inflation

1. The potential of inflation can be written as

\[ V(\phi, \chi) = \frac{\lambda_\phi}{p} \phi^p + \frac{\lambda_\chi}{4} (\chi^2 - m^2 - \alpha^2(\phi - \phi_0)^2)^2 \]

2. The initial value of \( \chi \) is in one of the minima. The mass of \( \chi \) is large at the beginning or end of inflation.

3. The tension of domain wall is a time dependent variable. At the time \( \alpha^2(\phi - \phi_0)^2 \) vanishes, tunneling possibility reaches the maximum.

4. The comoving radius of domain walls and thus the PBH mass is sensitive to \( \phi_0 \). By tuning \( \phi_0 \) we can get a large mass range.
PBH mass function

For subcritical case, given the comoving radius of the domain walls, we need to know the time it enters the horizon and evaluate the mass. The number in an interval $dt_0$ is changed to that in an interval $dM$

$$\frac{1}{a(t_0)H(t_0)} = \frac{1}{a(t_H)H(t_H)} = \frac{2t}{(t/t_e)^{1/2}} = \sqrt{2R_Ht_e}$$

$$\frac{dH}{dt}(t_0)a(t_0) \ll \frac{da}{dt}(t_0)H(t_0) \quad \left| \frac{dt_0}{dR_H} \right| = t_eH(t_0)a^2(t_0) \quad M \approx 4\pi\sigma R^2_H$$

$$\rho_{cri,0} \Omega_{DM} f(M) = \frac{d\rho}{d\ln M} = \frac{M^2dN}{dV_{phy}(t_*)dM} = \frac{H^4(t_0)e^{-S(t_0)}}{8\pi R_H\sigma} \left| \frac{dt_0}{dR_H} \right|$$

$$\rho_{cri,0} \Omega_{DM} f(M) = H^5(t_0)e^{-2\pi^2\sigma(t_0)H^{-3}(t_0)} \pi \left( \frac{M}{4\pi\sigma(t_0)} \right)^{3/2} \sigma \sqrt{\frac{24\pi G\rho}{\lambda_\phi}} a^2(t_0) \left( \frac{a(t_0)}{a(t_*)} \right)^3$$

Where $a(t_*)$ is the scale factor at present.
PBH mass function

For supercritical case, the radius of domain walls at the end of inflation should be determined to relate $d t_0$ to $d M$. The horizon mass at the radiation matter equality is about $10^{50}$ g, much larger than the supermassive black hole so we only consider PBHs formed in radiation dominant era.

\[ M = 5.6 \left( R H_i \right)^2 \quad R = \frac{1}{H_0} \frac{a(t_e)}{a(t_0)} \]

\[ \frac{dH}{dt}(t_0)a(t_0) \ll \frac{da}{dt}(t_0)H(t_0) \quad \left| \frac{dt_0}{dR_H} \right| = \frac{a(t_0)}{a(t_e)} \frac{\sqrt{G}}{11.2 R H(t_e)} \]

\[ \rho_{\text{crit},0} \Omega_{DM} f(M) = \frac{M^{5/2} e^{-S_E(t_0)} a^4(t_0) H(t_e)}{2G a^3(t_*)} \]
Primordial black holes as dark matter and current constrains

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<th>$\lambda_\chi$</th>
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Conclusion and discussion

1. Domain walls from quantum creation during inflation can collapse into PBHs with a large mass range avoiding cosmic domain wall problem.

2. In some multi-field inflation models the PBHs from cosmic domain walls can be the candidates of a part or all of the dark matter. The seed of super massive black hole can be provided through this mechanism.

3. The PBH mass is not directly relevant to the Hubble mass and the sensitivity of $\delta_c$ in gravitational collapse models can be avoided.

4. PBHs from domain walls do not require large scalar perturbation, the induced GWs can not give constraint to the mass function.

5. Tuning possibility exponentially depends the action and fine tuning of this model is still large.
Thanks for your attention