The large scale structure of the universe

Part 1:

- **Deciphering the large scale structure (LSS)**
  - **With statistics and physics**

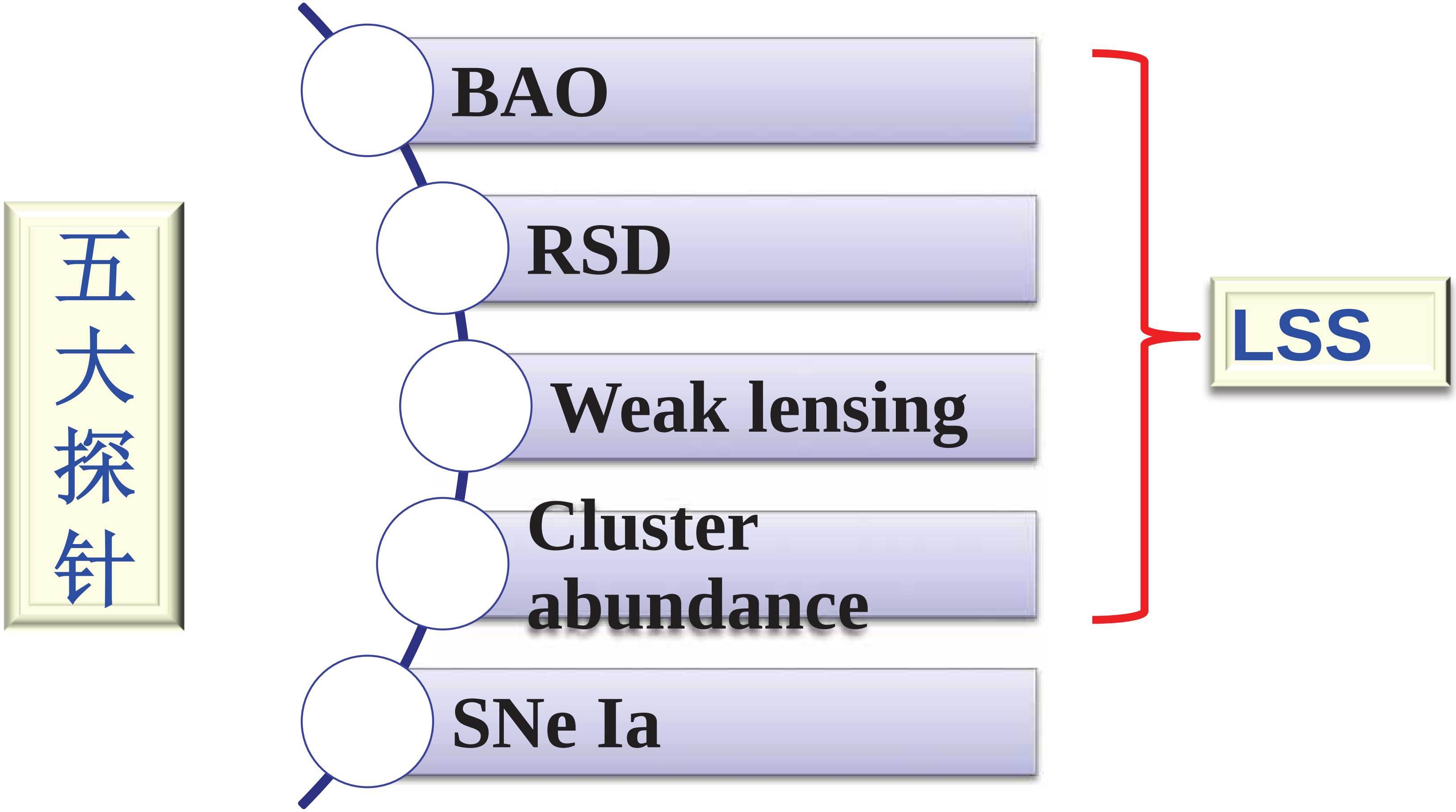
Part 2:

- **Tracers of LSS**
  - **Redshift distortion, weak lensing, broadband power spectrum, BAO, SZ effect, ISW, etc.**

Part 3

- **Synergies of LSS tracers**
  - **Probe DM, DE, MG, neutrino, etc.**
  - **Reduce statistical errors**
  - **Control systematic errors**

Major probes of dark energy and cosmic acceleration



五大探针

BAO

RSD

Weak lensing

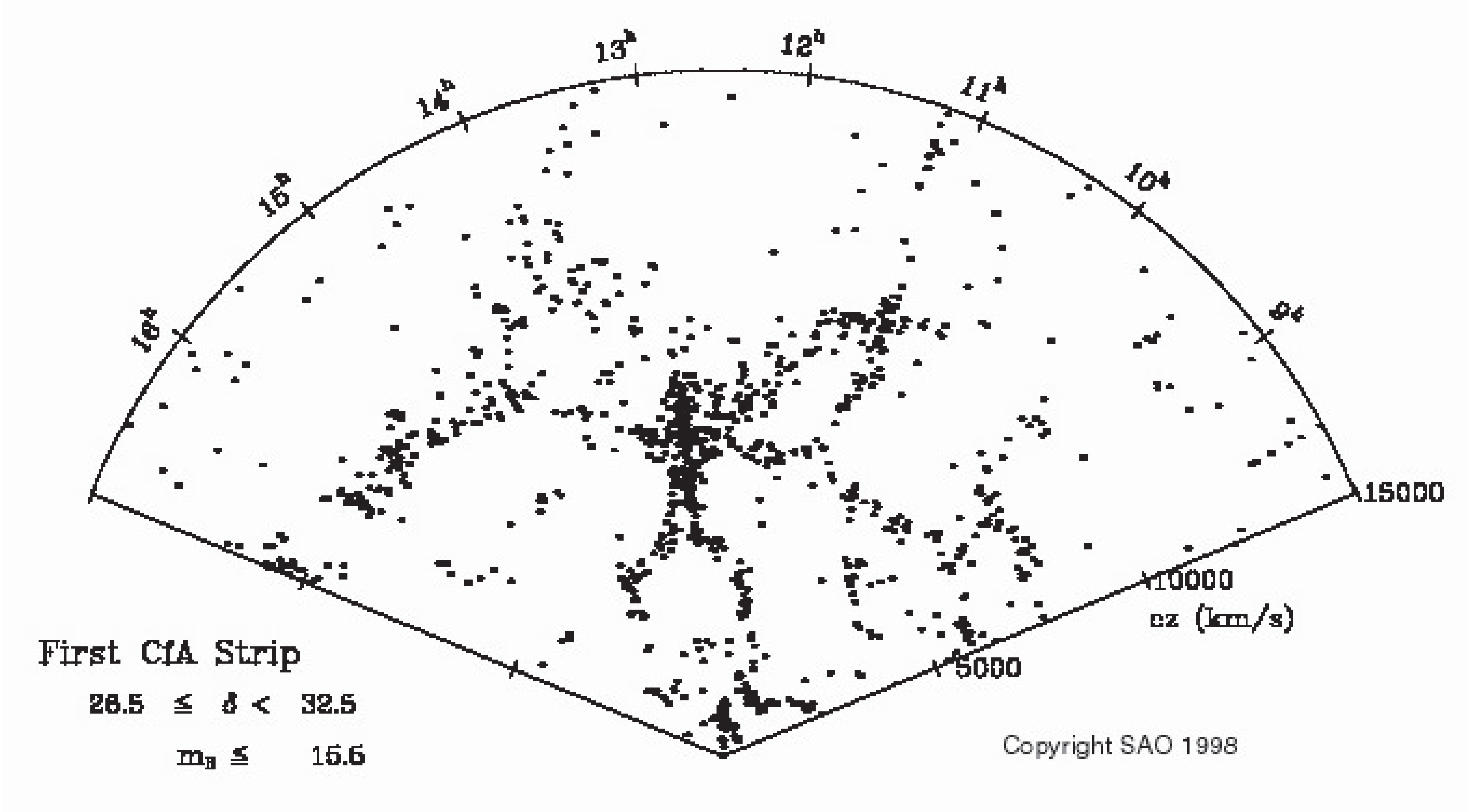
Cluster abundance

SNe Ia

LSS

- 暗能量特别工作组报告（2006）−DOE、NASA、NSF
- DOE/HEP 暗能量科学项目报告(2012)

Redshift Space Distortion Cosmology:
Promises and Challenges



Copyright SAO 1998

Noticed at least as early as 1972

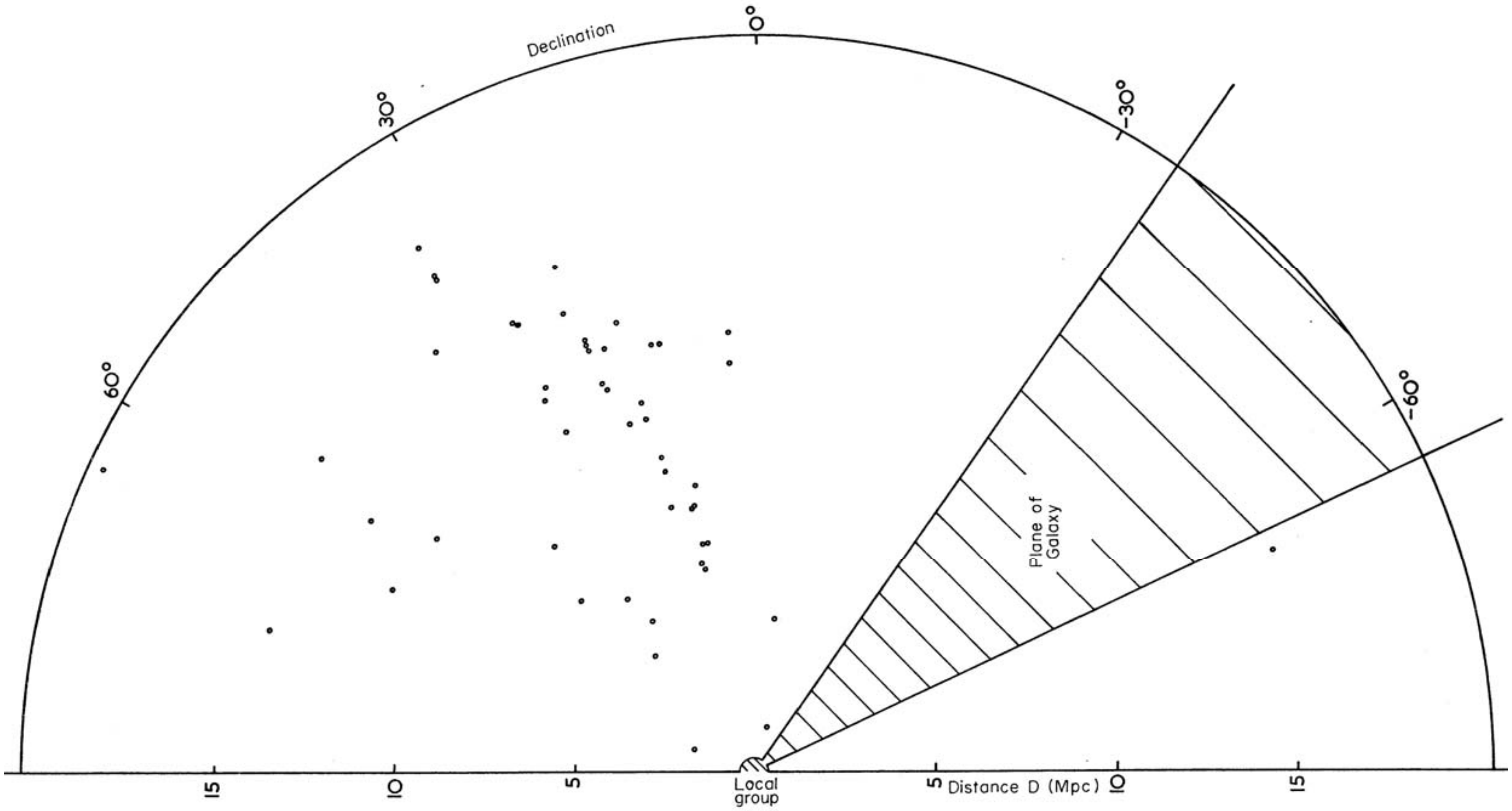


Fig. 1. *The apparent space distribution of nearby galaxies with* 10h ⩽ *RA* < 11*h, assuming radial velocity to be a good distance indicator.*

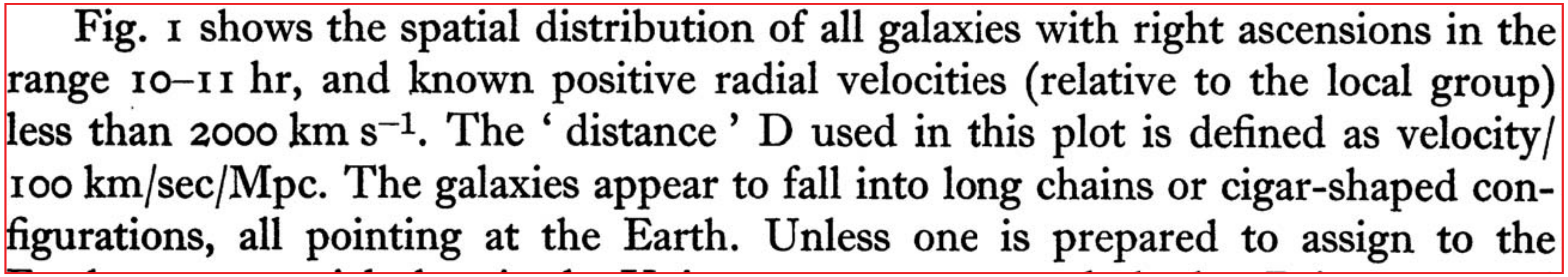
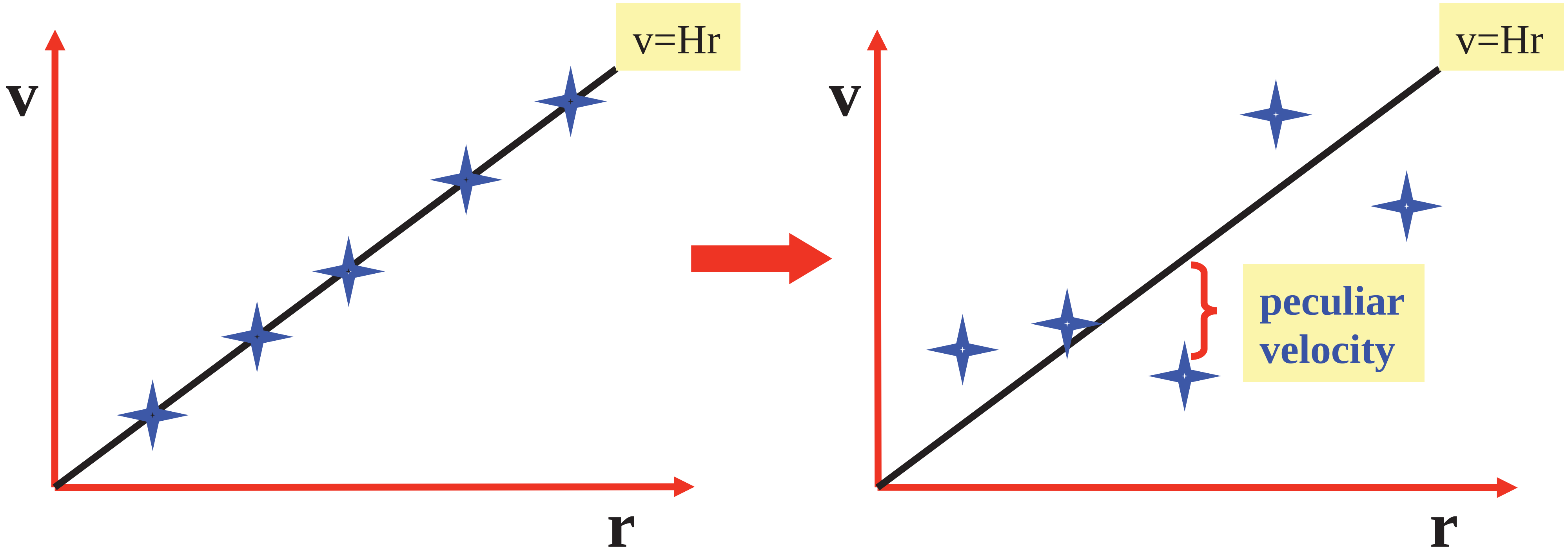


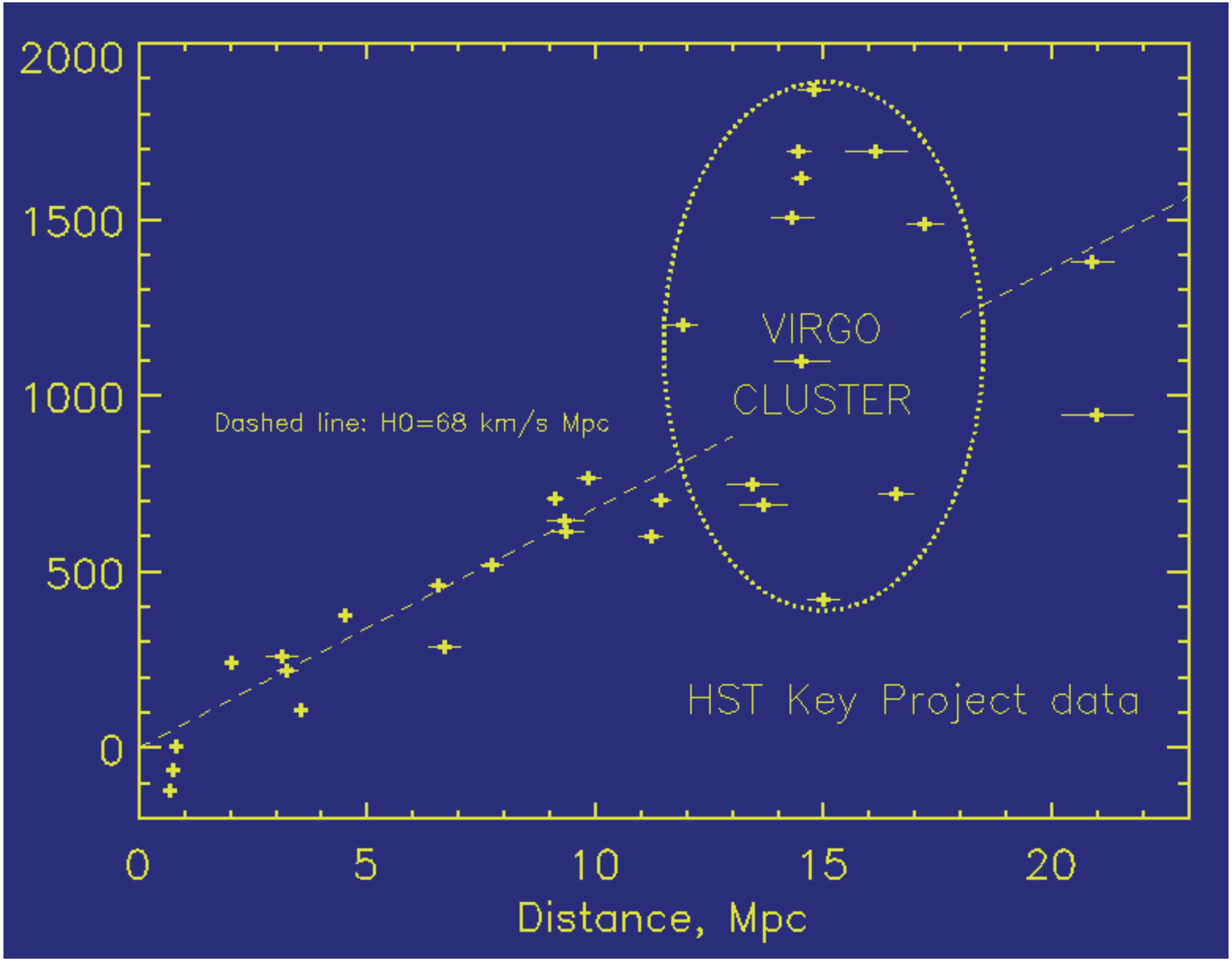
Fig. 1 shows the spatial distribution of all galaxies with right ascensions in the range 10–11 hr, and known positive radial velocities (relative to the local group) less than 2000 km s^{-1}. The 'distance' D used in this plot is defined as velocity/100 km/sec/Mpc. The galaxies appear to fall into long chains or cigar-shaped configurations, all pointing at the Earth. Unless one is prepared to assign to the

Jackson, 1972, MNRAS . **Finger of God effect**

Peculiar velocity: a window to the dark universe

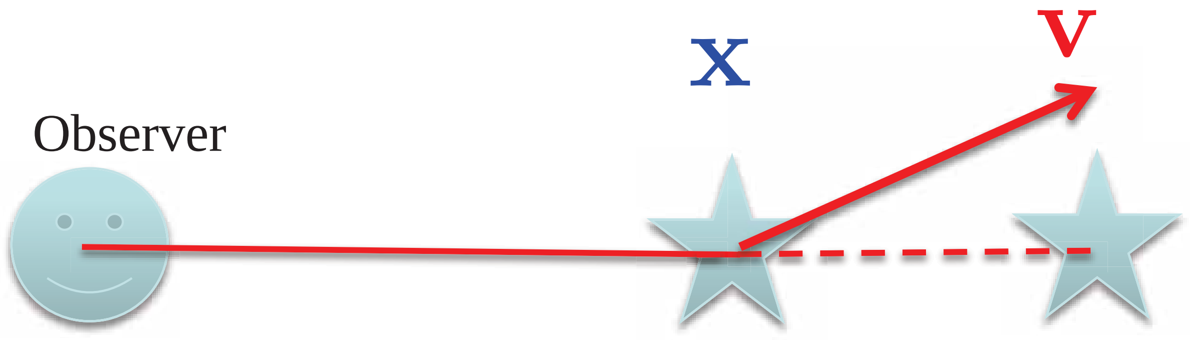
- Matter distribution in our universe is inhomogeneous
- Gravitational attraction arising from inhomogeneity perturbs galaxies and causes deviation from the Hubble flow





http://www.astr.ua.edu/keel/galaxies/distance.html

Redshift space distortion

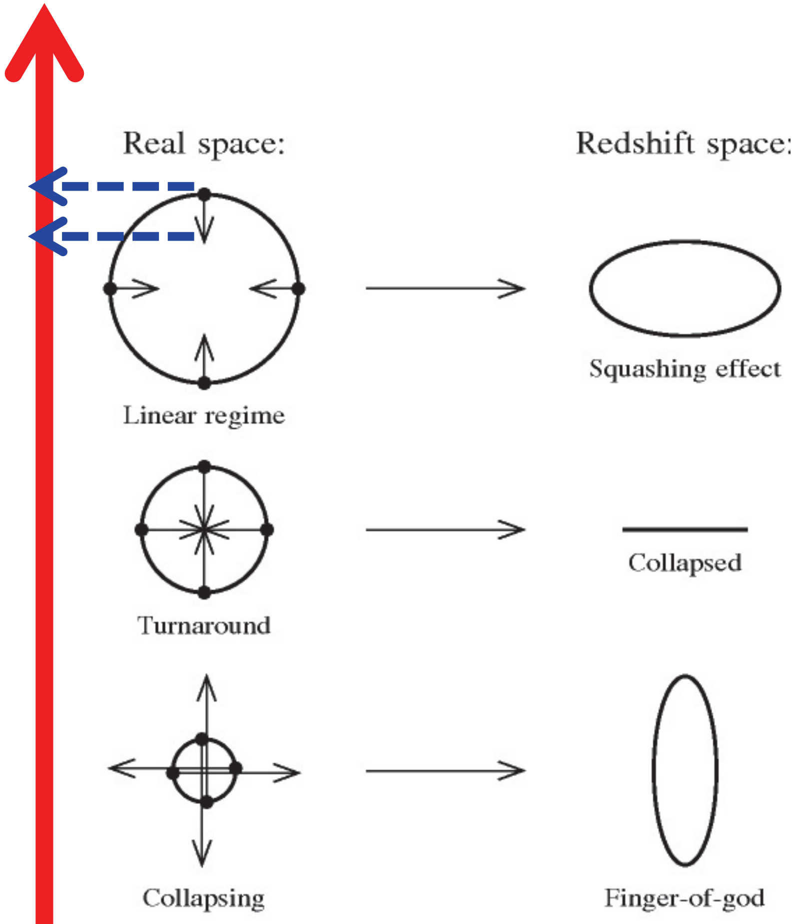


$$z^{\text{obs}} = z + \frac{\vec{v}}{c} \cdot \hat{x}$$

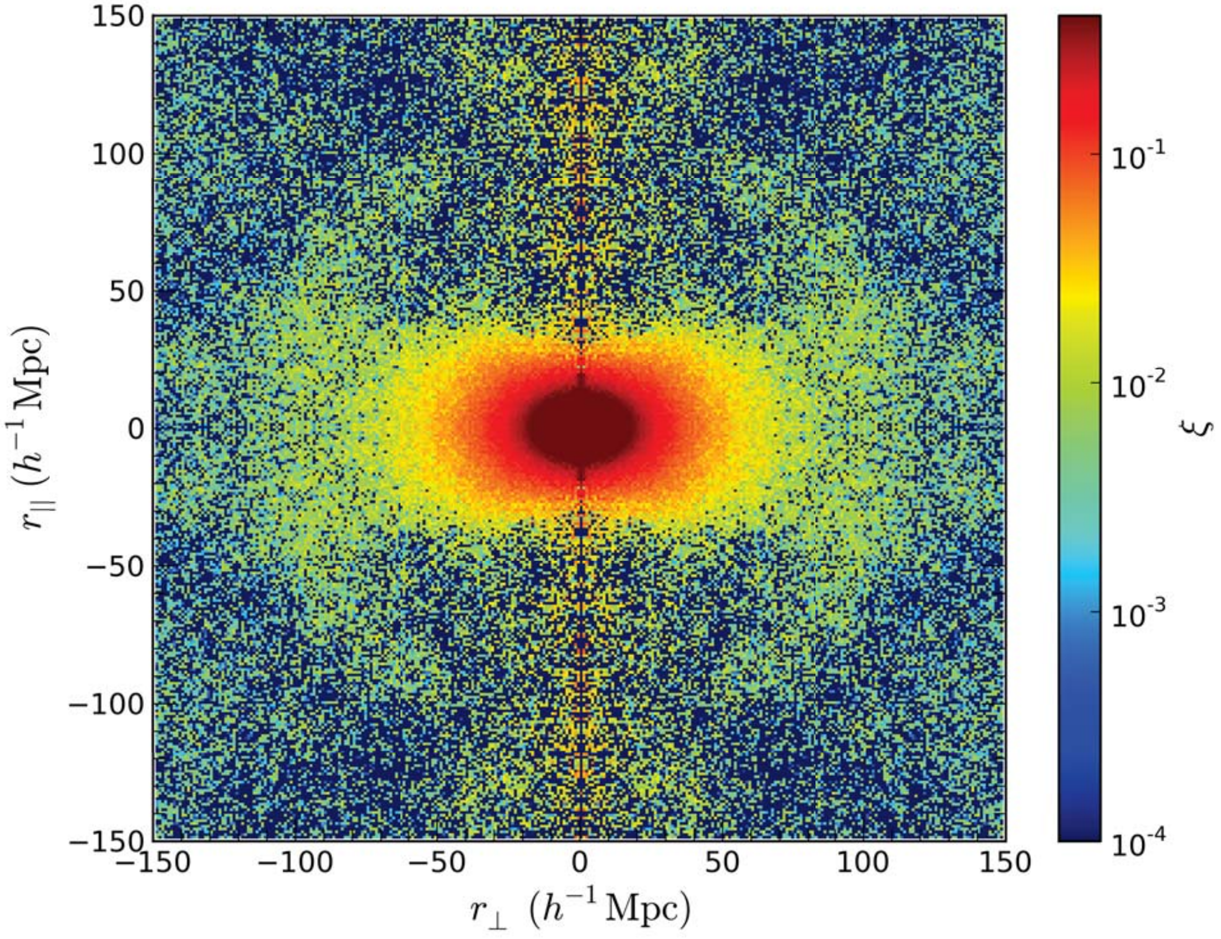
$$\vec{s} = \vec{x} + \frac{\vec{v} \cdot \hat{x}}{H(z)} \hat{x}$$

- **We observe galaxies in redshift space (namely infer their distances from their redshifts).**
- **Peculiar velocity changes the galaxy redshift (cosmological z+ Doppler z) and hence distorts the galaxy distribution in an anisotropic way**
  - **Galaxy clustering along the line of sight is different to that perpendicular to the line of sight**

Statistical isotropy to statistical anisotropy



A. J. S. Hamilton, atrophy/9708102



Observe along this direction

BOSS: Samushia et al. 2014.

Measure peculiar velocity at cosmological distance
heuristic approach

Peculiar velocity can be reconstructed!

Not only the average statistics, but also the 3D field

Real space
power spectrum

Redshift space
power spectrum

$$P(k) \xrightarrow{\mathbf{v}} P^s(\mathbf{k}_\perp, k_\parallel)$$

Also directly measurable!

Directly measurable

$$P(k) = P^s(k_\perp = k, k_\parallel = 0)$$

RSD and the structure growth

- RSD measures velocity, which is a specific combination of structure growth rate

Linearized mass conservation **Linearized evolution**

$$\frac{d\delta_m}{dt} + \nabla \cdot \vec{v} = 0 \qquad \delta_m(\vec{x}, t) = D(t)\delta_i(\vec{x})$$

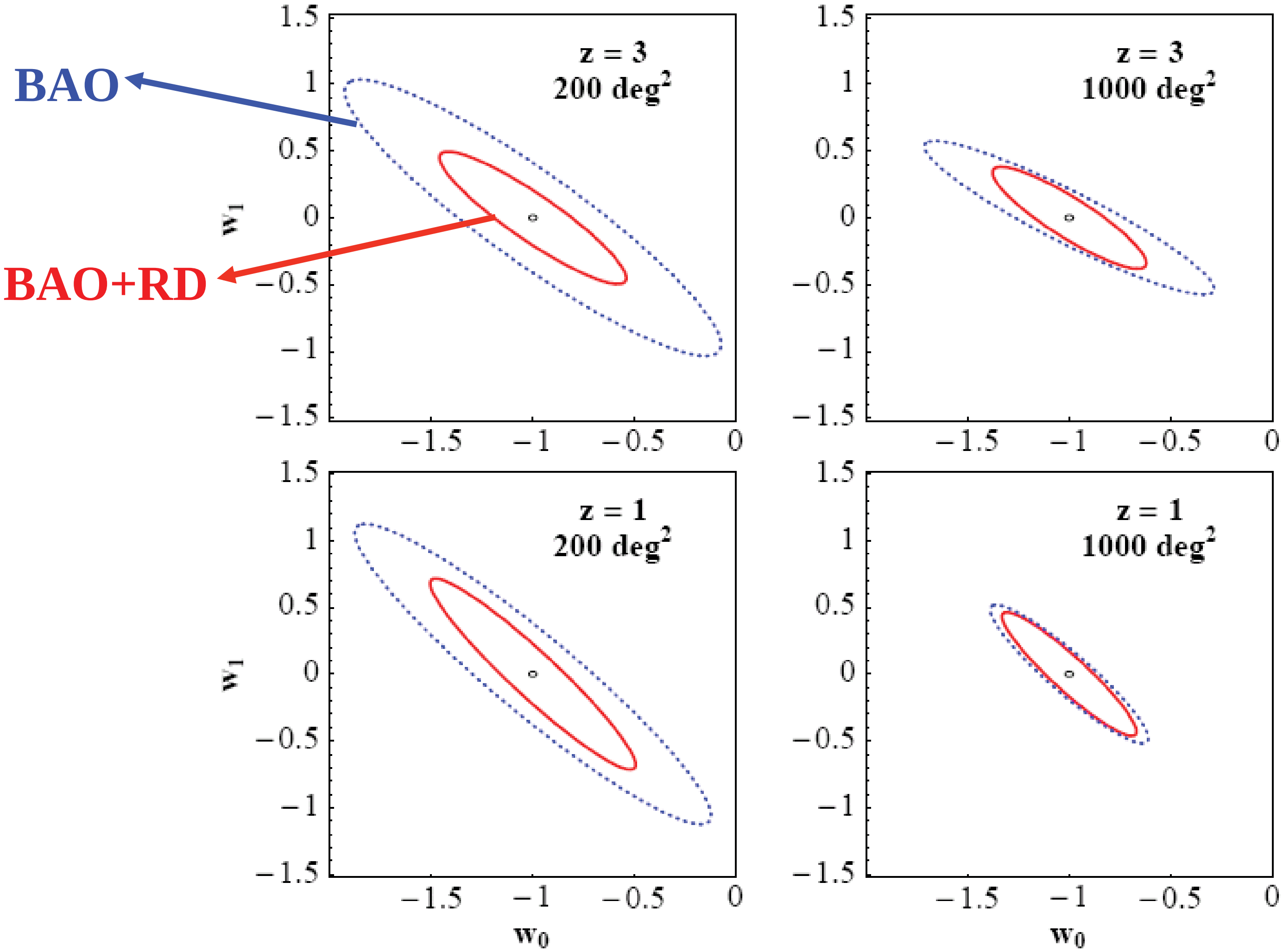
$$\boxed{\Rightarrow \vec{v} \propto fH\delta_m \propto fD \propto f\sigma_8}$$

$$f \equiv \frac{d \ln D(a)}{d \ln a} \propto \Omega_m^\gamma(a)$$

Applications (1): Constrain dark energy

Spectroscopic surveys

$$w \equiv \frac{P_{\rm DE}}{\rho_{\rm DE}} = w_0 + w_1(1-a)$$



✓RD helps to improve dark energy constraints

➢However, the improvement is not significant for future big surveys

➢Because if smooth dark energy, BAO and RD basically probes the same H(z)

Amendola, Quercellini &Giallongo 2004

More important application of peculiar velocity: constrain gravity

- ✓ **At scales larger than galaxy clusters, directly probes gravity (t-t component):** $ds^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)d\mathbf{x}^2$

$$\frac{d(a^2\vec{v})}{dt} = -\nabla\Psi$$

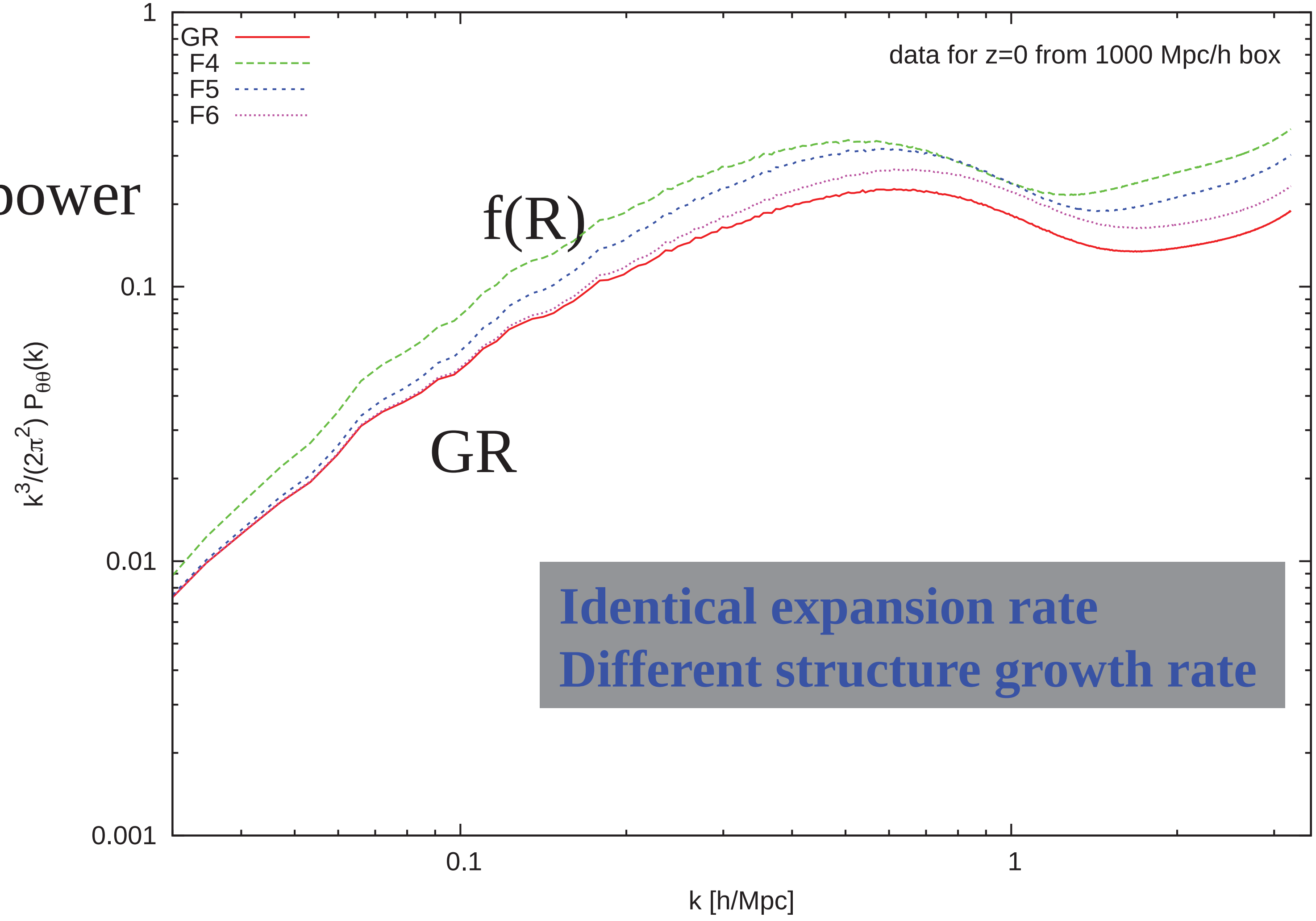
- ✓ **The most direct measure of the t-t metric perturbation (Jain & ZPJ, 2008)**

- ✓ **In combination with weak lensing, allows for direct measurement of a key parameter of gravity (ZPJ+, 2008)**

$$\eta \equiv -\frac{\Phi}{\Psi}$$

Applications (2): test GR

Velocity power spectrum



Identical expansion rate
Different structure growth rate

Jennings et al. 2012

Existing measurements of f*sigma_8: ~10% accuracy

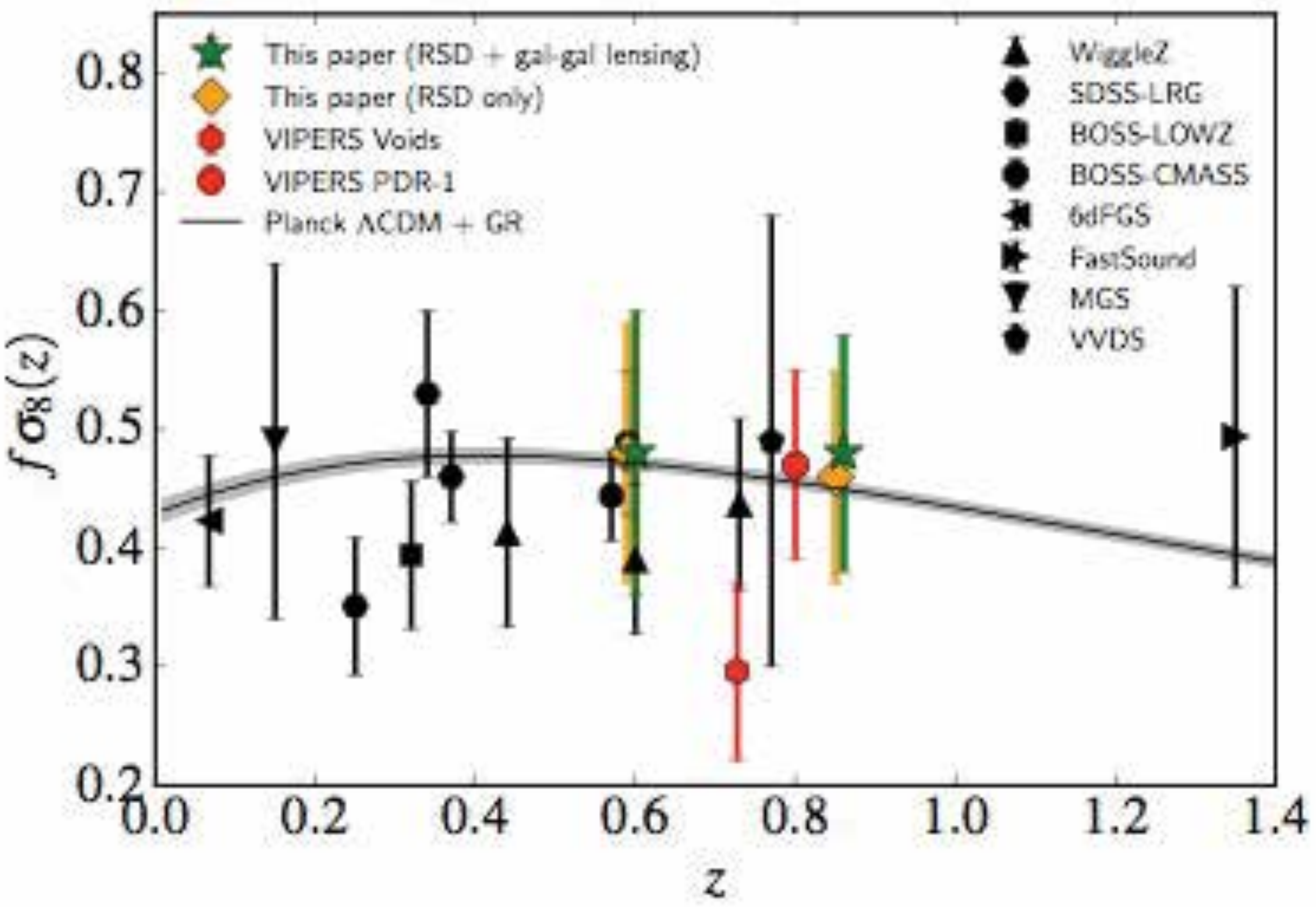


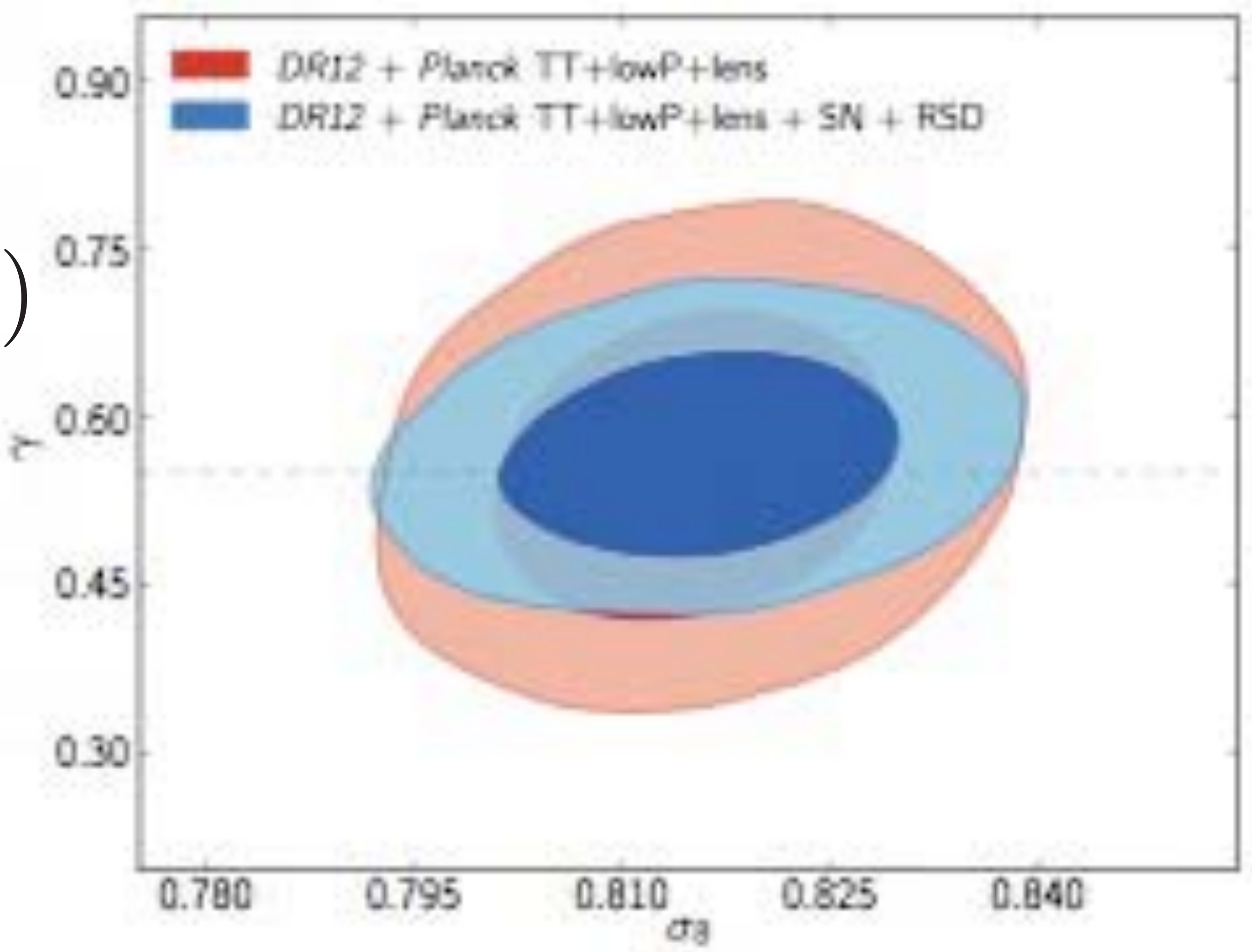
Fig. 12. $f\sigma_8$ as a function of redshift, showing VIPERS results contrasted with a compilation of recent measurements. The previous results from VVDS (Guzzo et al. 2008), SDSS LRG (Cabré & Gaztañaga 2009; Samushia et al. 2012), WiggleZ (Blake et al. 2012), 6dFGS (Beutler et al. 2012), VIPERS PDR-1 (de la Torre et al. 2013), MGS (Howlett et al. 2015), FastSound (Okumura et al. 2016), BOSS-LOWZ (Gil-Marín et al. 2016), BOSS-CMASS (Gil-Marín et al. 2016; Chuang et al. 2016), and VIPERS PDR-2 voids (Hawken et al. 2016) are shown with the different symbols (see labels). The solid curve and associated shaded area correspond to the expectations and 68% uncertainty for General Relativity in a ΛCDM background model set to TT+lowP+lensing Planck 2015 predictions (Planck Collaboration et al. 2015).

VIPERS, arXiv: 1612.05647

The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: constraining modified gravity

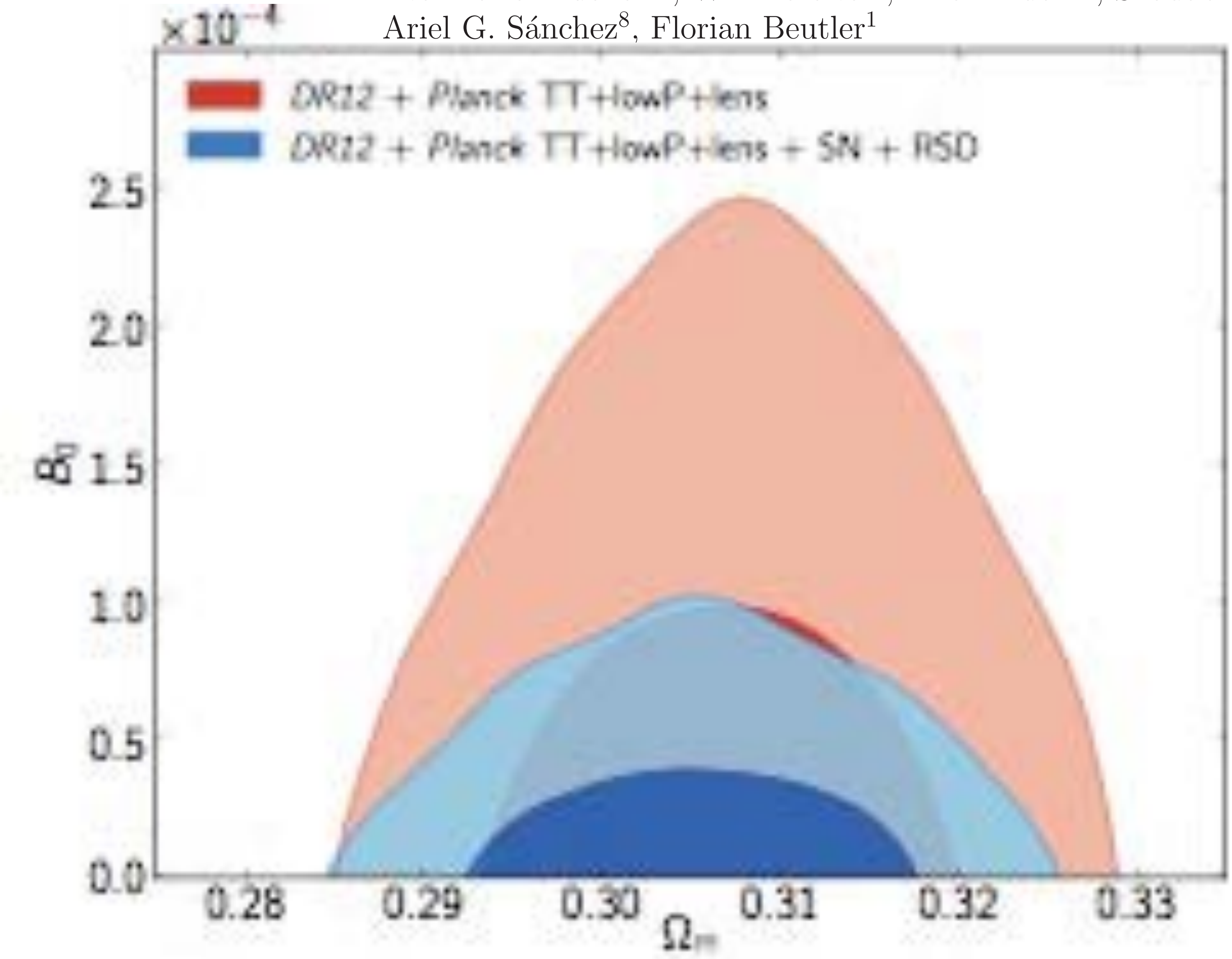
Eva-Maria Mueller[1⋆], Will Percival[1], Eric Linder[2,3], Shadab Alam[4,5,6], Gong-Bo Zhao[1,7],
Ariel G. Sánchez[8], Florian Beutler[1]

$$f \simeq \Omega_m^\gamma(z)$$

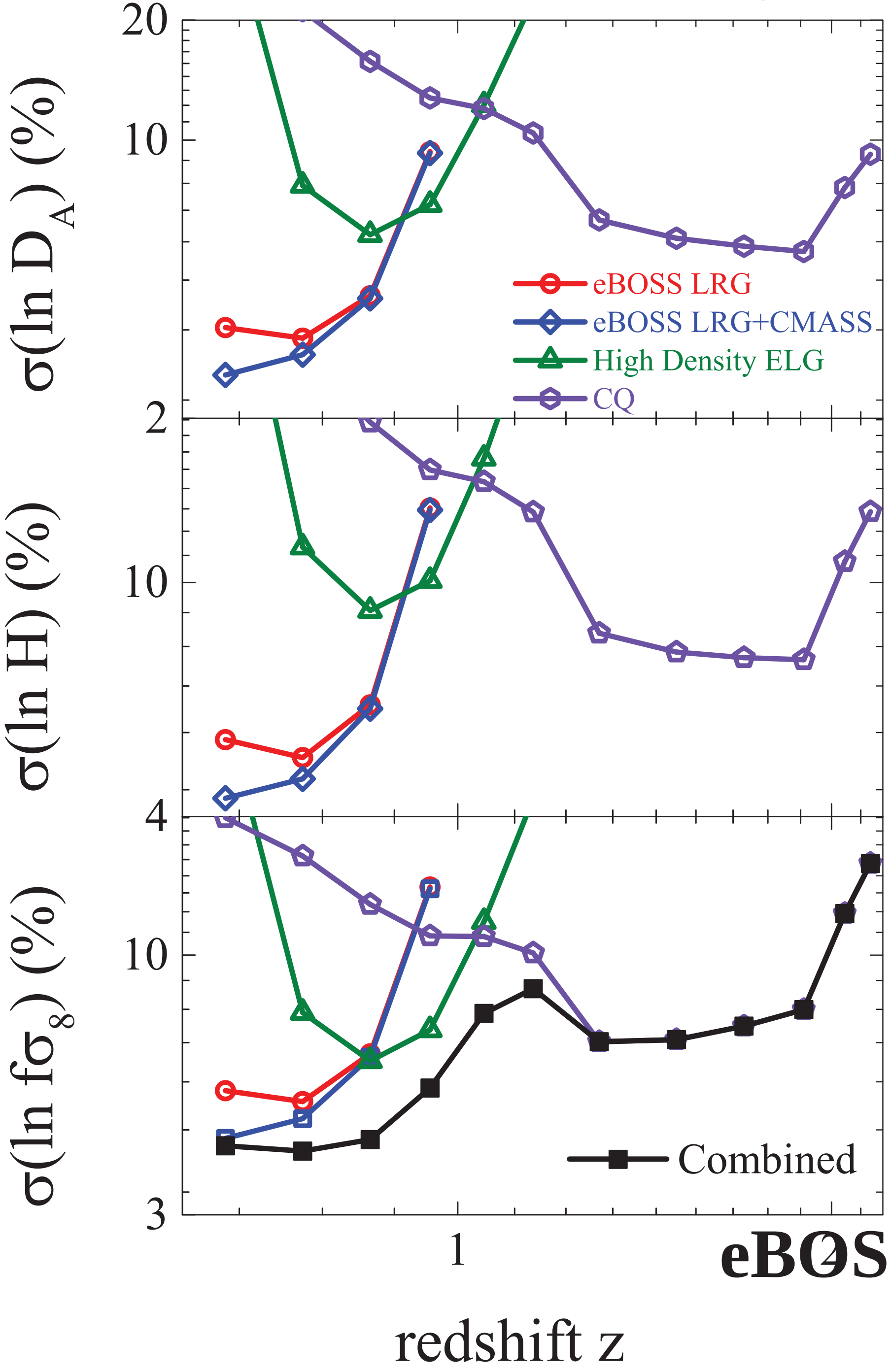


The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: constraining modified gravity

Eva-Maria Mueller[1⋆], Will Percival[1], Eric Linder[2,3], Shadab Alam[4,5,6], Gong-Bo Zhao[1,7],
Ariel G. Sánchez[8], Florian Beutler[1]



Bright future: 5% accuracy in 5 years



eBOSS (Zhao+, 2016)

Bright future: 1% accuracy in 10-20 years by stage IV (DESI, PFS, Euclid, WFIRST,SKA)

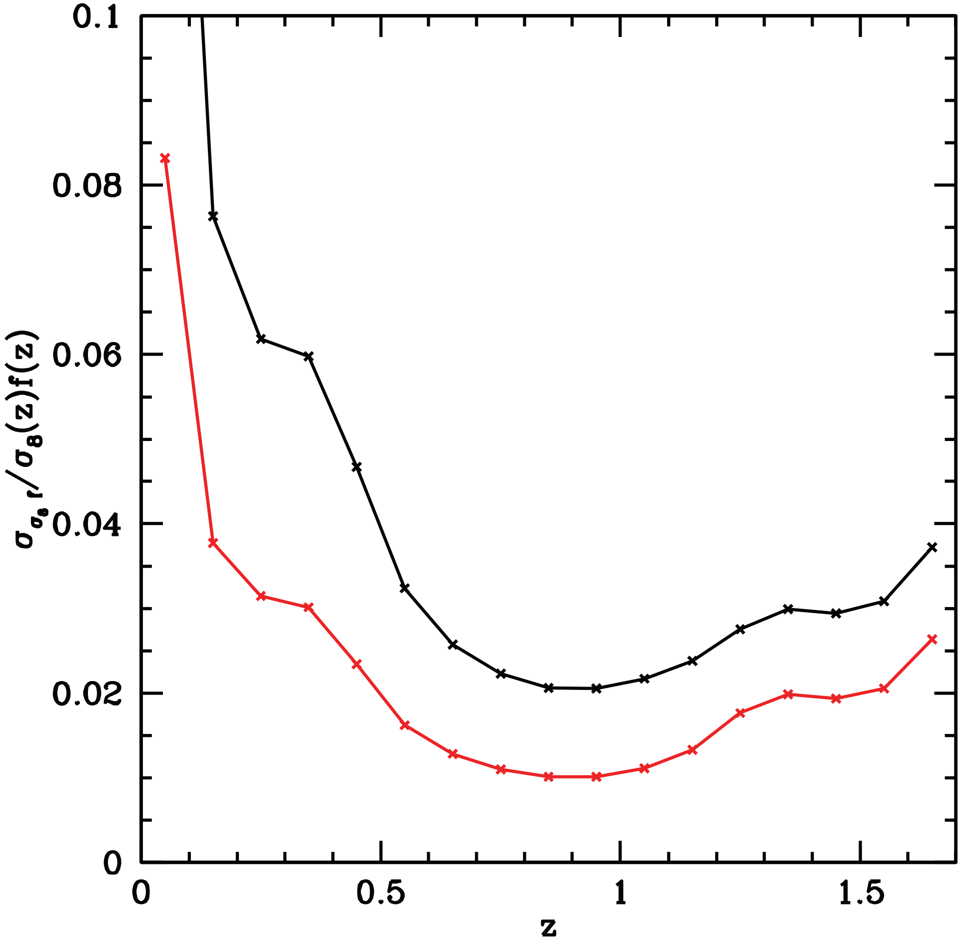


Figure 2.3: Constraints, derived from redshift-space distortions, on $\sigma_8(z)f(z)$ (amplitude of the power times growth rate), for bins of $\Delta z = 0.1$, for $k_{\max} = 0.1$ or $0.2\ h\mathrm{Mpc}^{-1}$ (upper and lower lines, respectively).

Problems in RSD cosmology

(=opportunities for young researchers)

- Major one: RSD modeling
- Minor one: RSD measurement

RSD modeling: Kaiser formula

$$\mathrm{Mapping}: \vec{x} \to \vec{s} = \vec{x} + \frac{v_z}{H}\hat{z}$$

$$\mathrm{Mapping}: (1+\delta)d^3x = (1+\delta^s)d^3s$$

$$\delta^s = \frac{1+\delta}{\left|\frac{ds}{dx}\right|} - 1 \qquad \left|\frac{ds}{dx}\right| = 1 + \nabla_z v_z/H$$

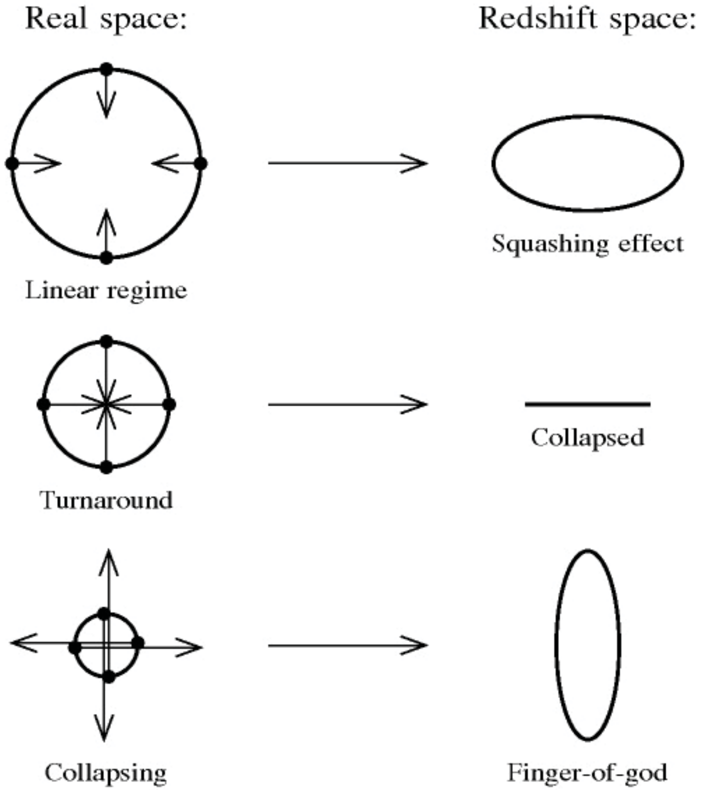
$$\boxed{\delta^s \simeq \delta - \nabla_z v_z/H}$$

$$\boxed{\begin{aligned} &\Rightarrow \delta^s(\vec{k}) \simeq \delta(\vec{k})(1+fu^2) \\ &\Rightarrow P^s(\vec{k}) = P(k)(1+fu^2)^2 \end{aligned}}$$

$$u \equiv \frac{k_z}{k}$$

$z:$ line of sight

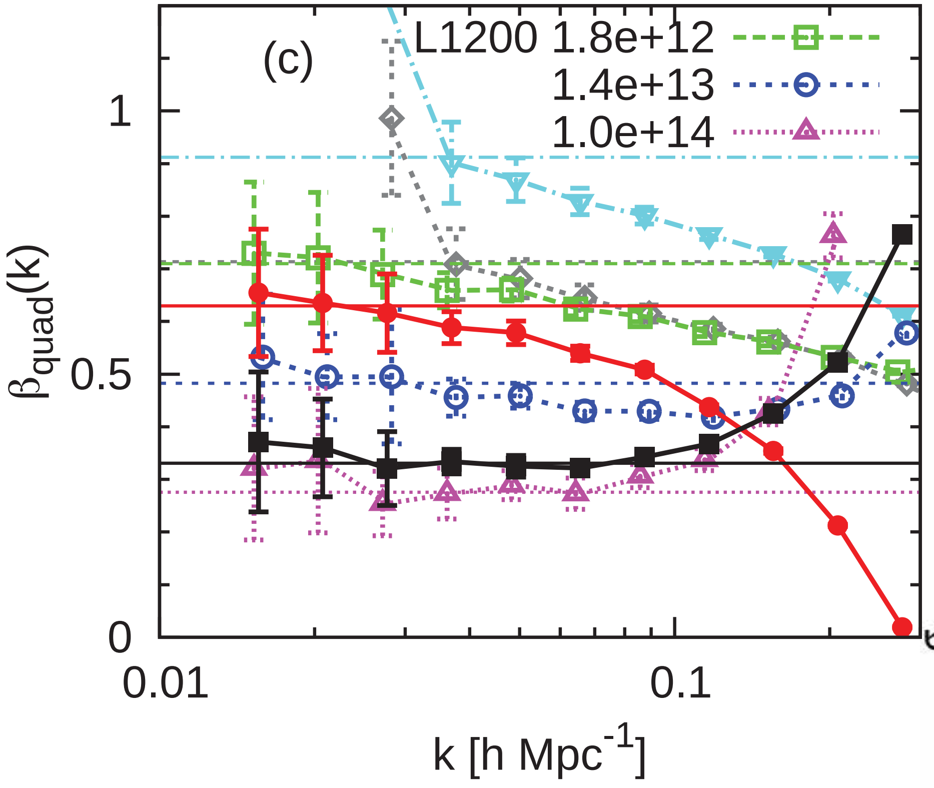
Including Finger of God



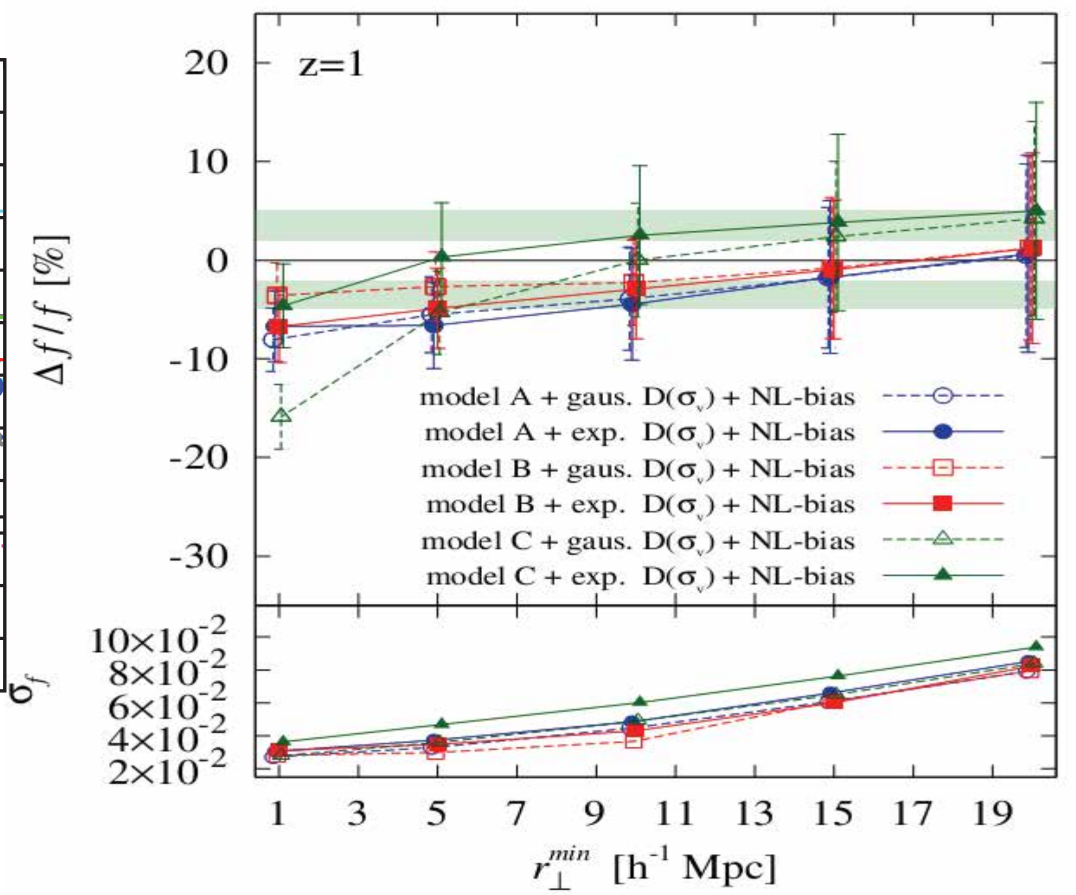
$$\boxed{P^s(k,u) = P(k)(1+fu^2)F(y \equiv ku\sigma_v/H)}$$

$$F(y) = \exp(-y^2) \text{ or } (1+y^2)^{-1}$$

Systematic error detected at 10% level



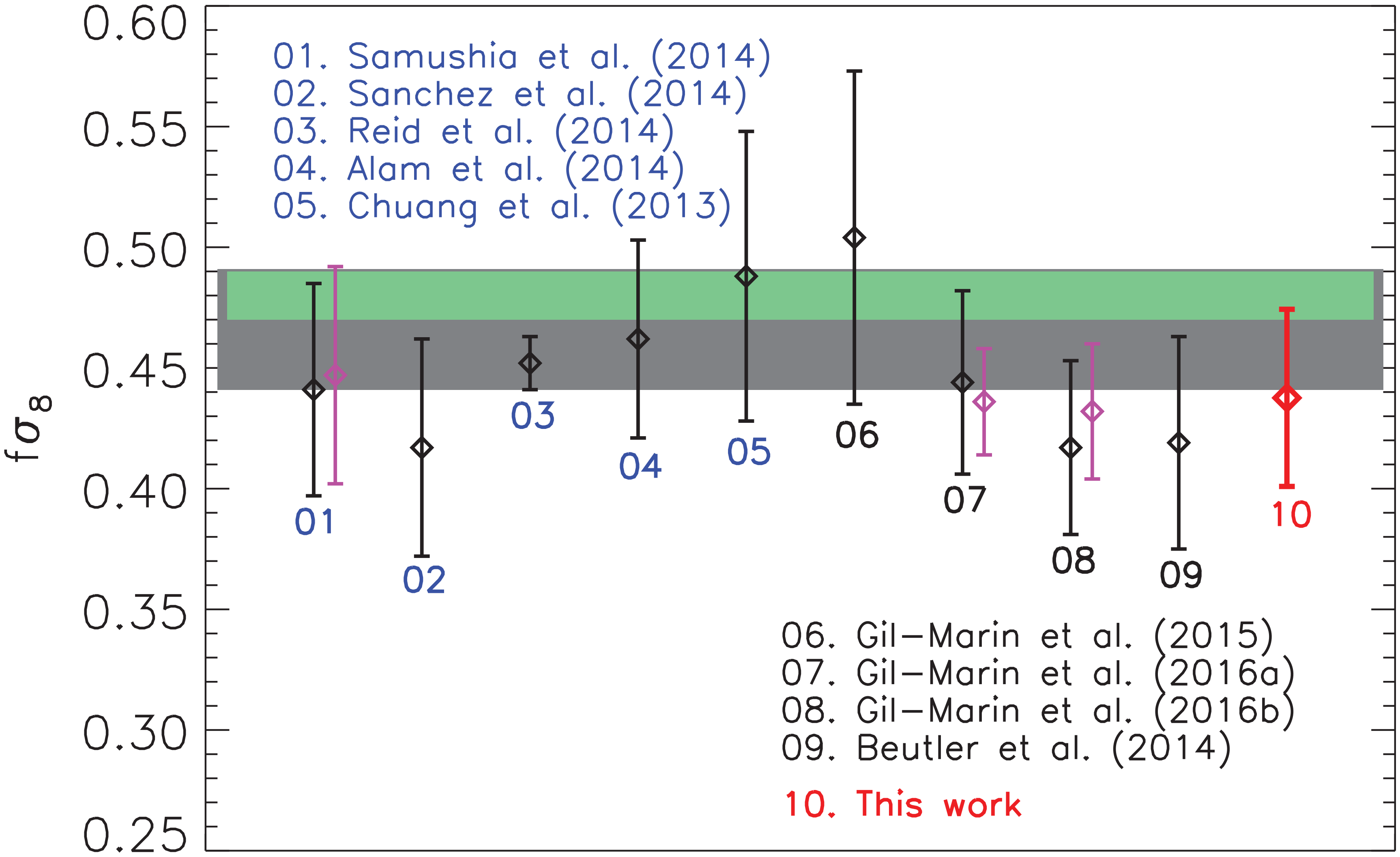
Okumura & Jing, 2010



Torre & Guzzo, 1202.5559

And many more works

Results vary between different groups



Li+, 2017, ApJ

Although data shows deviation from GR, no one has confidence to defeat Einstein!

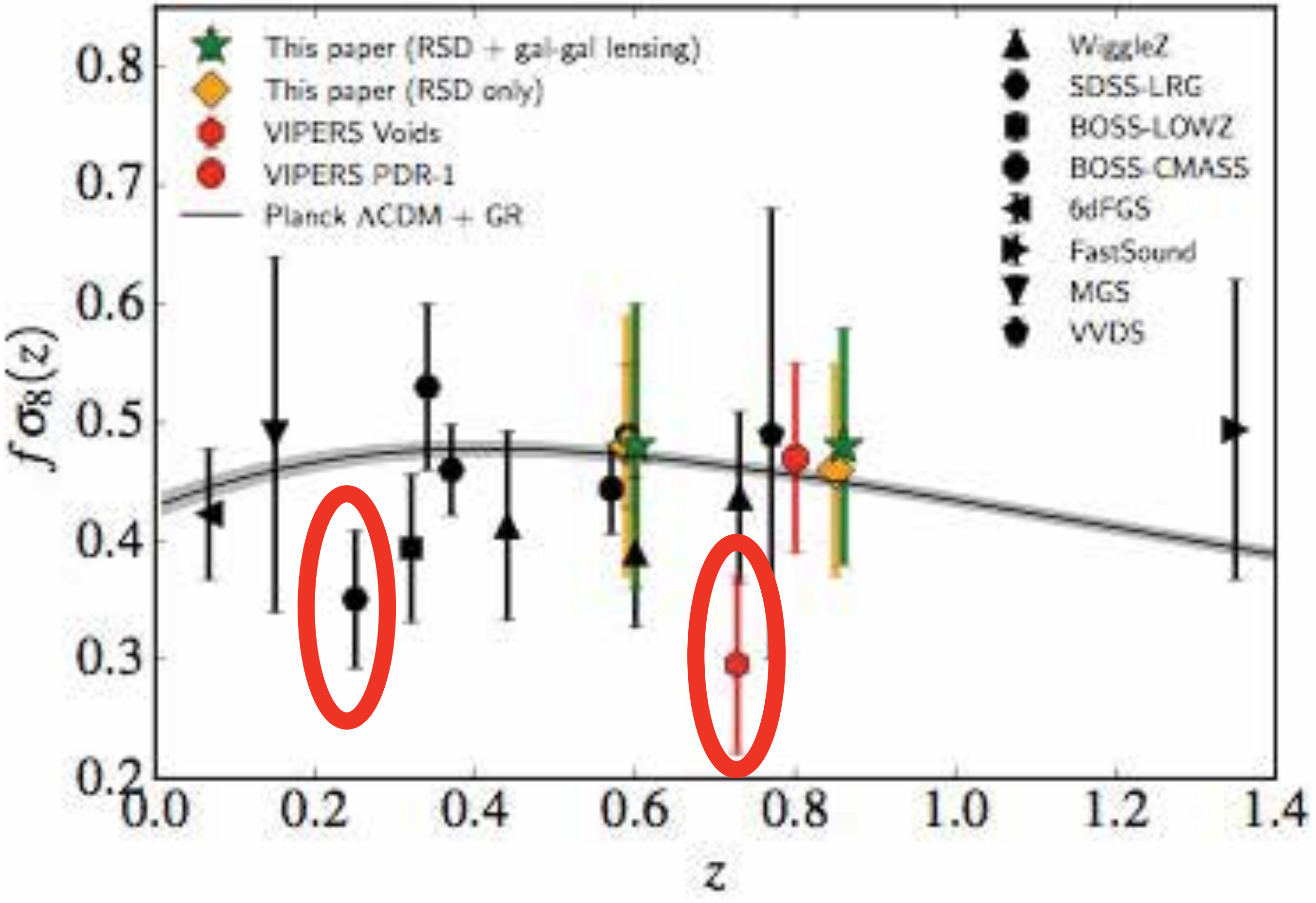


Fig. 12. $f\sigma_8$ as a function of redshift, showing VIPERS results contrasted with a compilation of recent measurements. The previous results from VVDS (Guzzo et al. 2008), SDSS LRG (Cabré & Gaztañaga 2009; Samushia et al. 2012), WiggleZ (Blake et al. 2012), 6dFGS (Beutler et al. 2012), VIPERS PDR-1 (de la Torre et al. 2013), MGS (Howlett et al. 2015), FastSound (Okumura et al. 2016), BOSS-LOWZ (Gil-Marín et al. 2016), BOSS-CMASS (Gil-Marín et al. 2016; Chuang et al. 2016), and VIPERS PDR-2 voids (Hawken et al. 2016) are shown with the different symbols (see labels). The solid curve and associated shaded area correspond to the expectations and 68% uncertainty for General Relativity in a ΛCDM background model set to TT+lowP+lensing Planck 2015 predictions (Planck Collaboration et al. 2015).

VIPERS, arXiv: 1612.05647

What go wrong?

Incomplete list of approximations/simplifications in RSD modeling

$$\mathbf{x}^s = \mathbf{x} + \frac{\mathbf{v}\cdot\hat{x}}{H}\hat{x} \xrightarrow{\text{Distant observer}} \mathbf{x}^s = \mathbf{x} + \frac{v_z}{H}\hat{z}$$

Neglecting AP/relativistic effects/lensing distortion

Single streaming
No magnification bias

$$P^s(\mathbf{k}) = \int \langle (1+\delta_1)(1+\delta_2)e^{ik_z(v_{1z}-v_{2z})/H}\rangle e^{i\mathbf{k}\cdot\mathbf{r}}d^3x$$

Cumulant expansion theorem

$$P_g^s(k,u) = \left[P_g(k)(1+\beta\tilde{W}(k)u^2)^2 + u^4P_{\theta_S\theta_S}(k) + \cdots\right]D^{\mathrm{FOG}}(ku)$$

Linear density-velocity relation, no velocity bias

Deterministic density-velocity

Negligible high order corrections

$$P_g^s(k,u) = P_g(k)(1+\beta u^2)^2 D^{\mathrm{FOG}}(ku)$$

Further approximations often used in observations

- **Scale independent galaxy density bias**
- **D^{FOG}**: Gaussian, Lorentz, more complicated? Meaning of σ_v?

Challenging to achieve 1% accuracy

- Many works to improve RSD model
  - e.g. Peebles 1980,…, Kaiser 1987,…, Hamilton 1992,…, Scoccimarro, 2000, …, White 2001,…, Yang et al. 2002,…, Kang et al. 2002,…, Szapudi 2004,…, Zu et al. 2007,…, Tinker 2007, …, Matsubara 2008,…,Taruya et al. 2010,…, Kwan et al. 2011,…, Seljak & McDonald, 2011,..,Reid & White 2011,…Jennings et al. 2012,…
- **Entangled complexities**
  - **Nonlinear mapping between real and redshift space**
    - **Redshift space 2pt is the sum of all N-pt in real space**
  - **Nonlinearity in the dark matter density and velocity statistics**
    - **Non-Gaussianity, no compact expression of redshift space ps**
    - **Stochastic velocity-density relation**
  - **Nonlinear galaxy-dark matter relation**
    - **Stochastic scale dependence density bias**
    - **Velocity bias**
- Disentangle RSD!
  - **ZPJ, Pan & Zheng, 2013; Zheng et al. 2013; ZPJ, Zheng & Jing, 2015, Zheng et al., 2015a ,2015b; Yu et al. 2015; 2016; Zheng et al. 2016**

Disentangle RSD (1): real space-redshift space mapping

$$\lambda \equiv ik_z(v_{1z} - v_{2z})/H.$$

$$P^s_{\delta\delta}(\mathbf{k}) = \int \langle (1+\delta_1)(1+\delta_2)\exp\lambda\rangle \exp(-i\mathbf{k}\cdot\mathbf{r})d^3\mathbf{r},$$

$$D^{\rm FOG}_\delta(k_z)\left[1 + \langle\delta_1\delta_2\rangle + i\frac{k_z}{H}\langle\delta_2 v_1 - \delta_1 v_2\rangle + \frac{k_z^2}{H^2}\langle v_1 v_2\rangle + C_{NG}(\mathbf{r},k_z) + C_G(\mathbf{r},k_z)\right].$$

$$\begin{aligned} C_{NG} &= \sum_{j\geq 3} C_{NG,j}(\mathbf{r},k_z) \\ &= \left[i\frac{k_z\langle\delta_1\delta_2(v_1-v_2)\rangle_c}{H} - \frac{k_z^2\langle(v_1-v_2)^2(\delta_1+\delta_2)\rangle_c}{2H^2}\right] \\ &\quad + \cdots. \end{aligned} \tag{A8}$$

$$\begin{aligned} C_G(\mathbf{r},k_z) = &\left[\exp\left(\frac{k_z^2\langle v_1v_2\rangle}{H^2}\right) - 1\right] \\ &\times\left(\langle\delta_1\delta_2\rangle + i\frac{k_z}{H}\langle\delta_2 v_1 - \delta_1 v_2\rangle\right) \\ &+ \frac{k_z^2}{H^2}\langle\delta_1 v_2\rangle\langle\delta_2 v_1\rangle\exp\left(\frac{k_z^2\langle v_1v_2\rangle}{H^2}\right) \\ &+ \left[\exp\left(\frac{k_z^2\langle v_1v_2\rangle}{H^2}\right) - \frac{k_z^2}{H^2}\langle v_1v_2 - 1\rangle\right]. \end{aligned}$$

$$P^s(k,u) = \left[P(k)(1+\underbrace{f\tilde{W}(k)u^2}_{\mathbf{v}_\delta})^2 + \underbrace{P_{\theta_s\theta_s}(k)u^4}_{\mathbf{v}_S} + \underbrace{C_G(k,u) + C_{NG,3}(k,u)}_{\mathbf{v}_\delta}\right]\underbrace{D^{\rm FOG}(ku)}_{\mathbf{v}_\delta}$$

ZPJ, Pan, Zheng, 2013

The cumulant expansion theorem

$$\langle e^f \rangle = \exp\left[\sum_{N=1}^{\infty} \frac{\langle f^N \rangle_c}{N!}\right] \qquad \langle f \rangle = 0$$

$$\langle f^N \rangle_c = \langle f^N \rangle - \langle f^N \rangle_G$$

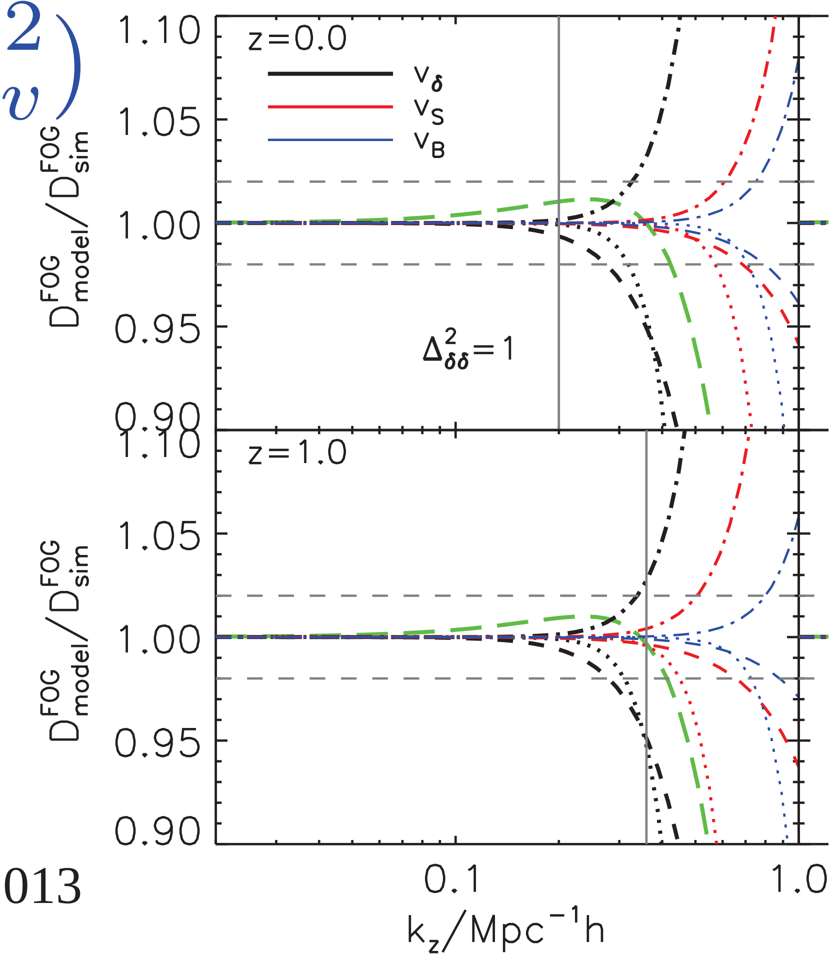
Disentangle RSD: Finger of God

$$P^s(k,u) = \left[P(k)(1+f\tilde{W}(k)u^2)^2 + P_{\theta_s\theta_s}(k)u^4 + C_G(k,u) + C_{NG,3}(k,u)\right] D^{\rm FOG}(ku)$$

$$D^{\rm FOG} = \langle \exp ik_z v_z/H \rangle^2$$

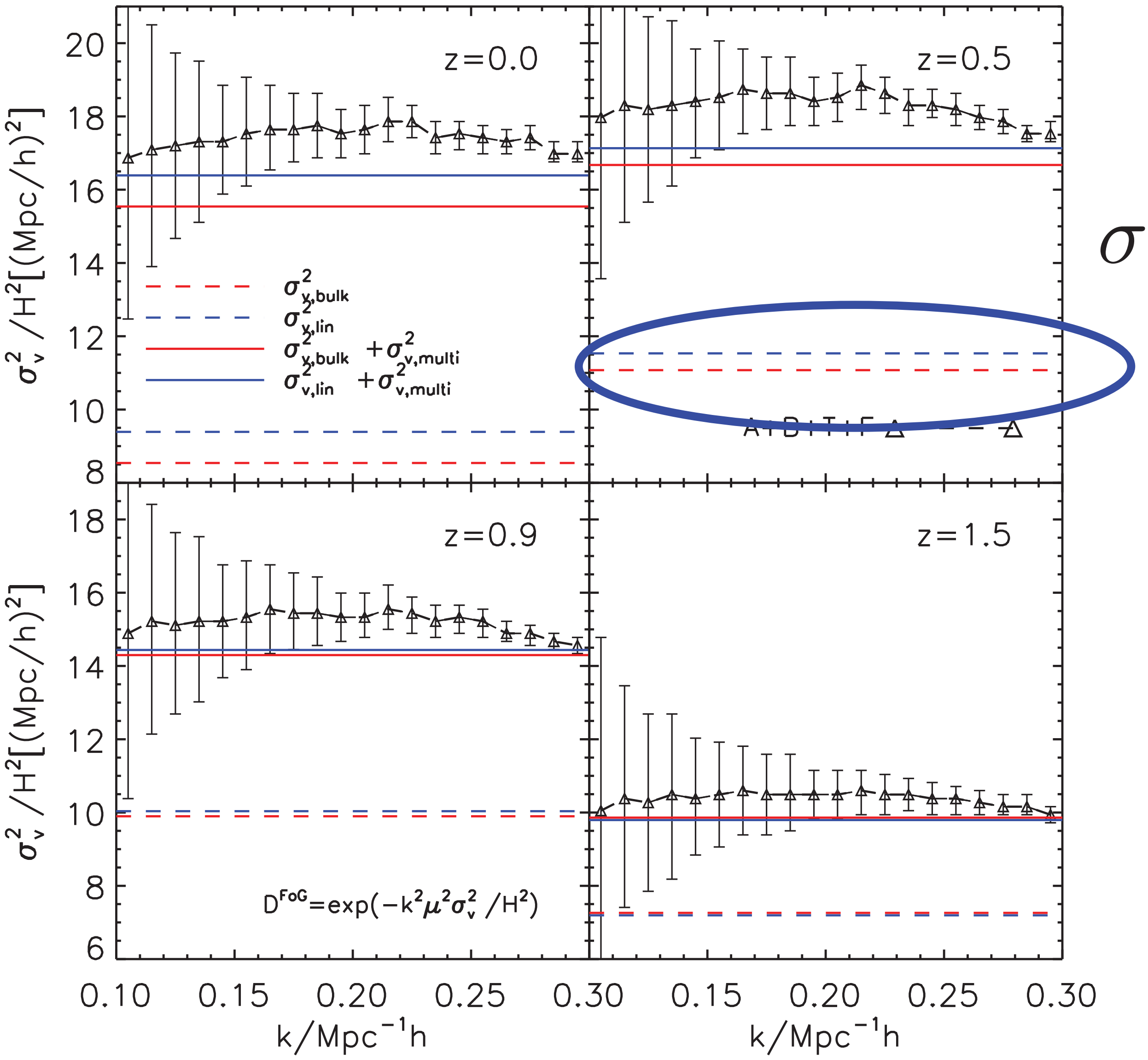
$$\simeq \exp(-k_z^2\sigma_v^2)$$

- All velocity components contribute!
- Large scale bulk flow (instead of small scale random motion) dominates!
- Well approximated by a Gaussian profile **(1% at k<0.3h/Mpc)**



Zheng et al. 2013

Problem: the measured FOG is larger than the theoretical expetcation!



$$\sigma_v^2 = \frac{1}{3}\int dk \frac{k^3}{2\pi^2} P_v(k)$$

Theoretical expectation under single streaming

Zheng & Song, 2016

First quantification of multi-steaming in RSD

$$\vec{s} = \vec{x} + \frac{v_z}{H}\hat{z}$$

Single streaming

$$(1+\delta)d^3x = (1+\delta^s)d^3s$$

$$\delta^s_{\rm single}(\mathbf{k}) = \int d^3\mathbf{x}\,\exp(i\mathbf{k}\cdot\mathbf{x})[1+\delta(\mathbf{x})]\exp\left(i\frac{k_z v_z}{H}\right).$$

The density power spectrum in redshift space is then

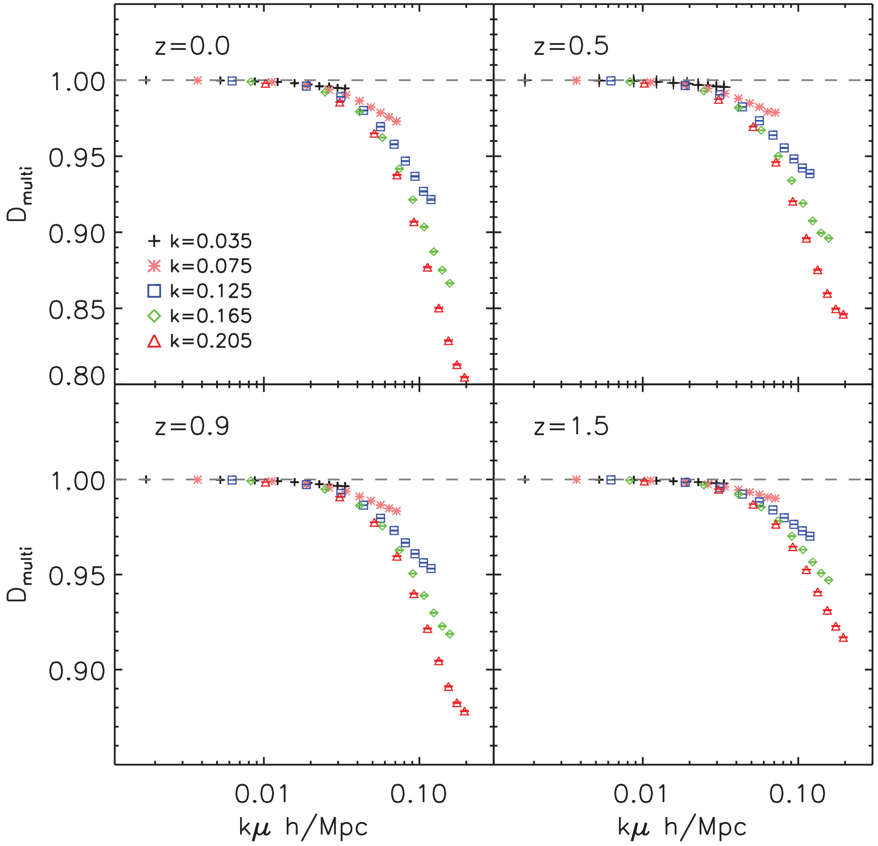
$$P^s_{\rm single} = \int d^3\mathbf{r}\,\exp(i\mathbf{k}\cdot\mathbf{r})\left\langle(1+\delta_1)e^{i\frac{k_z v_{1z}}{H}}(1+\delta_2)e^{-i\frac{k_z v_{2z}}{H}}\right\rangle$$

$$D_{\rm multi}(\mathbf{k}) \equiv \frac{P^s_{\rm multi}(\mathbf{k})}{P^s_{\rm single}(\mathbf{k})} < 1$$

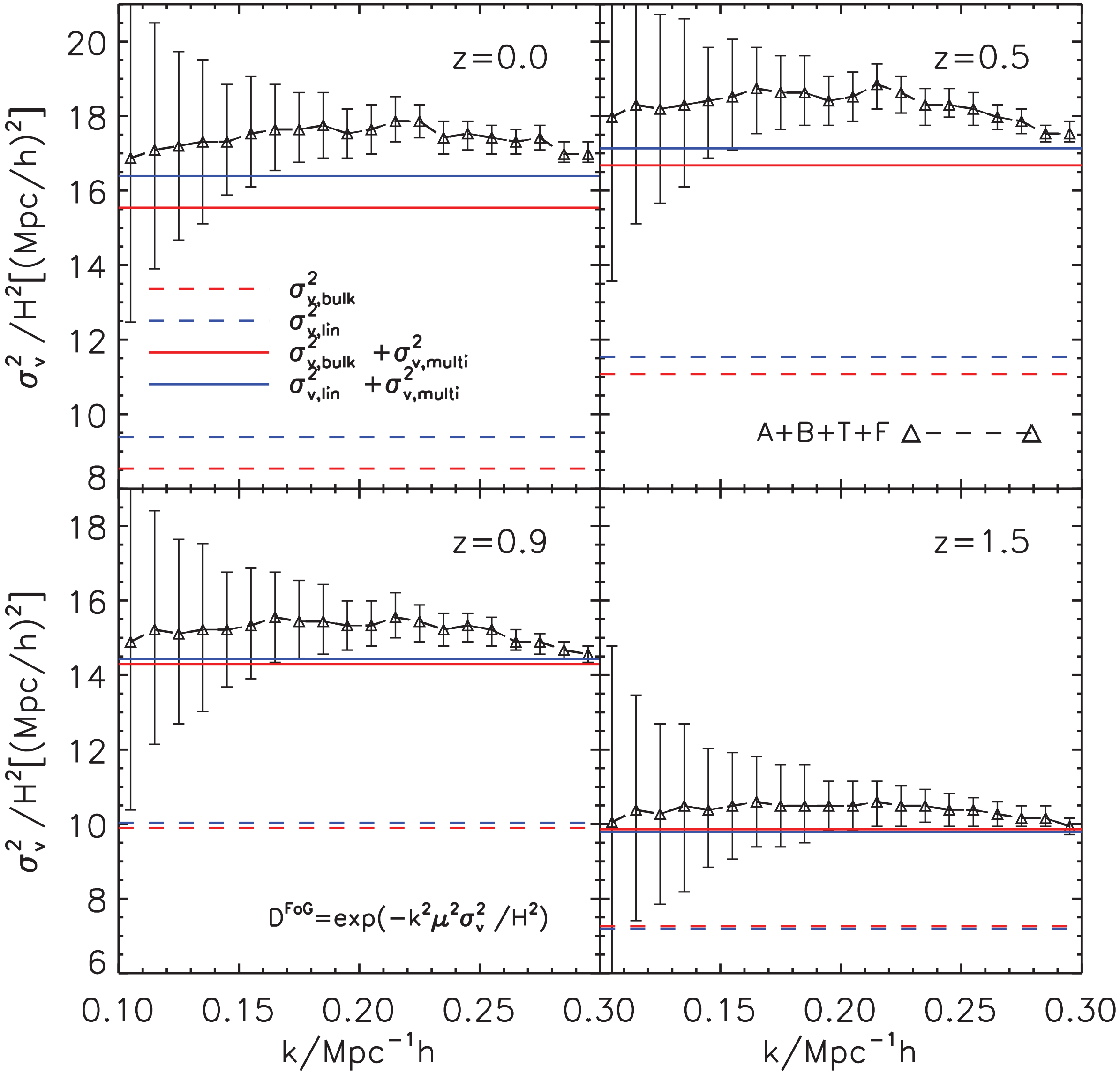
Zheng, ZPJ & Oh, 2016

Multi-streaming

Detection in simulations



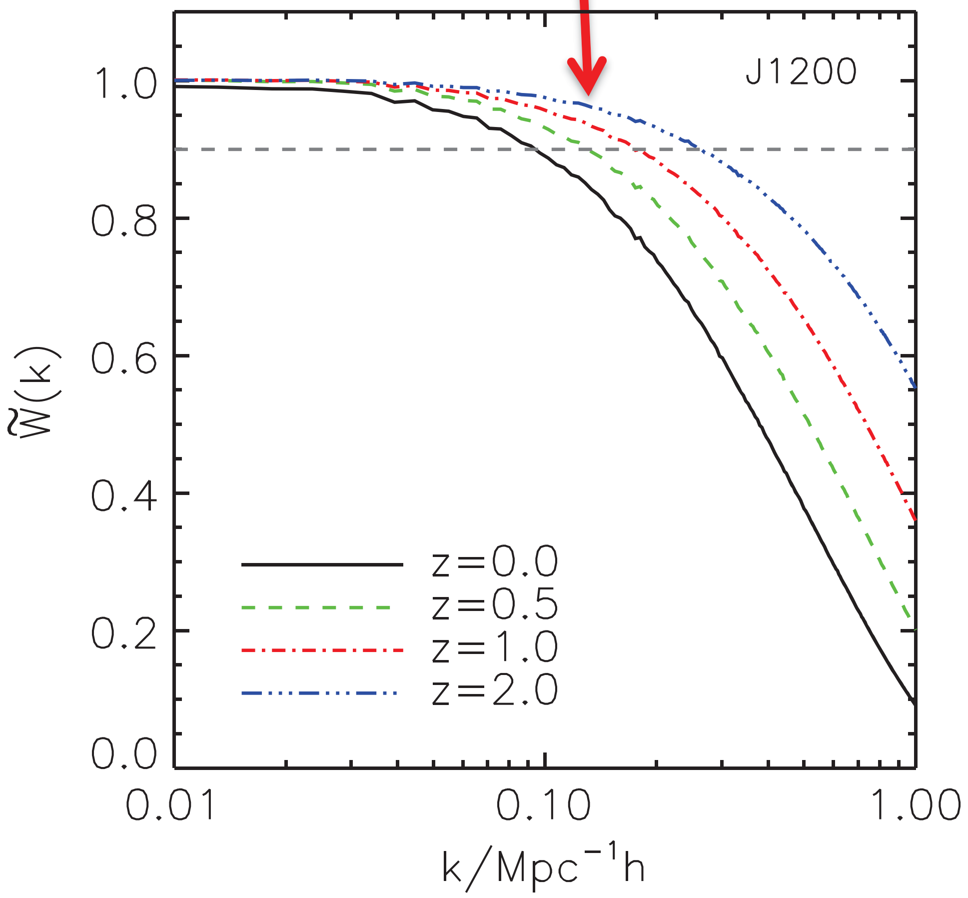
It is the missing link in explaining FOG



Zheng, ZPJ & Oh, 2016

Disentangle RSD (2): Nonlinear evolution in the DM density and velocity

$$P^s(k,u) = \left[P(k)(1+f\tilde{W}(k)u^2)^2 + P_{\theta_s\theta_s}(k)u^4 + C_G(k,u) + C_{NG,3}(k,u)\right] D^{\mathrm{FOG}}(ku)$$



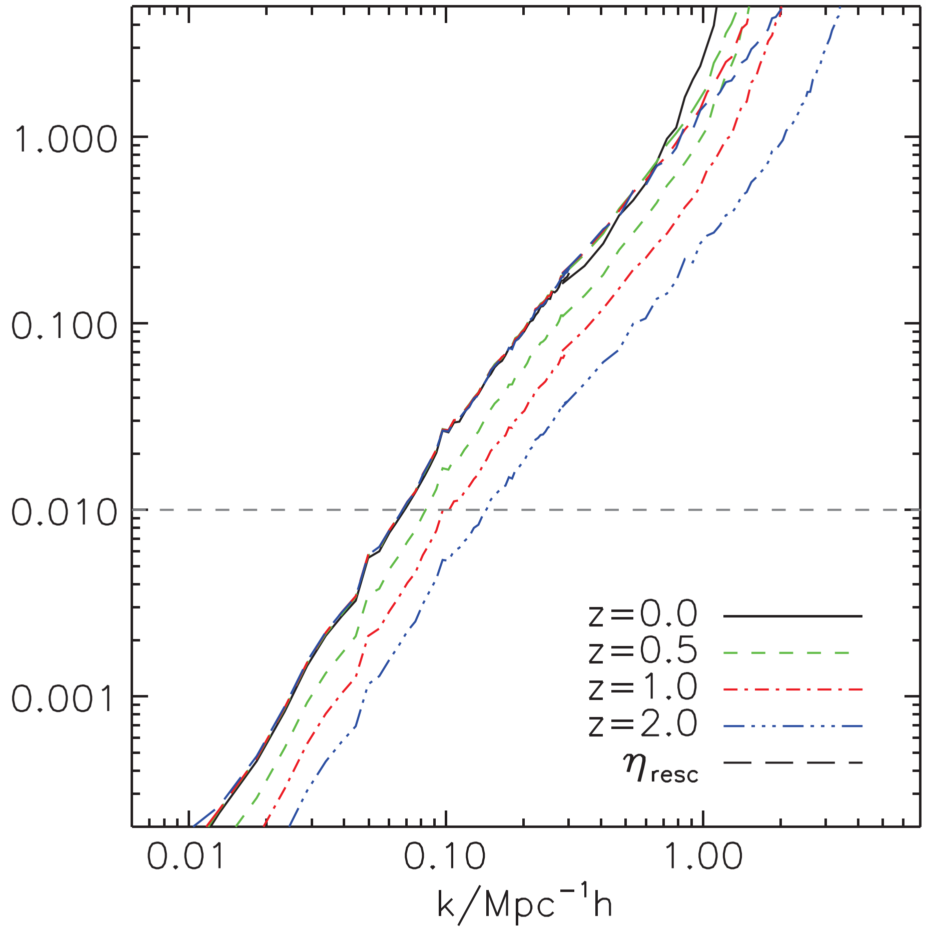
- Velocity growth is suppressed w.r.t density.
- Leading order correction to the Kaiser formula
- **10% at k=0.1h/Mpc and z=0**

Zheng et al. 2013

Nonlinear evolution: stochastic velocity-density

$$P^s(k,u) = \left[P(k)(1+f\tilde{W}(k)u^2)^2 + \boxed{P_{\theta_s\theta_s}(k)u^4} + C_G(k,u) + C_{NG,3}(k,u)\right] D^{\rm FOG}(ku)$$

$$P_{\theta_s\theta_s}/P_{\theta_\delta\theta_\delta}$$



Zheng et al. 2013

- ➤ Stochastic velocity v_S has a leading order contribution with **u^4** directional dependence

- ➤ **O(1%) effect at k=0.1h /Mpc and z=0-2.**

Disentangle RSD (3) : velocity bias

- Most cosmological constraints based on RSD assume no velocity bias at large scale (e.g. k=0.1h/Mpc)
  - Refer to Guo Hong’s talk on velocity bias at small scale

- But in reality velocity bias exists!
  - Environmental effect: halos locate at density peaks which are correlated with velocity
    - 10% velocity bias at k=0.1h/Mpc was predicted (BBKS 1986; Desjacques & Sheth 2010)
  - Gastrophysics (e.g. ram pressure)

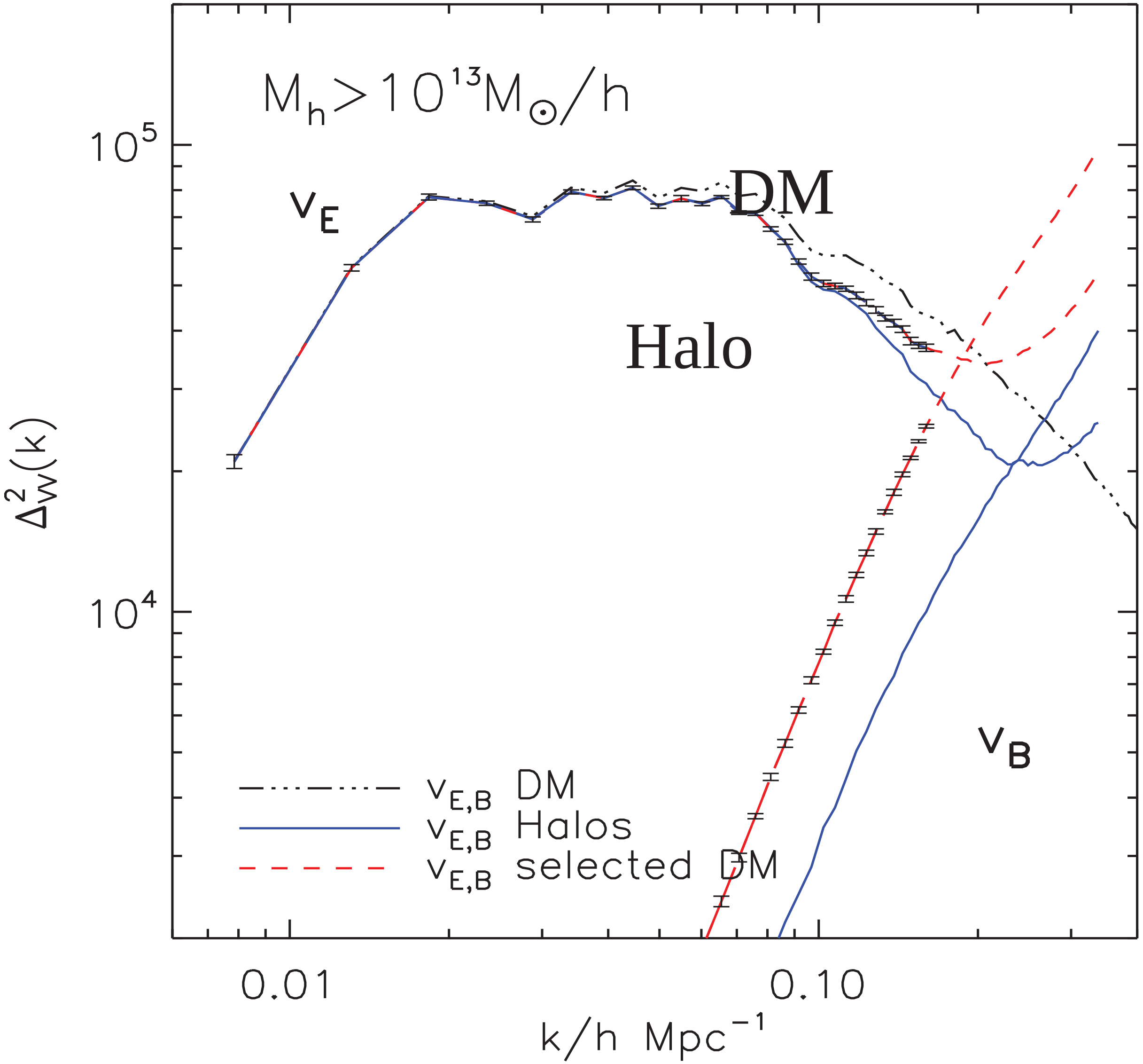
- How large?
  - Have to understand the velocity bias to 1% level accuracy at k~0.1h /Mpc.

Severe numerical artifacts

$$b_v \equiv \sqrt{\frac{P_{h,vv}}{P_{DM,vv}}}$$

- The apparent $b_v<1$ is caused by the **sampling artifact** in measuring the volume weighted velocity statistics.
  - Velocity where is no particles is ill defined/sampled.
  - Where there is no galaxy, velocity can be large
  - This sampling artifact depends on the particle number density and clustering

- No velocity bias detected after correcting sampling artifact! **(Preliminary)**



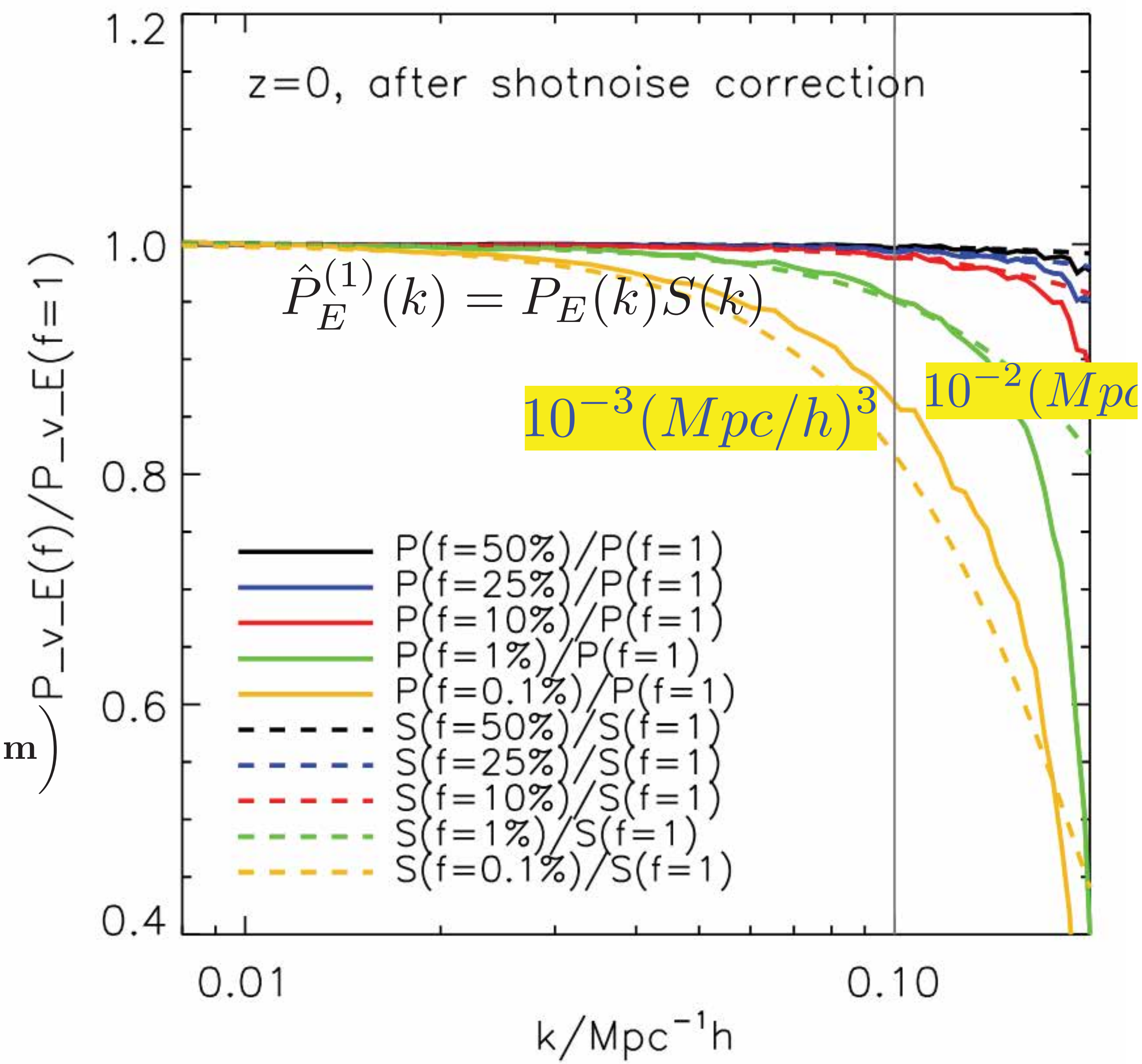
Zheng et al. 2015b

Understand and self-calibrate sampling artifacts

- We develop a theoretical model to understand and self-calibrate sampling artifact
  - ZPJ, Zheng & Jing, 2015
- Leading order approximation works to a few percent
  - Zheng et al. 2015
- Higher order corrections also derived.

$$\hat{P}_E^{(2)}(\mathbf{k}) \simeq \sum_{\mathbf{m}\neq\mathbf{0}} P_E(q_\mathbf{m})\cos^2\theta_\mathbf{m} W\left(\mathbf{q}_\mathbf{m}, \frac{2\pi}{L_{\rm box}}\mathbf{m}\right)$$

- Promising to measure 1% level velocity bias
- **Stay tuned on the halo velocity bias**



$$\hat{P}_E^{(1)}(k) = P_E(k)S(k)$$

$10^{-3}(Mpc/h)^3$

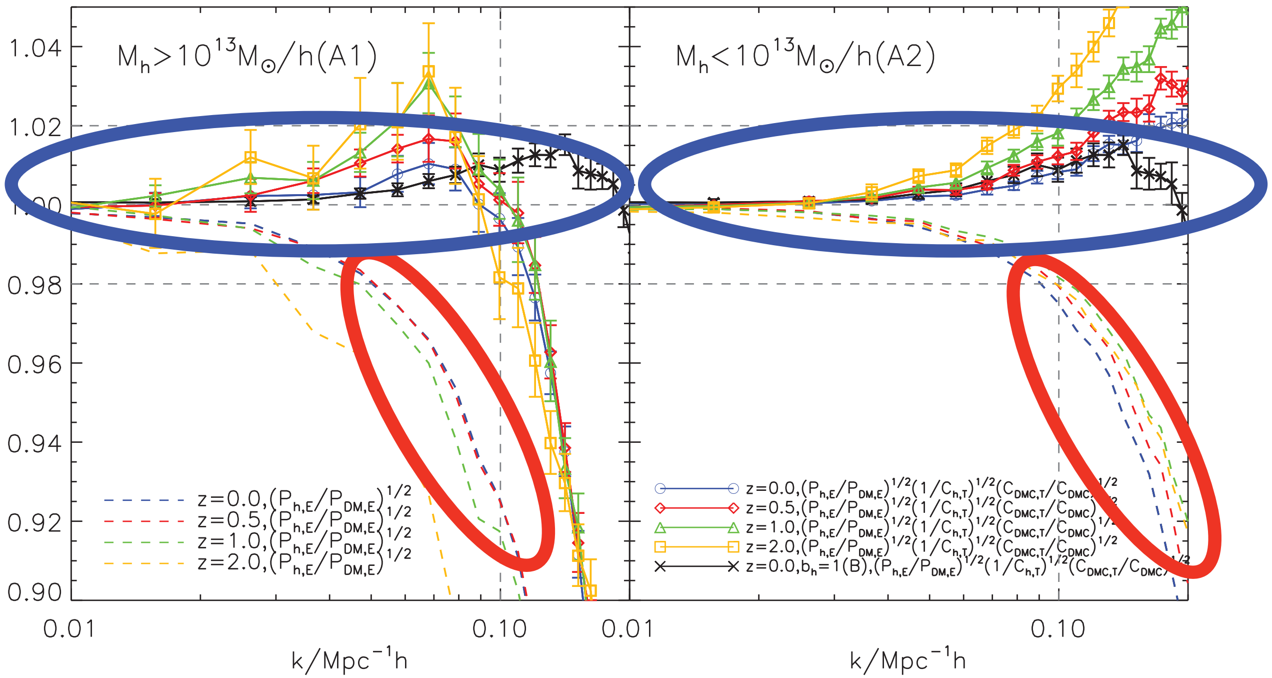
$10^{-2}(Mpc$

Zheng et al. 2015a

Halo velocity bias, after sampling artifact correction

Halo velocity

DM v



Need significantly improved understanding of sampling artifact or significantly improved velocity assignment method to reach 1%

Problems in RSD cosmology

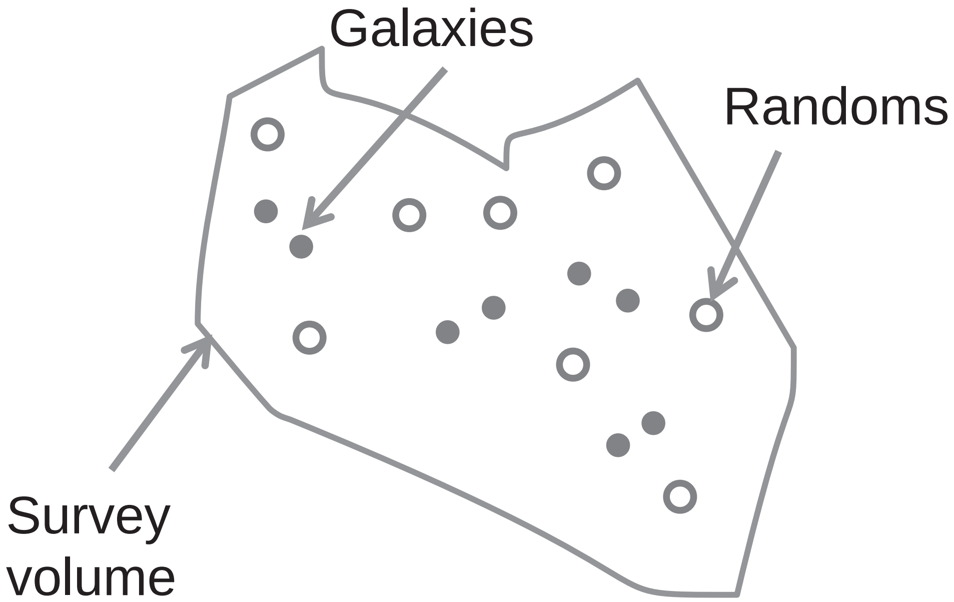
(=opportunities for young researchers)

- Major one: RSD modeling
  - fail at ~10%
- Minor one: RSD measurement
  - needs better data analysis method

RSD correlation function measurement

DD = number of galaxy-galaxy pairs
DR = number of galaxy-random pairs
RR = number of random-random pairs

All calculated as a function of separation **and** direction to LOS



$$\xi = \frac{DD}{RR} - 1$$

$$\xi = \frac{DD}{DR} - 1$$

$$\xi = \frac{DD\,RR}{DR^2} - 1$$

$$\xi = \frac{DD - 2DR}{RR} + 1$$

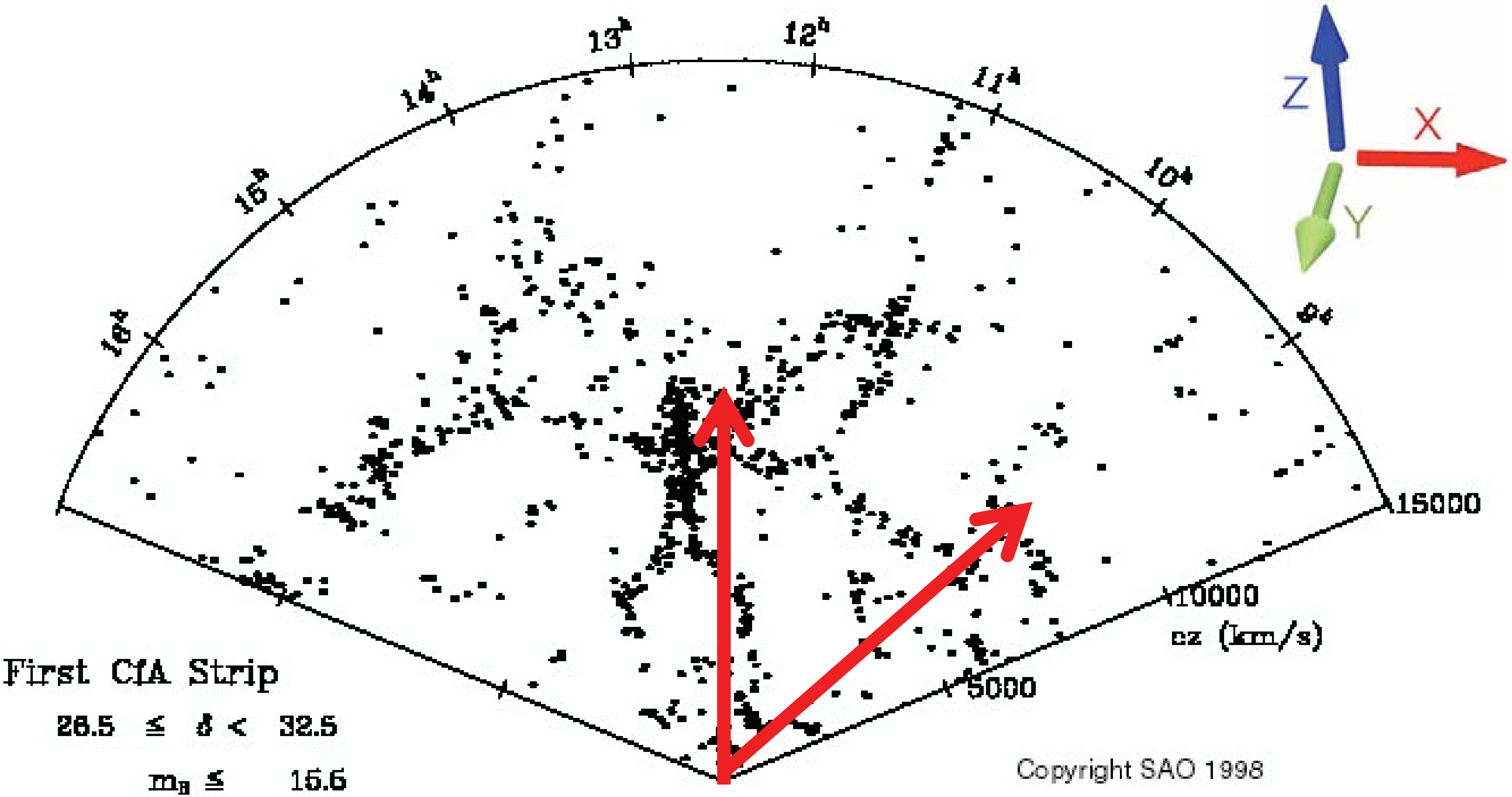
Landy & Szalay (1993) considered noise from these estimators, and showed that this has the best noise properties

Landy & Szalay 1993; ApJ 412, 64

Coutesy of Percival's lecture

RSD power spectrum measurement

Ordinary FFT does not work since it mixes different lines of sight and then smooths away the RSD anisotropy.



Power spectrum measurement in redshift space

- **Spherical Fourier Bessel (SFB) decomposition.**

$$\vec{x} = (x, \theta, \varphi) \qquad (\theta, \varphi) \leftrightarrow Y_{lm}(\theta, \varphi)$$

$$x \leftrightarrow j_\ell(x)$$

- **Moving l.o.s. approximation and RSD moments measurement with FFT (Yamamoto + 2005; Bianci + 2015; Scoccimarro 2015)**
- **Correlation function -> power spectrum (Jing & Borner 2001; Li+, 2016).**
  - **Window/mask deconvolved**
  - **Unbiased moments measurement**
  - **Can be as fast as FFT methods**

We have tested our method and applied to BOSS DR11,12

The redshift-space galaxy correlation function is measured using the Landy-Szalay estimator,

$$\xi_g^s(s,\mu_s) = \frac{\mathrm{DD} - 2\mathrm{DR} + \mathrm{RR}}{\mathrm{RR}} \ . \qquad (2)$$

$$\begin{aligned} P_g^s(k,\mu) &= \int \xi_g^s(s_\perp, s_\parallel) e^{i\mathbf{k}\cdot\mathbf{s}} d^3\mathbf{s} \\ &= \int \xi_g^s(s_\perp, s_\parallel) e^{i(k_\parallel s_\parallel + k_\perp s_\perp \cos(\phi))} s_\perp ds_\perp d\phi ds_\parallel \\ &= \int \xi_g^s(s_\perp, s_\parallel) K(k_\perp, k_\parallel; s_\perp, s_\parallel) s_\perp ds_\perp ds_\parallel \ , \quad (3) \end{aligned}$$

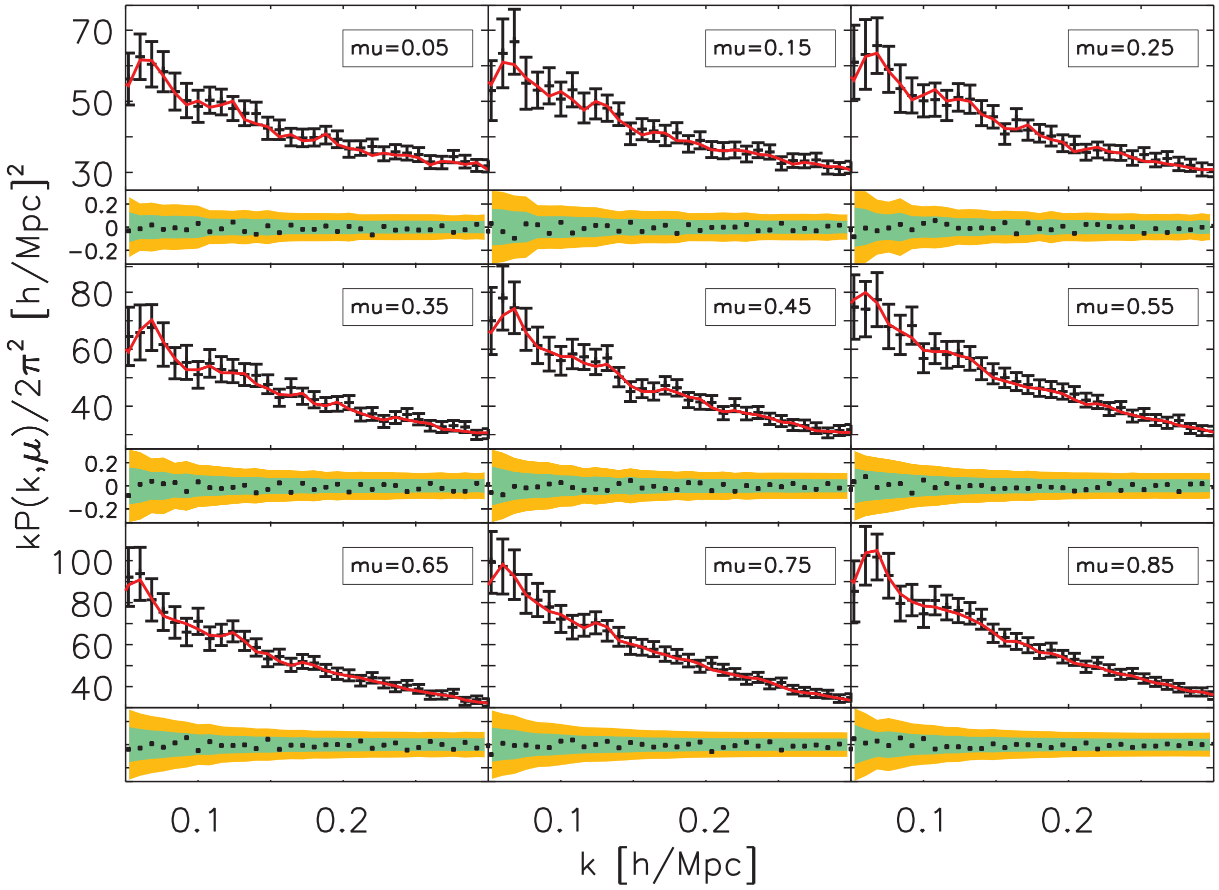
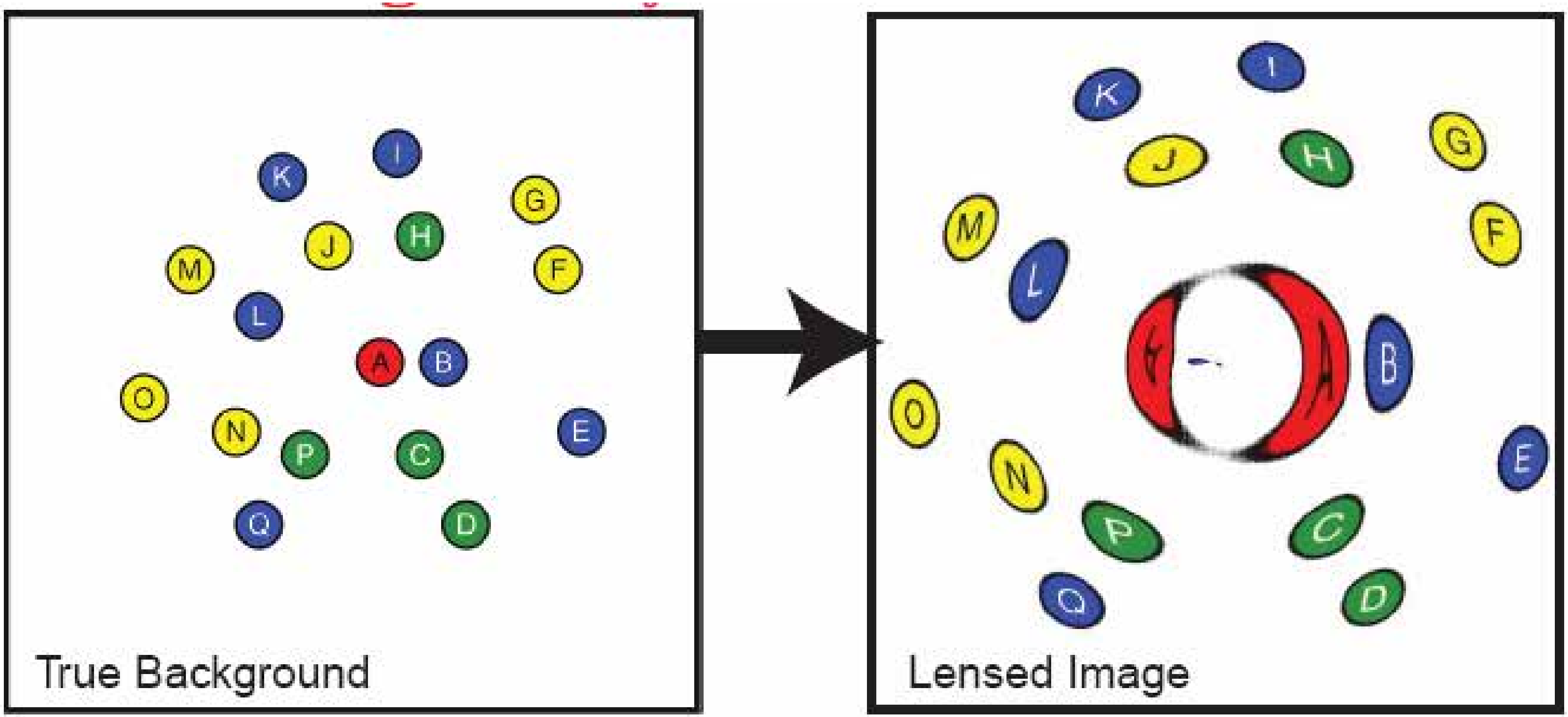


FIG. 1.— 2D power spectrum of mock galaxies in the CosmicGrowth simulation. The nine panels show $kP(k,\mu)/2\pi^2$ distribution as a function of k, for specific μ bins. The power spectrum we obtained with our method are shown as red solid lines and those using FFT method are shown as black plus signs in the top sub-box of each panel. The error bars for the data points of FFT method are estimated according to Eq. 6. In the bottom sub-box of each panel shows the difference between the two methods, $(P(k,\mu) - P_{\rm FFT}(k,\mu))/P_{\rm FFT}(k,\mu)$. The Celadon green (Chrome yellow) band shows $1\,\sigma$ $(2\,\sigma)$ confidence level. The two methods show good consistency. The differences between them are well within the $1\,\sigma$ level.

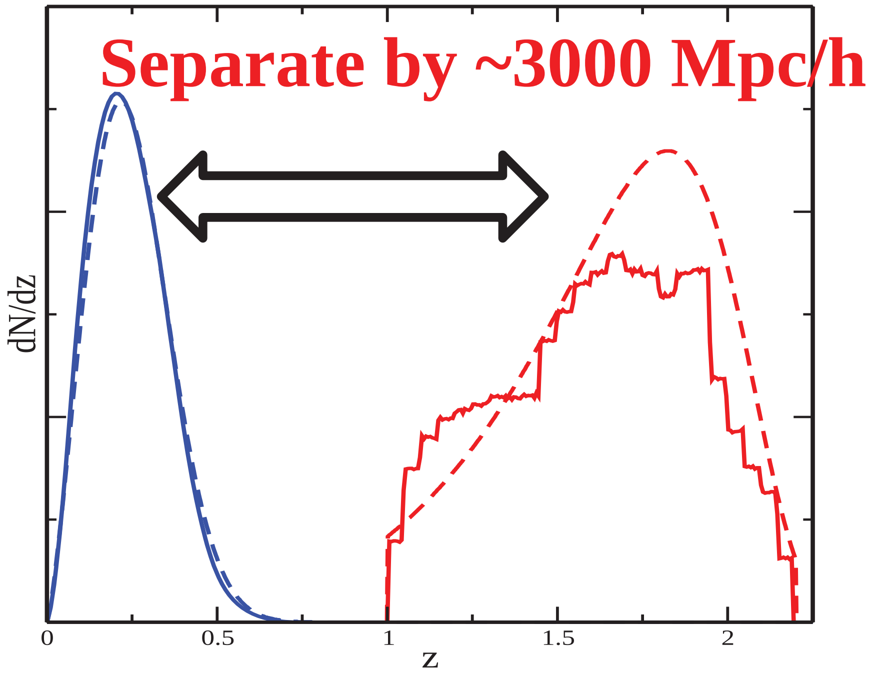
Another problem

- Mocks for RSD analysis
- We need of the order 10^3 mocks of 100 Gpc^3 volume to well understand the error bars, selection effect, and various other issues.
- Existing mock generation methods have various problems to overcome

Weak lensing Cosmology:
Promises and Challenges



Weak lensing: the cause of "abnormal" correlation at horizon scales

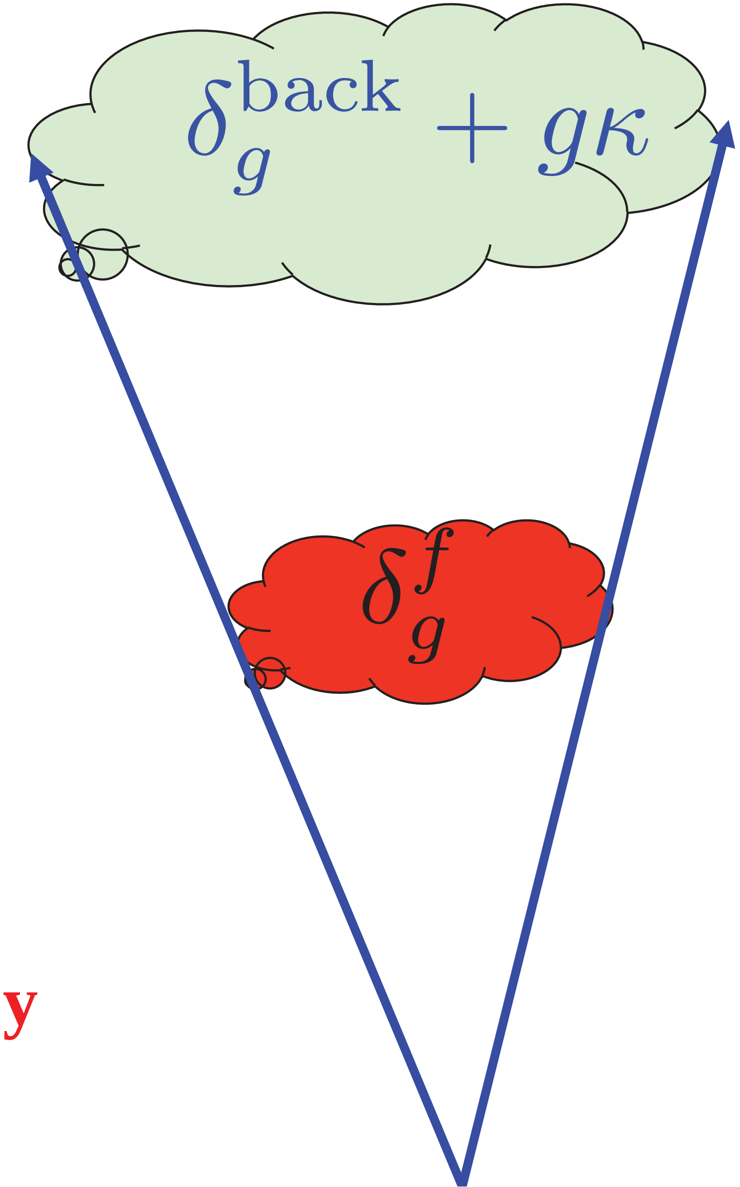


Separate by ~3000 Mpc/h

FIG. 1.— Galaxy redshift distribution from applying our $17 < r < 21$ magnitude limit to the CNOC2 luminosity function and quasar redshift distribution inferred from quasar photometric redshifts (solid lines). The fitted redshift distributions from Equation 8 are shown with dashed lines. In all cases, the amplitude scaling is arbitrary.

Non-vanishing correlation! First detected by Scranton+, 2005 at ~10sigma and then by other data

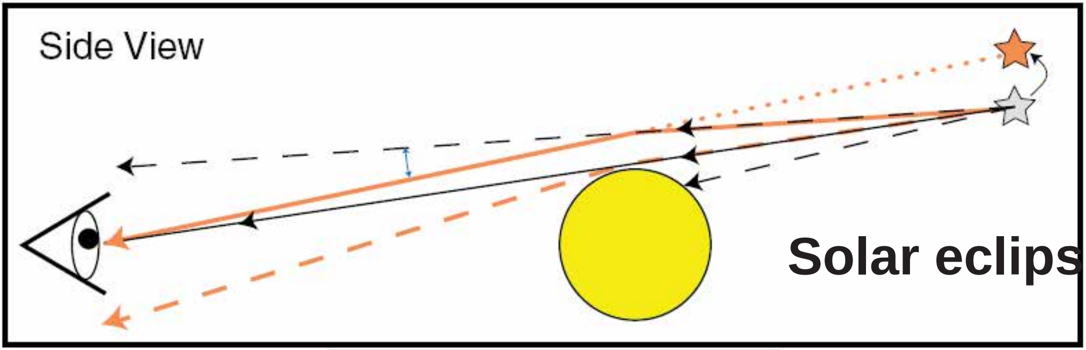
$$\boxed{\kappa = \sum w_i \delta_i = w^{\mathrm{f}} \delta_g^{\mathrm{f}} + \cdots}$$



Gravitational lensing: generic consequence of metric gravity

EP

Light deflection in curved spacetime

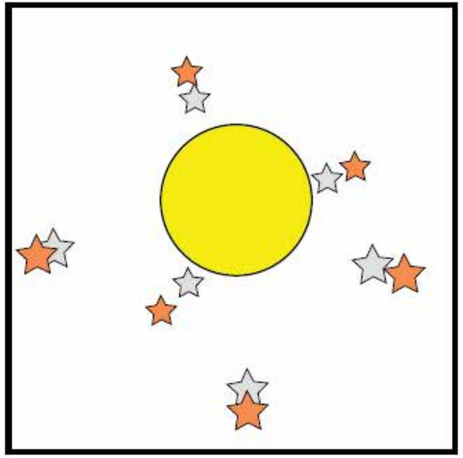


Solar eclipse (1919)

GR field equations

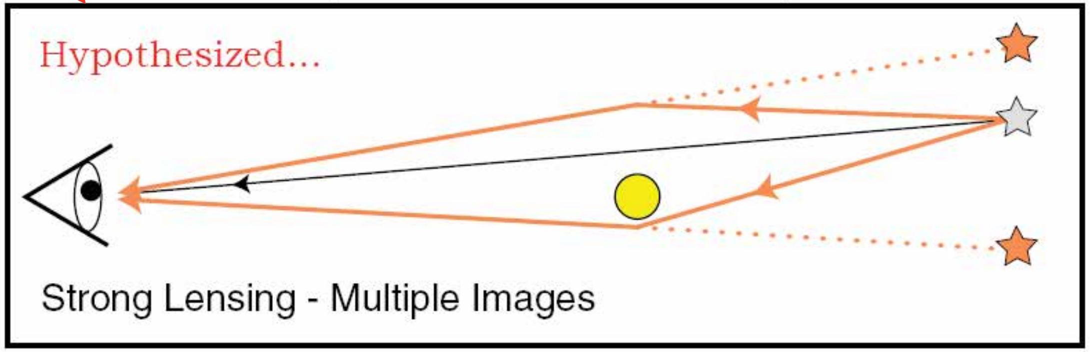
Curvature by matter/energy

View from Earth



Weak lensing: Galaxy clusters (1990) Blank sky (2000)

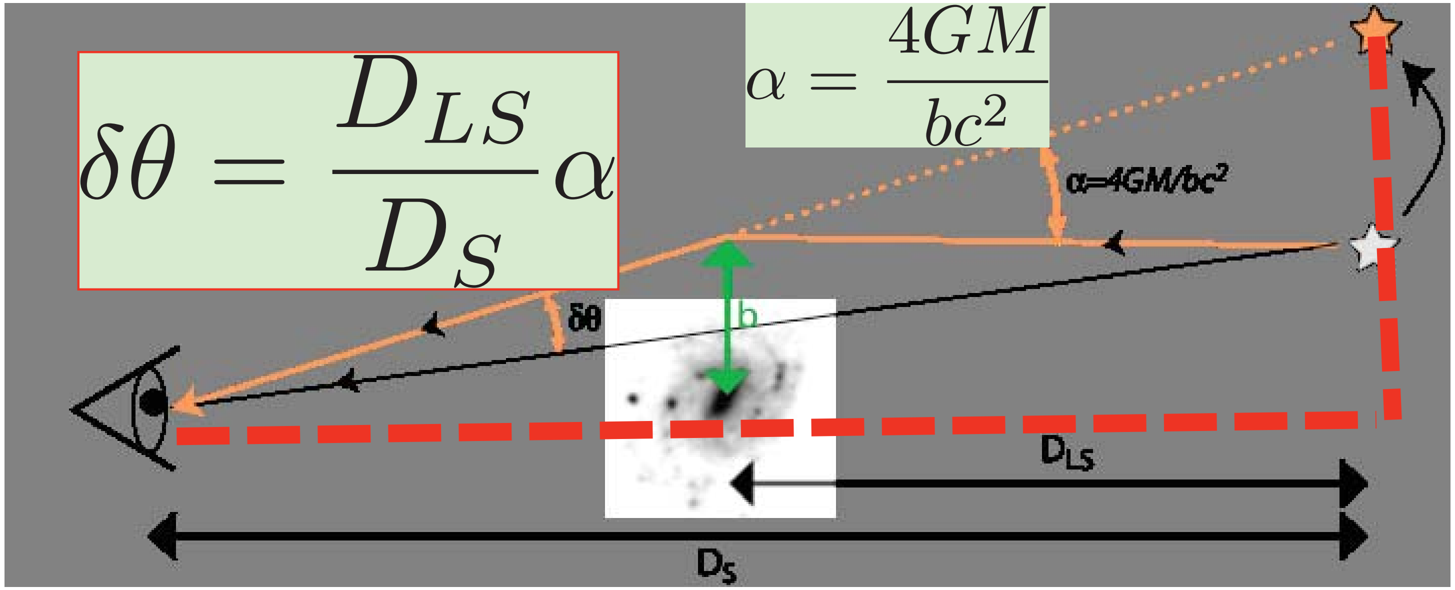
Strong lensing (1979)



Microlensing – probe DM

Gravitational lensing （single lens)

α depends on only the lens.
$\delta\theta$ depends on also the geometry



$$\delta\theta = \frac{D_{LS}}{D_S}\alpha$$

$$\alpha = \frac{4GM}{bc^2}$$

θ^I Image plane/sphere

θ^S Source plane/sphere

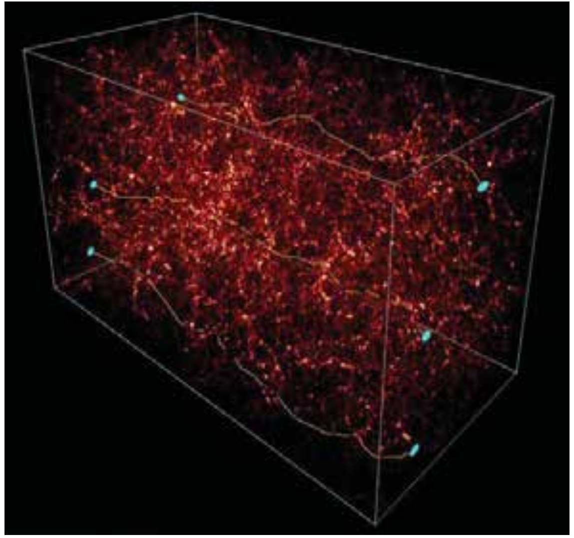
$$D_S\theta^I = D_S\theta^S + D_{LS}\alpha$$

$$\theta^I = \theta^S + \frac{D_{LS}}{D_S}\alpha$$

Flat geometry

$$\frac{D_{LS}}{D_S} \to 1 - \frac{\chi^L}{\chi_S}$$

Relativistic view of weak lensing



$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Phi)a^2 dX^2$$

$$\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0\ .$$

$$\theta^i_S = \theta^i + 2\int_0^\chi d\chi' \overset{\text{Structure growth}}{\boxed{\Phi_{,i}\left(\vec{x}(\chi')\right)}}\ \overset{\text{Geometry}}{\boxed{\left(1-\frac{\chi'}{\chi}\right)}}\ .$$

$$A_{ij} \equiv \frac{\partial\theta^i_S}{\partial\theta^j}$$

$$A_{ij} - \delta_{ij} = \begin{pmatrix} -\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & -\kappa+\gamma_1 \end{pmatrix} = 2\int_0^\chi d\chi'\Phi_{,ij}\left(\vec{x}(\chi')\right)\chi'\left(1-\frac{\chi'}{\chi}\right)$$

e.g. Modern cosmology by Scott Dodelson

Weak lensing statistics

- **Weak lensing is completely described by the transformation matrix. Symmetric (3 apparent DOFs)**
- **Lensing signal: Only one real DOF (the lensing convergence kappa, the so called E-mode)**

$$\kappa(\hat{n}) = \int_0^{z_s} (-\nabla^2\Phi(\chi(z_L), \hat{n}))W(z_L, z_s)dz_L$$

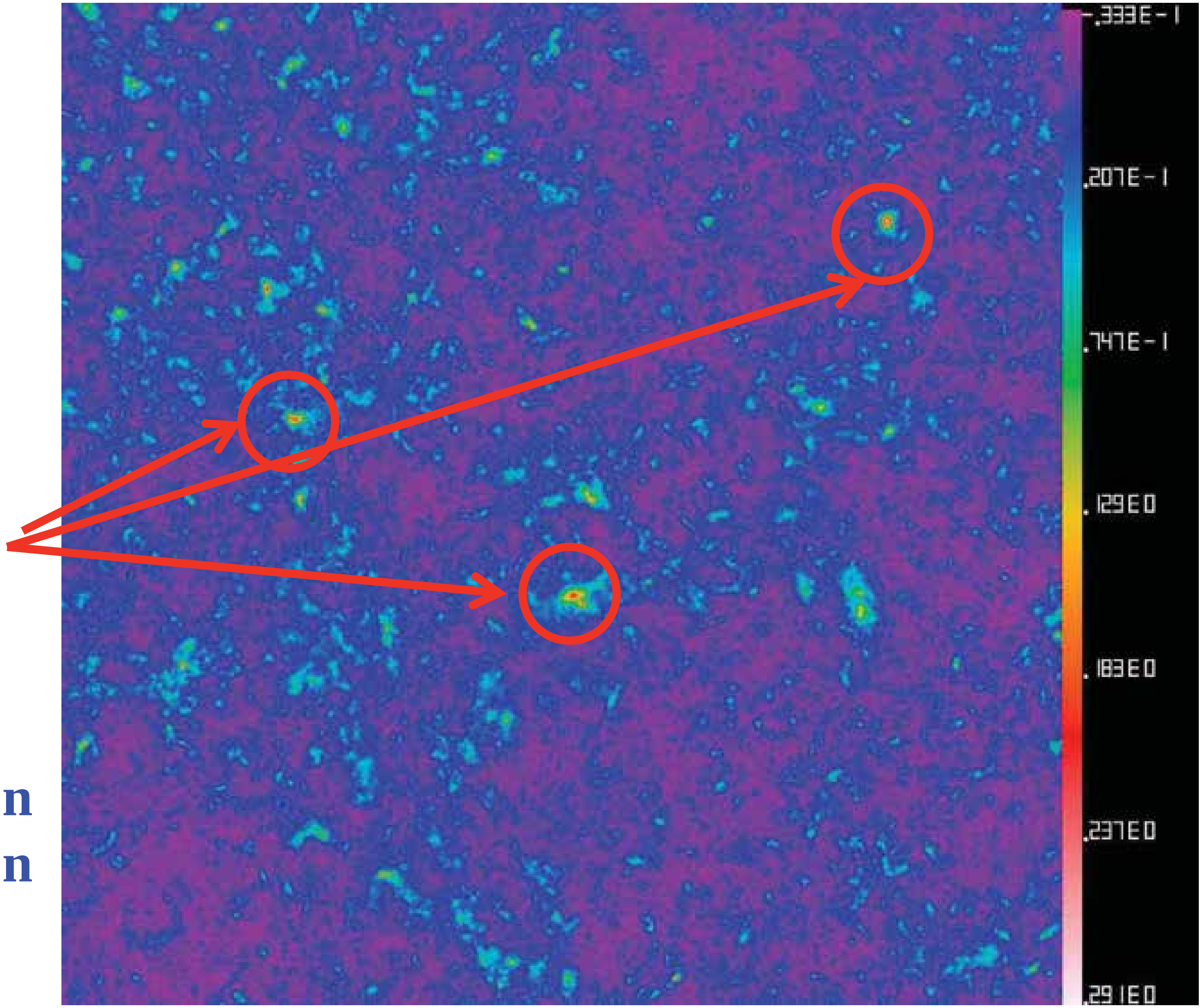
- **In GR, with the Poisson equation,**

$$\boxed{\kappa(\hat{n}) = \int_0^{z_s} \delta(\chi(z_L), \hat{n})W(z_L, z_s)dz_L}$$

$$W(z_s, z_L) = \frac{3}{2}\Omega_m(1+z_L)\frac{\chi_L}{c/H_0}(1-\frac{\chi_L}{\chi_s})\frac{H_0}{H(z_L)}$$

Map of lensing convergence

Peak analysis



2pt correlation
3pt correlation

Fig. 1.— An initial noise-free κ map in the N-body simulation of a Ω_m =0.3 ΛCDM cosmology with a map width of 3.02 degrees and 2048^2 pixels, and the scale is in units of κ.

Zhang, Tongjie + 2003

The Limber approximation

The 2D field y is the 3D field delta projected along the line of sight

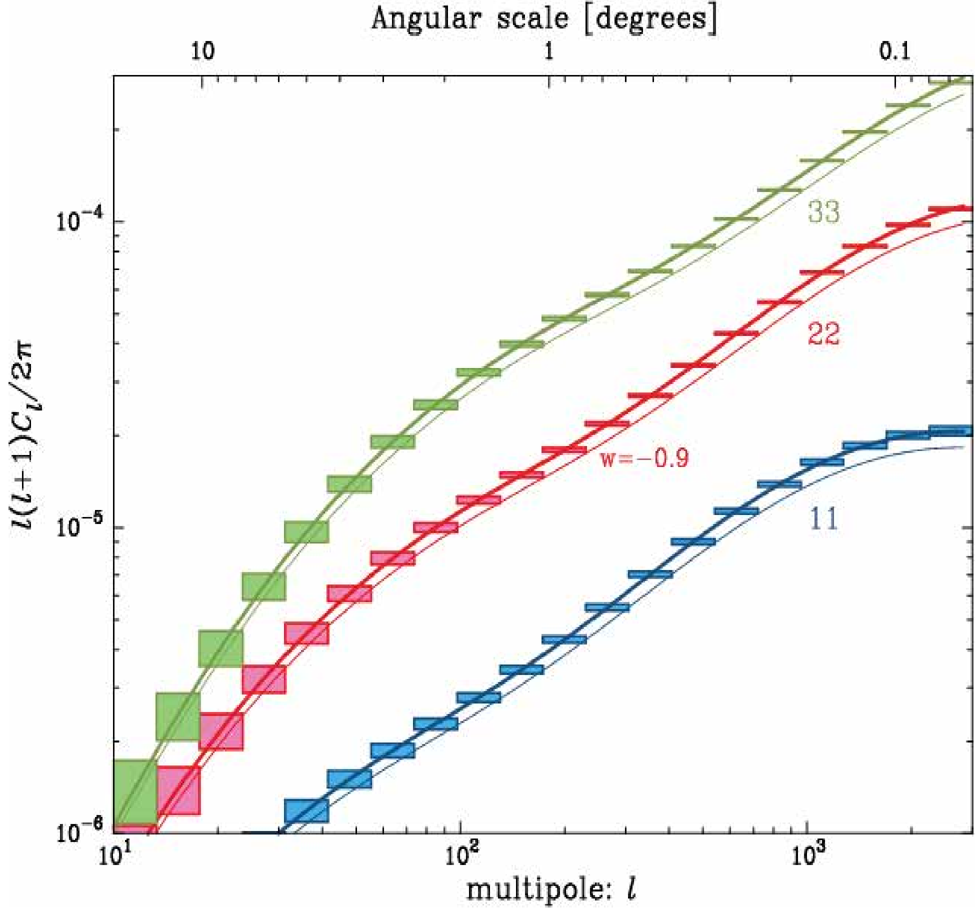
$$y(\hat{n}) = \int \delta(\chi, \hat{n}) W(\chi) d\chi$$

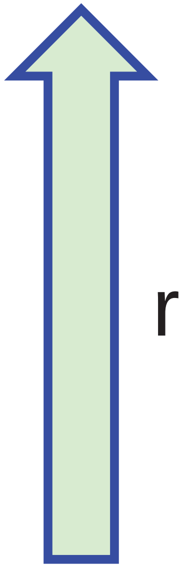
- **The power spectrum of y is the power spectrum of delta projected along the line of sight.**
- **When the correlation length of delta is much smaller than the projection length of y,**

$$\boxed{\frac{\ell^2 C(\ell)}{2\pi} = \frac{\pi}{\ell} \int \Delta_\delta^2 \left(k = \frac{\ell}{\chi}, z \right) W^2(\chi) \chi d\chi}$$

$$\chi \equiv \chi(z) : \ \text{distance to } z.$$

The weak lensing power spectrum

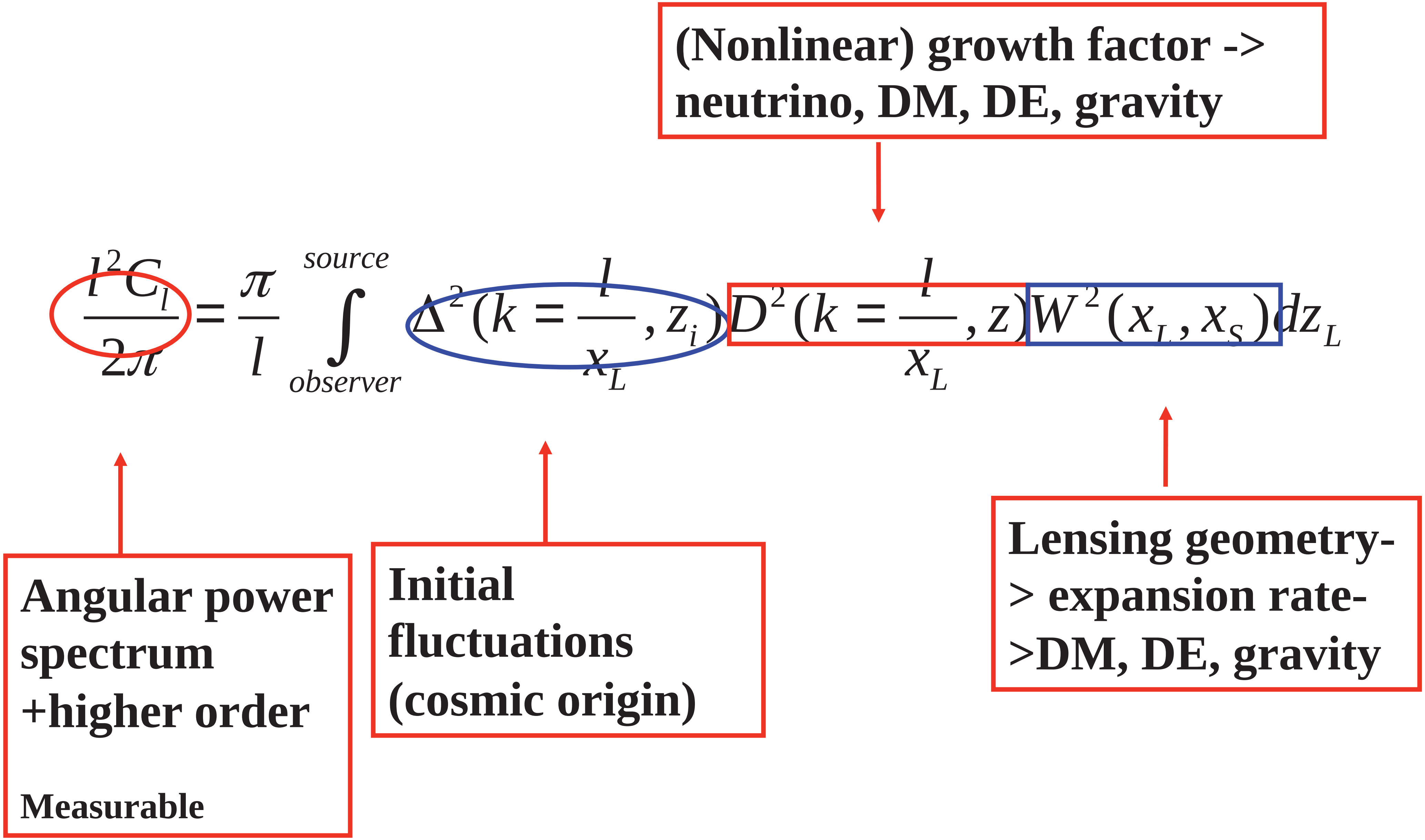




redshift

Figure 14.3: The lensing power spectra constructed from galaxies split into three broad redshift bins: $z < 0.7, 0.7 < z < 1.2$, and $1.2 < z < 3$. The solid curves are predictions for the fiducial ΛCDM model and include nonlinear evolution. The boxes show the expected measurement error due to the sample variance and intrinsic ellipticity errors (see text for details). The thin curves are the predictions for a dark energy model with $w = -0.9$. Clearly such a model can be distinguished at very high significance using information from all bins in ℓ and z. Note that many more redshift bins are expected from LSST than shown here, leading to over a hundred measured auto- and cross-power spectra.

Weak lensing to probe cosmology



(Nonlinear) growth factor -> neutrino, DM, DE, gravity

$$\frac{l^2 C_l}{2\pi} = \frac{\pi}{l} \int_{observer}^{source} \Delta^2 (k = \frac{l}{x_L}, z_i) \boxed{D^2 (k = \frac{l}{x_L}, z)} \boxed{W^2 (x_L, x_S)} dz_L$$

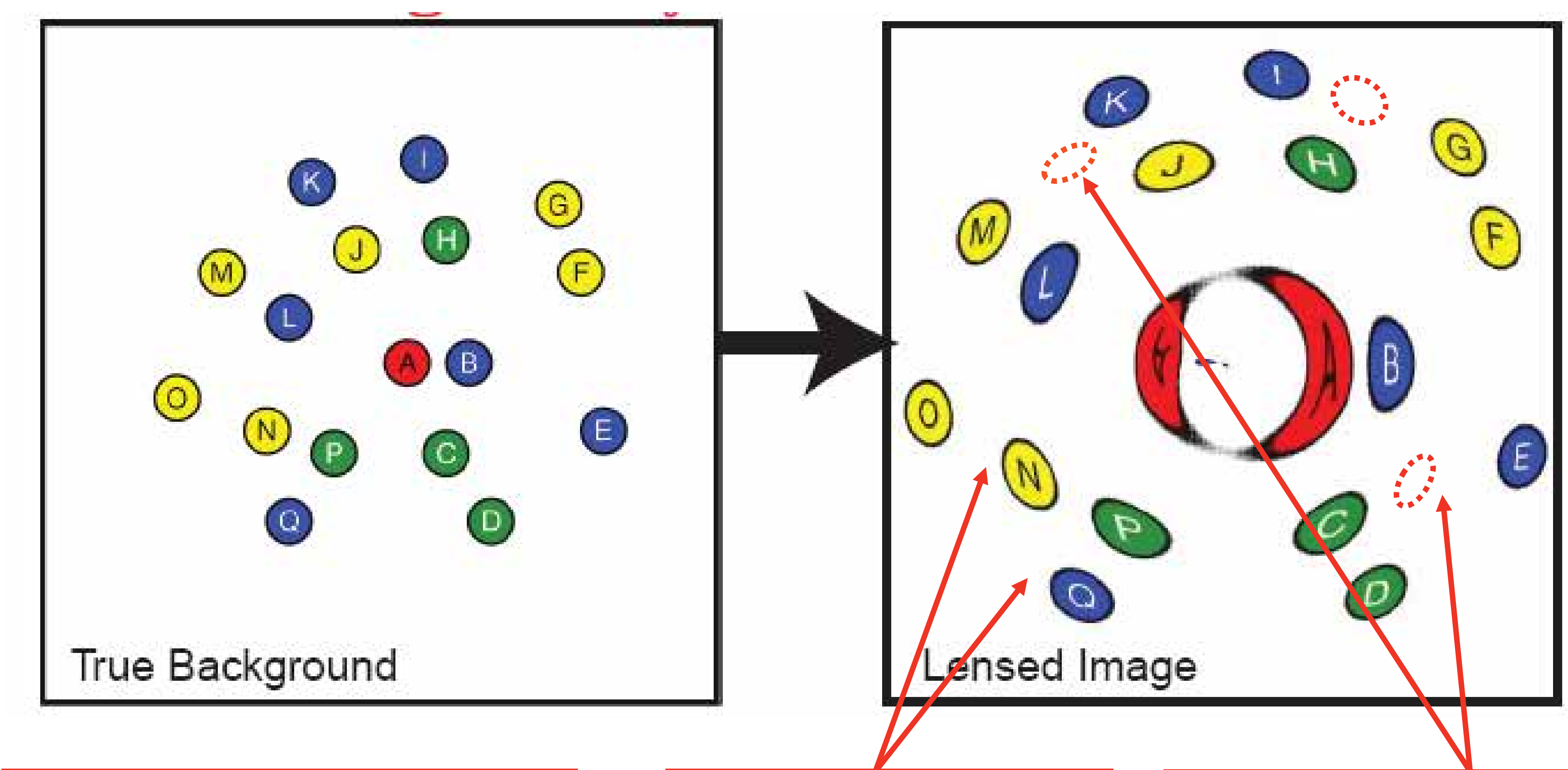
Angular power spectrum +higher order

Measurable

Initial fluctuations (cosmic origin)

Lensing geometry-> expansion rate->DM, DE, gravity

Observable consequences



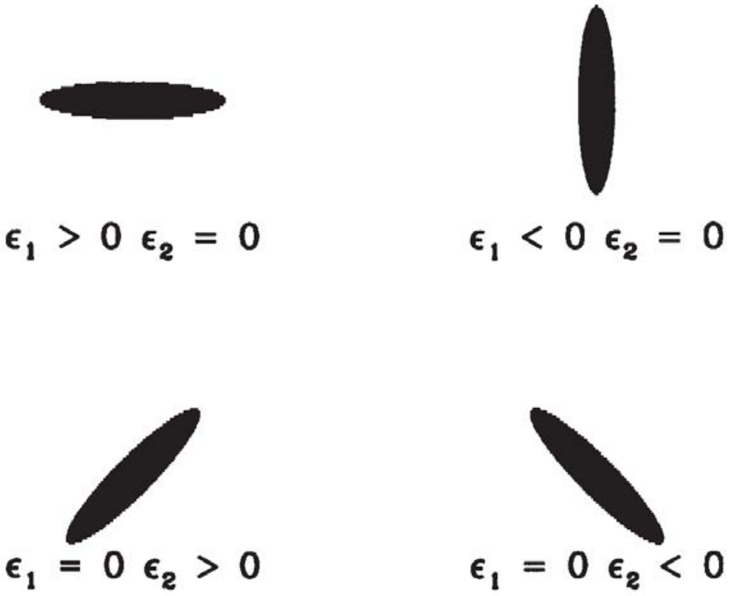
CMB lensing (anisotropies and non-Gaussianities in CMB, ACT/SPT/Planck, 2012-)

Cosmic shear (shape distortion, 2000-)

Cosmic magnification (number distortion, through galaxy-galaxy lensing, 2005−)

Weak lensing measurements: cosmic shear

$$\epsilon_1 \equiv \frac{q_{xx} - q_{yy}}{q_{xx} + q_{yy}}$$



$$\epsilon_2 \equiv \frac{2q_{xy}}{q_{xx} + q_{yy}}.$$

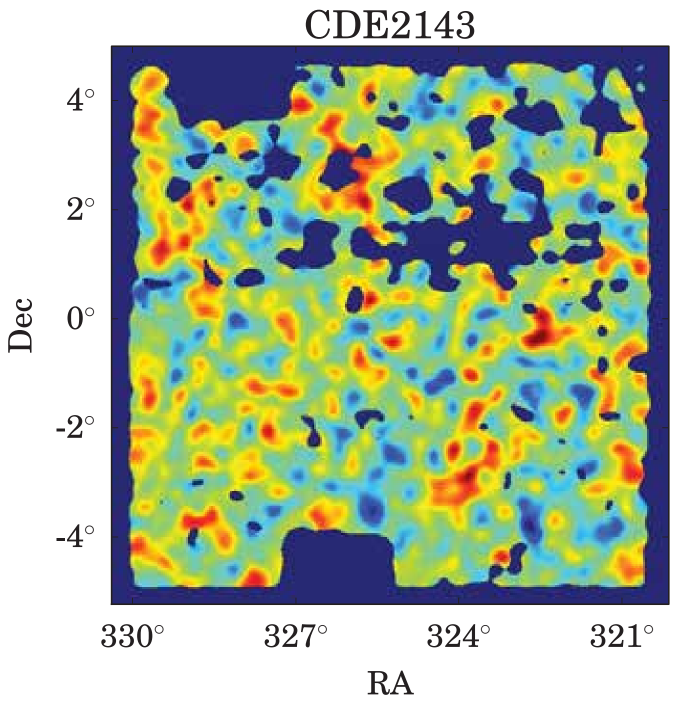
$$\boxed{\epsilon_i \rightarrow \epsilon_i + 2\gamma_i}$$

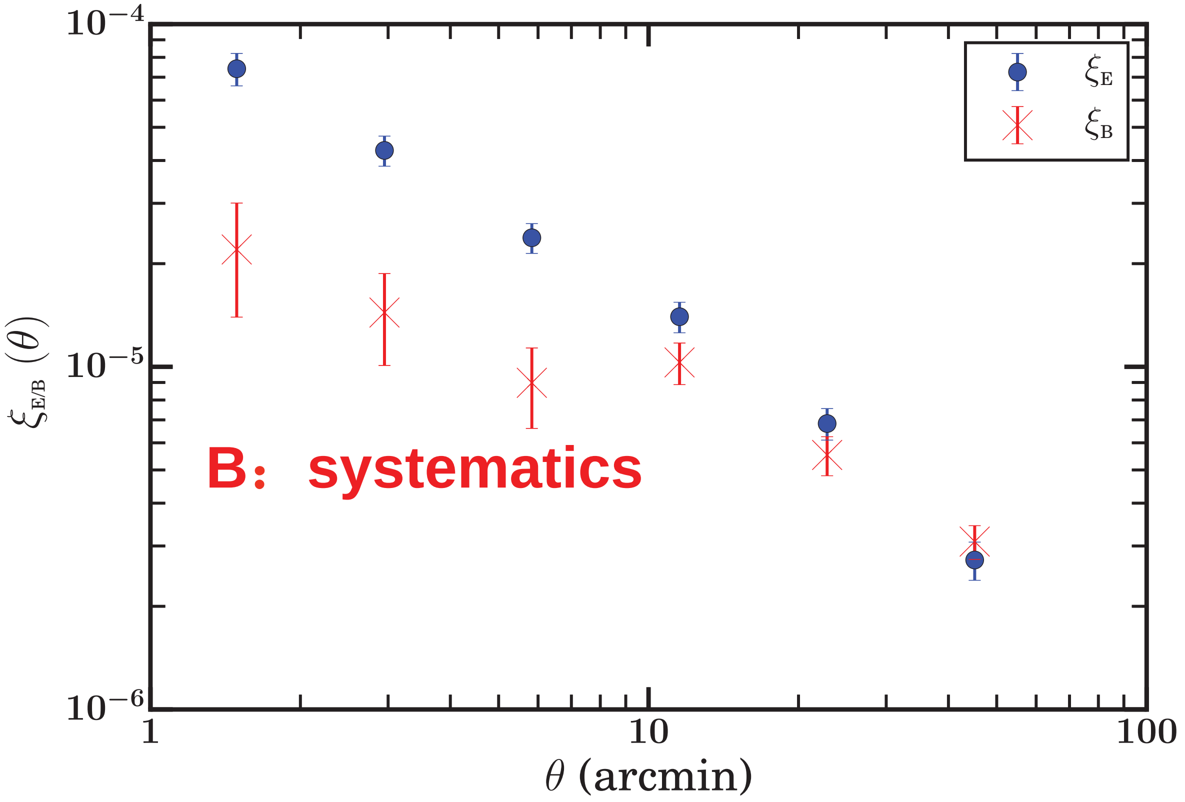
- First detections in 2000 by 4 group
- **G2. CFHTLenS, SDSS, COSMOS, etc.**
- **G3. KiDS, RCSLenS, DES,HSC**
- **G4. LSST, Euclid, WFIRST, etc.**

Weak lensing measurements: cosmic shear

RCSLenS: The Red Cluster Sequence Lensing Survey*

H. Hildebrandt,[1]† A. Choi,[2] C. Heymans,[2] C. Blake,[3] T. Erben,[1] L. Miller,[4]

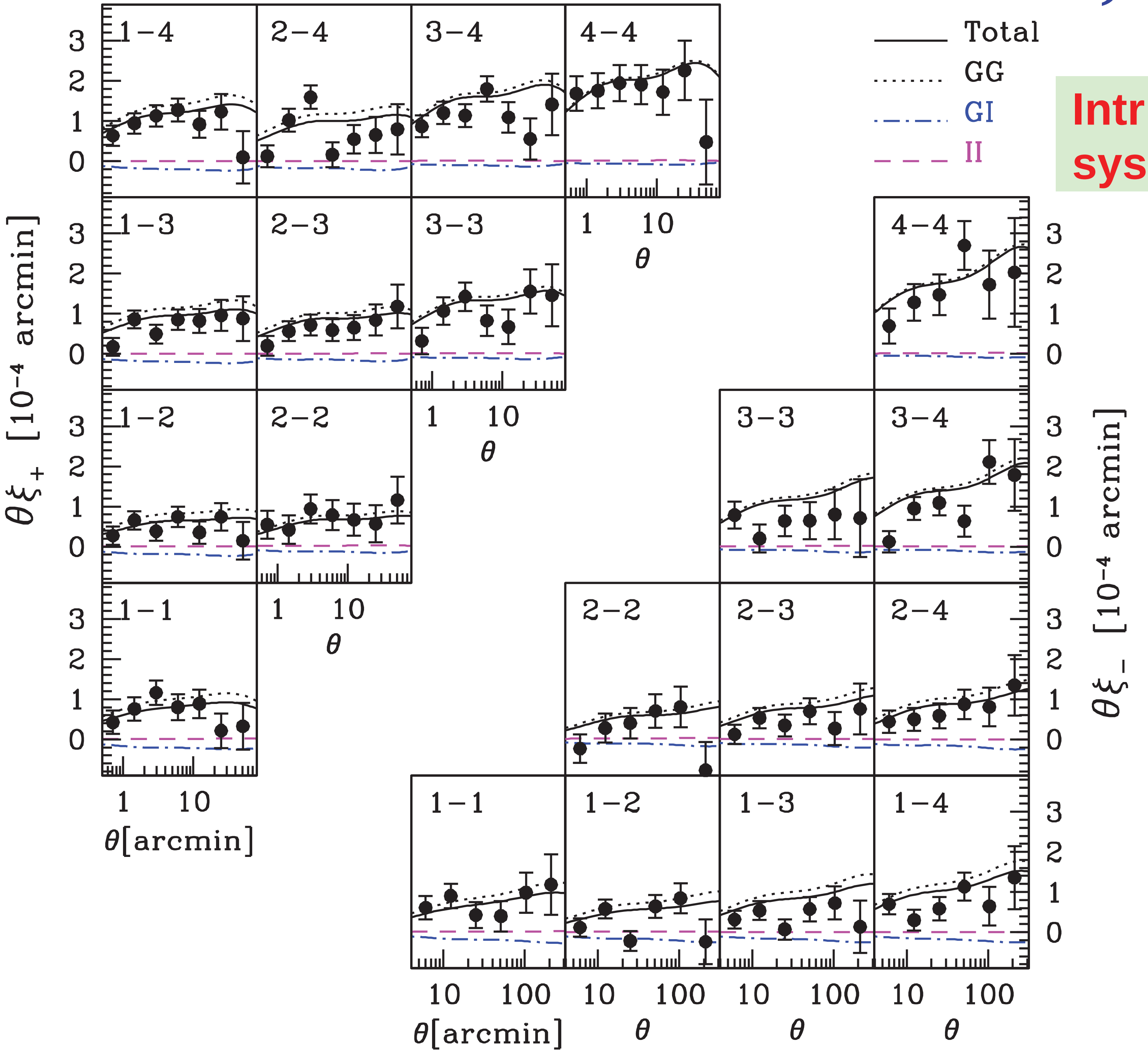




B：systematics

Figure 14. Two-point correlation functions ξ_E (blue circles) and ξ_B (red crosses) for the full RCSLenS shear catalogue.

KiDS-450. Hildebrandt +, 2016



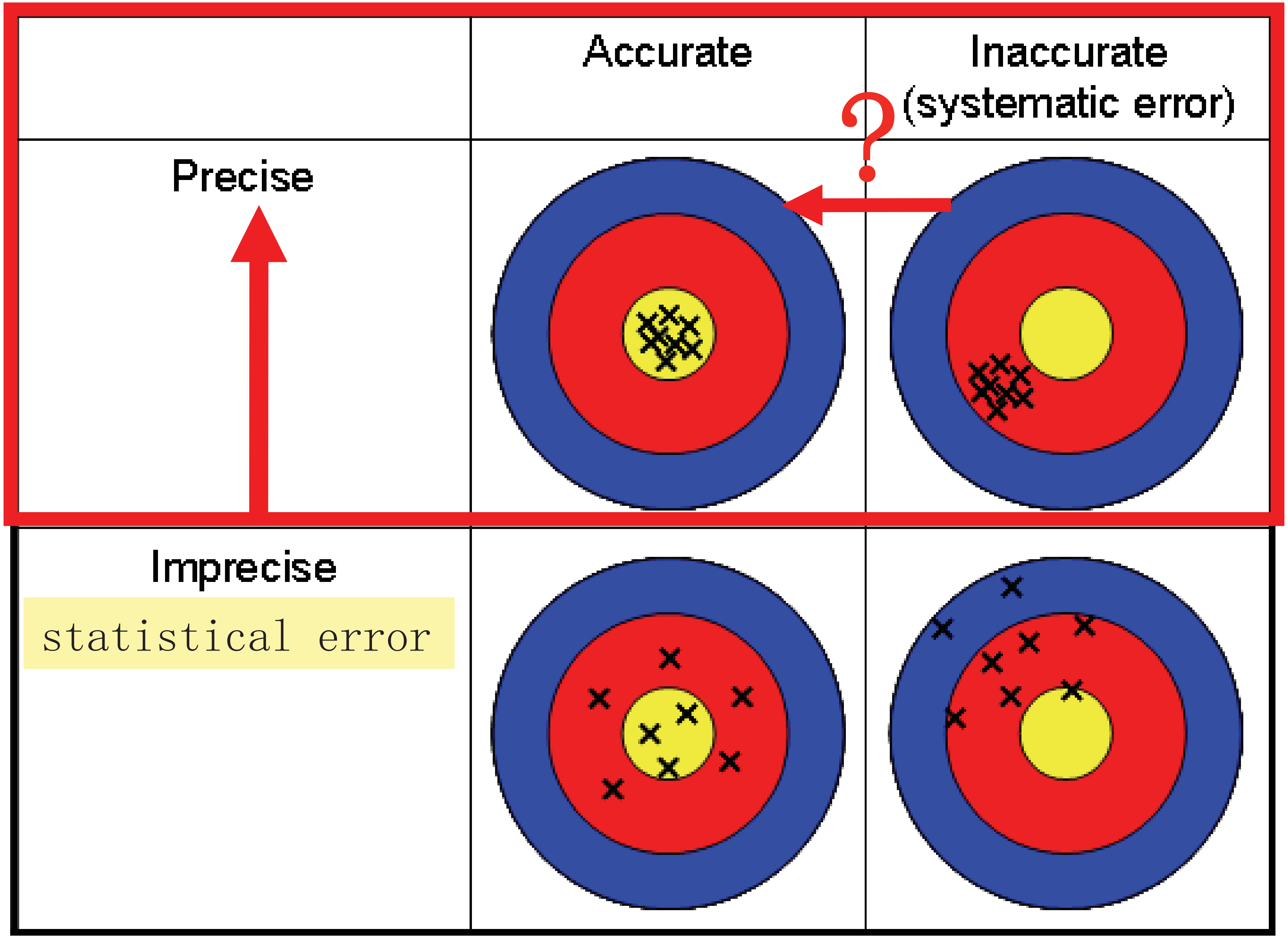
Intrinsic alignment systematic

More problems in weak lensing cosmology

(=better opportunities for young researchers)

- How to measure it accurately?
- How to model it accurately?
- Aim: 1% or better!

Precise ≠ accurate!



	Accurate	Inaccurate (systematic error)
Precise		
Imprecise statistical error		

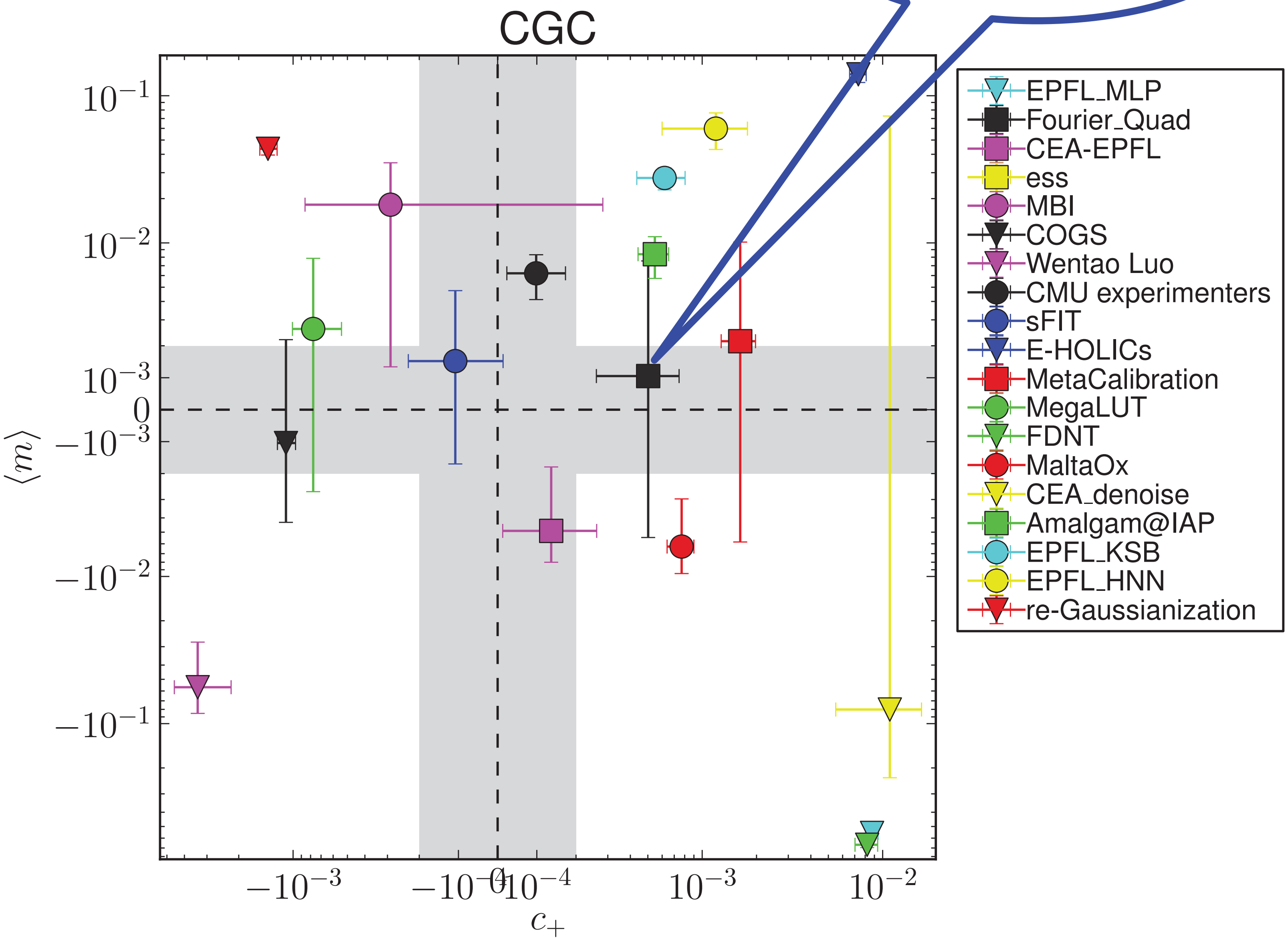
Challenges to cosmic shear cosmology

- **Systematic errors in observational measurement**
  - **Galaxy shape measurement (e.g. GREAT3 test)**
  - **Galaxy intrinsic alignment (e.g. Troxel et al. 2015 review)**
  - **Photo-z error (in particular outliers)**
  - **Many more (e.g. LSST science book)**

- **Systematic errors in theoretical modeling**
  - **Baryon effects（non-gravitational processes such as gas cooling, SN and AGN feedback, affect the matter clustering)**
  - **Nonlinear and non-Gaussian evolution**
  - **Second order corrections: source-lens clustering, Born deviation, lens-lens coupling, reduced shear, etc.**
  - **Many more**

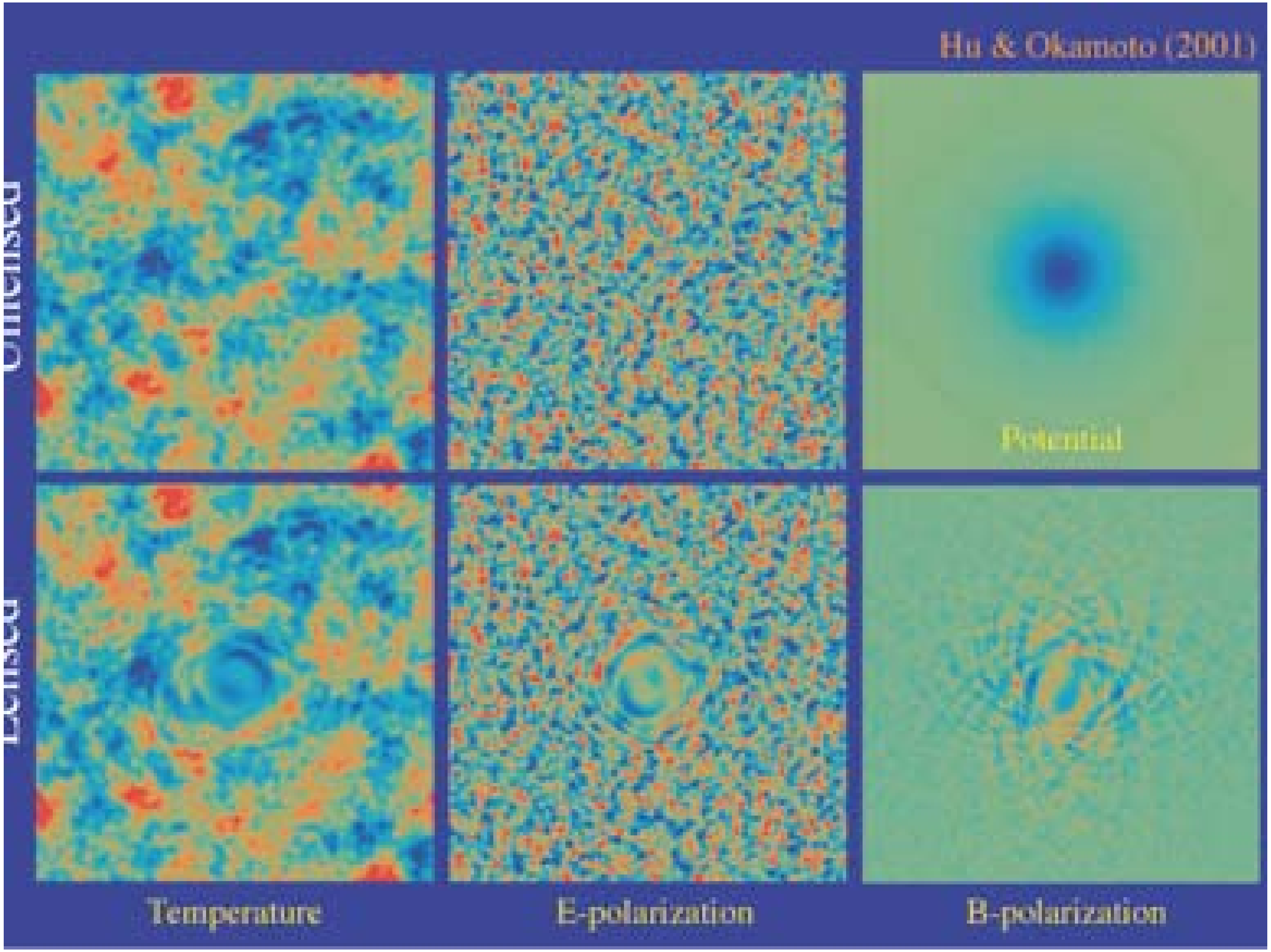
e.g. GREAT3 test results for constant

$$g_i^{\mathrm{obs}} - g_i^{\mathrm{true}} = m_i g_i^{\mathrm{true}} + c_i$$



张骏（交大）
的方法

CMB lensing

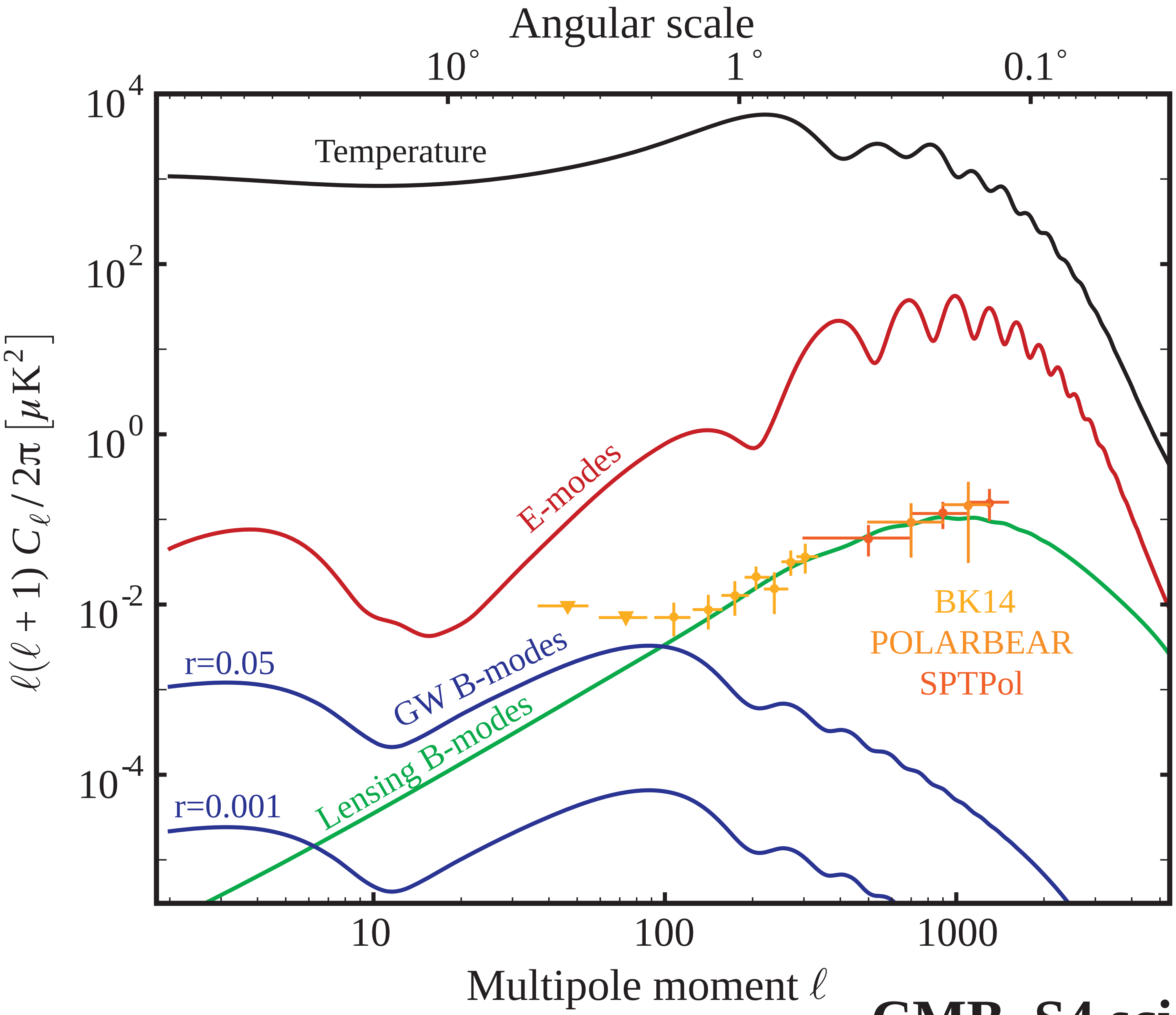

Hu & Okamoto (2001)

$$T^L(\theta^L) = T(\theta$$

$$E(\theta^L) \neq E(\theta$$

$$B(\theta^L) \neq B(\theta$$

Lensing changes CMB E-mode into B-mode
Contamination to gravitational wave B-mode



CMB−S4 science book

CMB lensing reconstruction

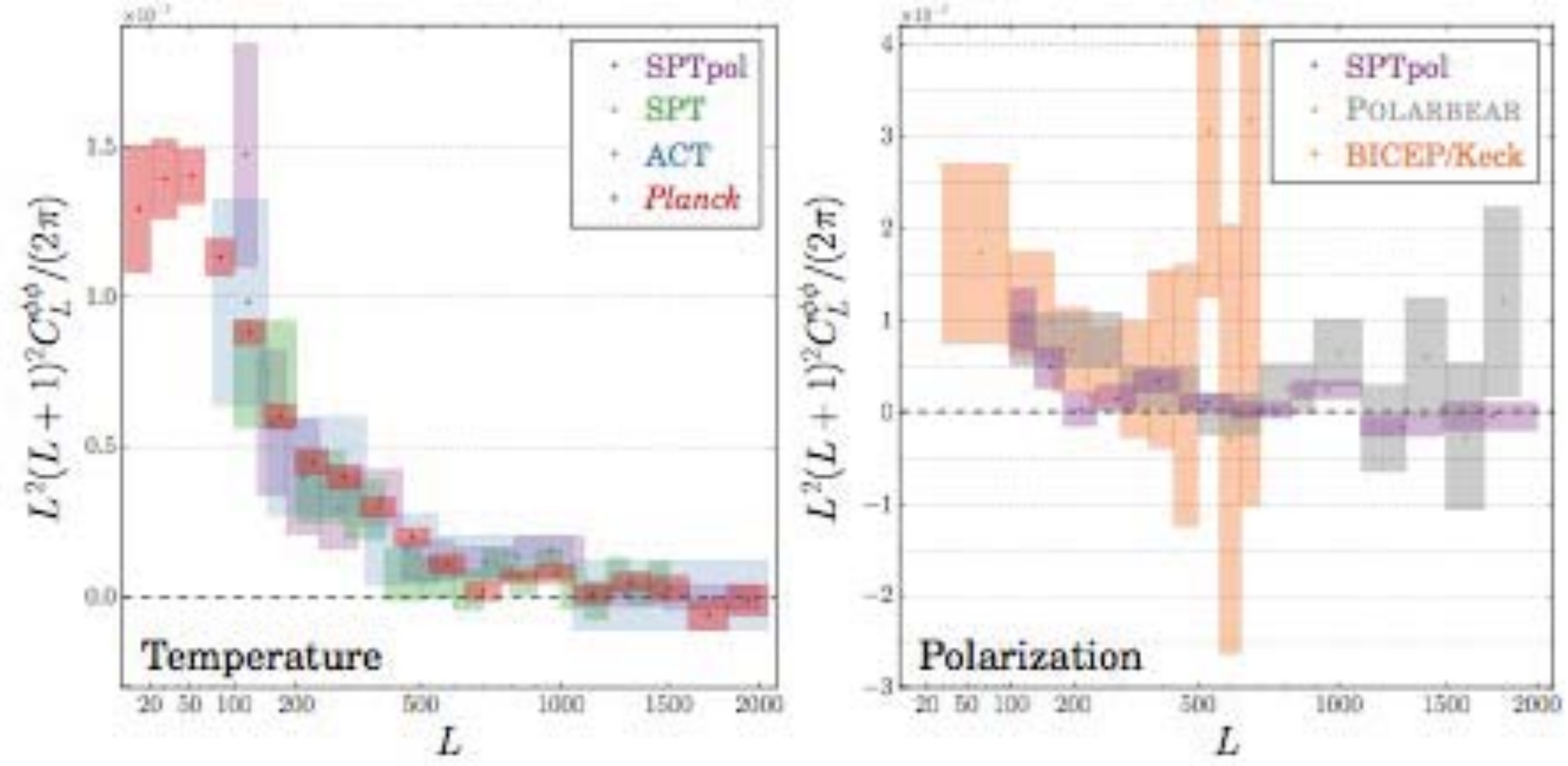
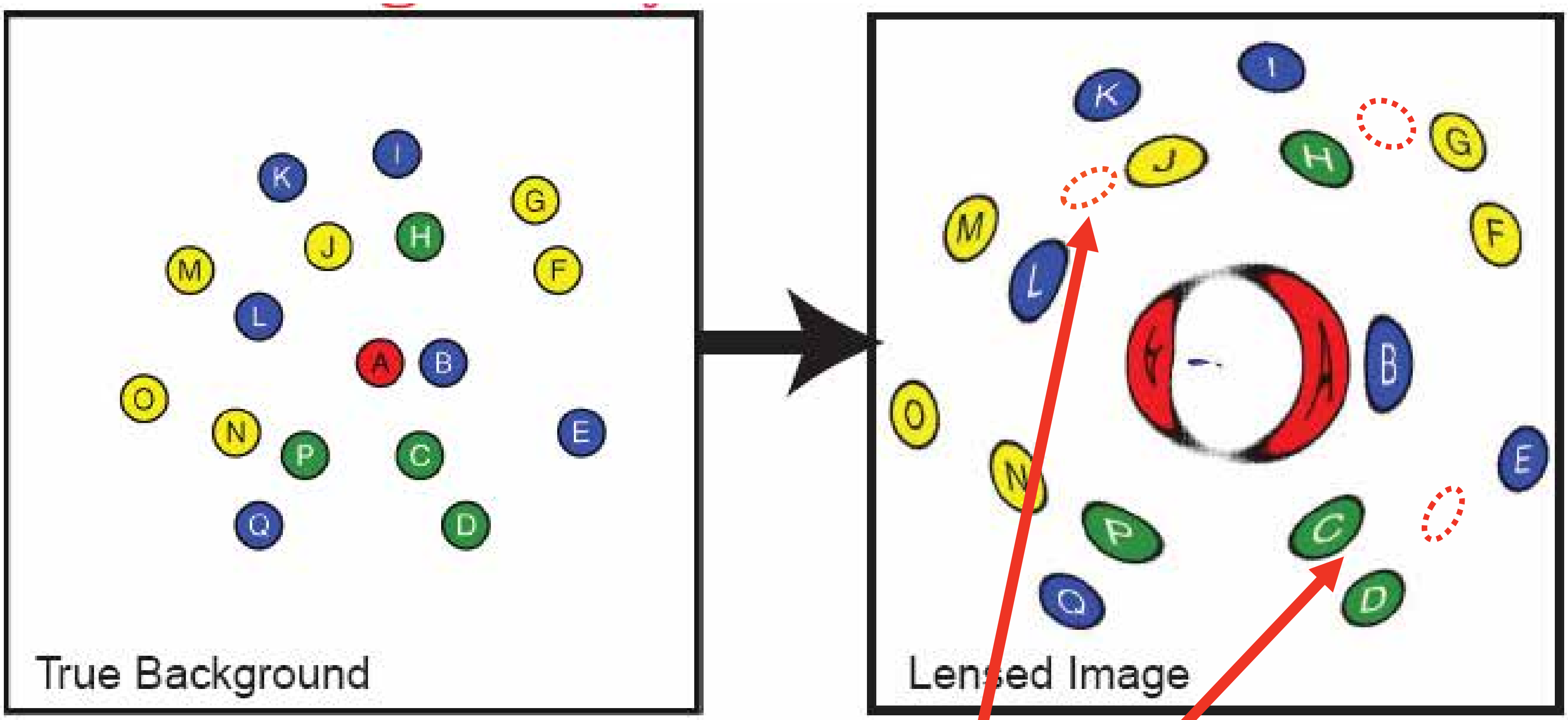


Figure 48. *Compendium of lensing power spectrum measurements since first measurements in 2011.*

Weak lensing: cosmic Magnification



$$\boxed{\begin{aligned} N^L(>F) &= N(>F/\mu)/\mu \\ &\simeq N(>F)(1+2(\alpha-1)\kappa) \end{aligned}}$$

$$\boxed{\alpha = -d\ln N(>F)/d\ln F}$$

Cosmic magnification vs. cosmic shear

Observable signal noise

$\delta_g \gg \kappa$

$$\delta_g^L = g\kappa + \delta_g$$

intrinsic clustering

$$\gamma^o = \gamma + I$$

intrinsic alignment

$$\boxed{g \equiv 2(\alpha - 1)}$$

Cosmic magnification statistics

$$\delta_g^L = 2(\alpha - 1)\kappa + \delta_g$$

$\delta_g \gg \kappa$

Cross correlation of foreground and background galaxies

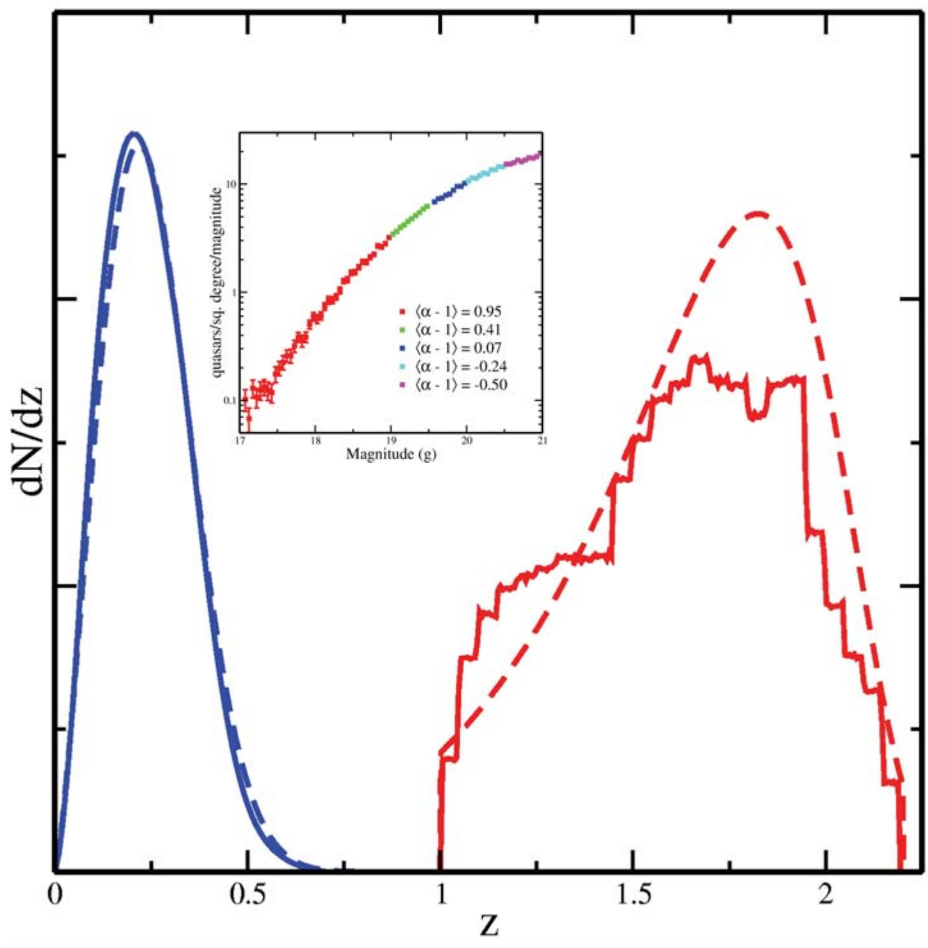
$$\begin{aligned} w(\theta) &= \langle \delta^f(\mathbf{x})\delta^b(\mathbf{x}+\theta)\rangle \\ &\simeq \langle \delta_g^f 2(\alpha - 1)\kappa^b\rangle \end{aligned}$$

SDSS galaxy-quasar cross correlation

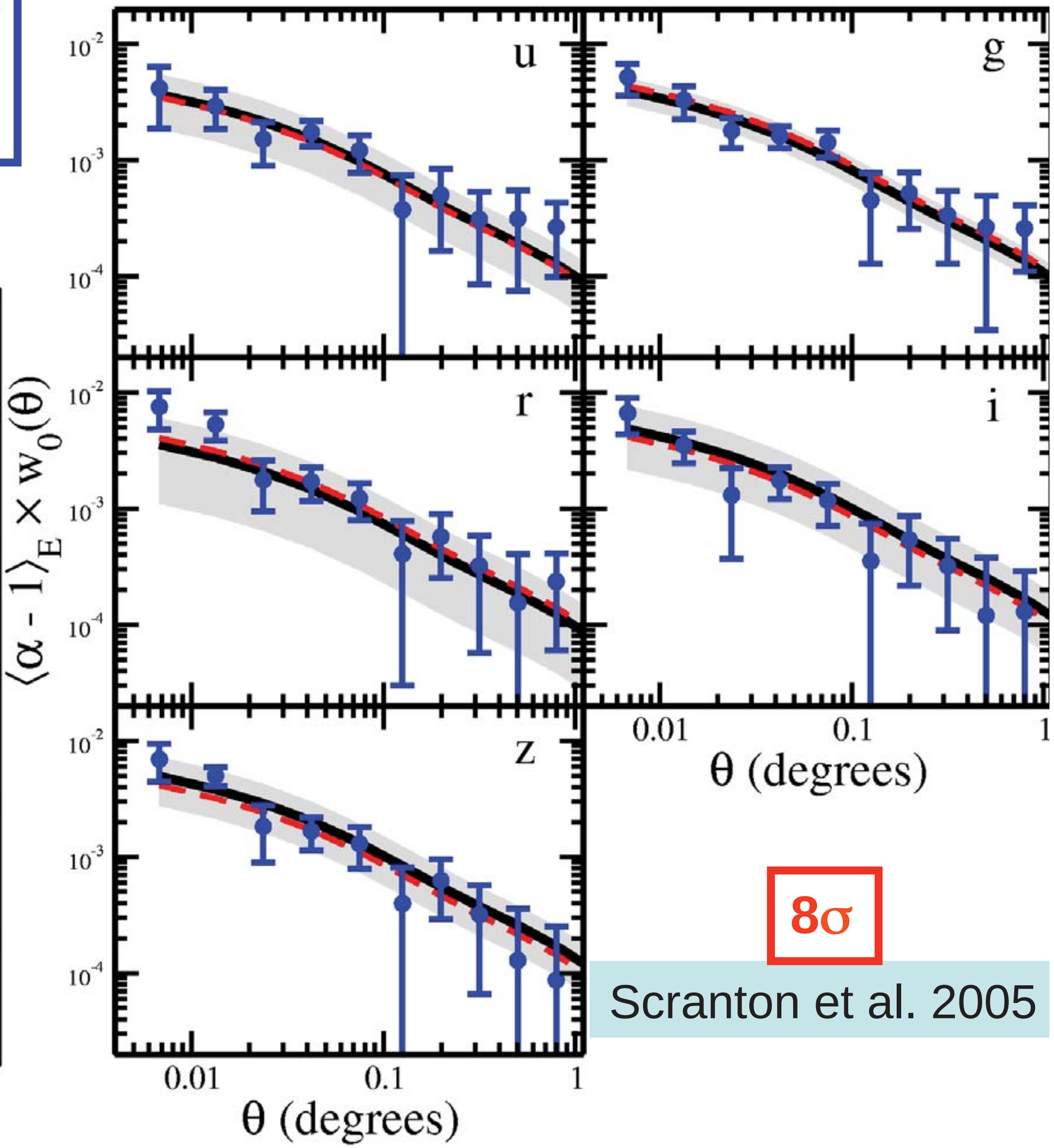
$$w(\theta) = \langle \delta^f(\mathbf{x})\delta^b(\mathbf{x}+\theta)\rangle$$
$$\simeq \langle \delta_g^f 2(\alpha-1)\kappa^b \rangle$$

1.3E7 galaxies

2E4 quasars



Weighting quasars by their α-1 (Menard & Bartelmann 2002)



8σ

Scranton et al. 2005

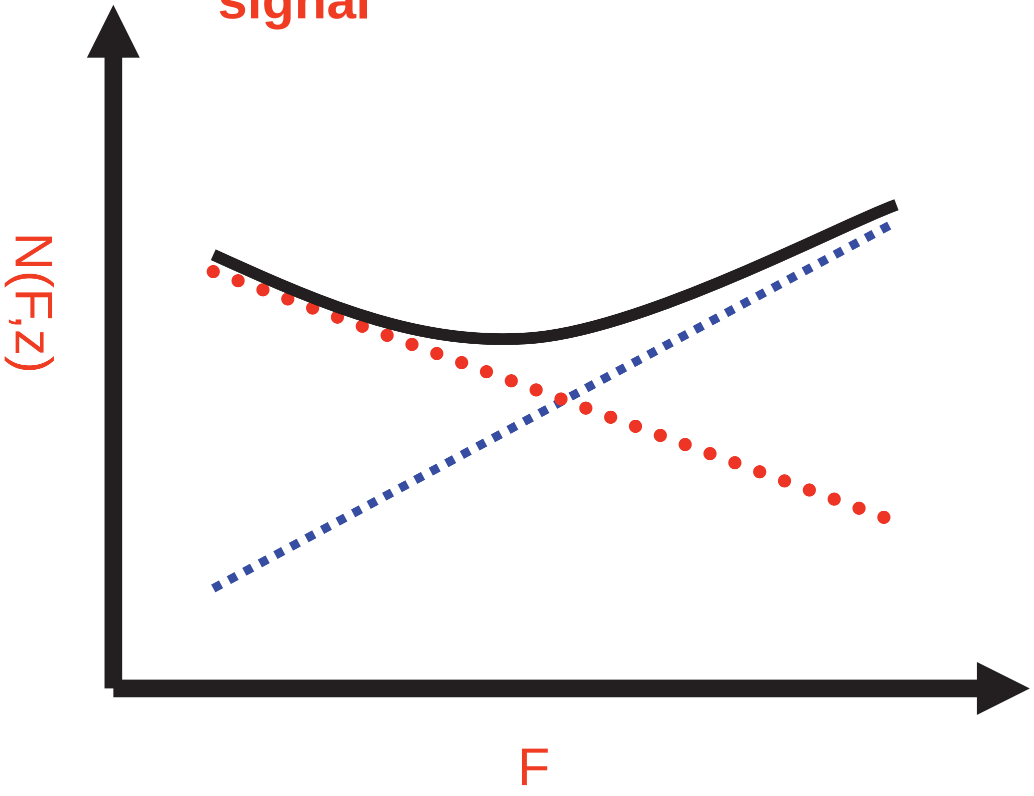
Seperate the cosmic magnification from the intrinsic clustering

$$\delta_g^L = 2(\alpha - 1)\kappa + \delta_g$$

signal

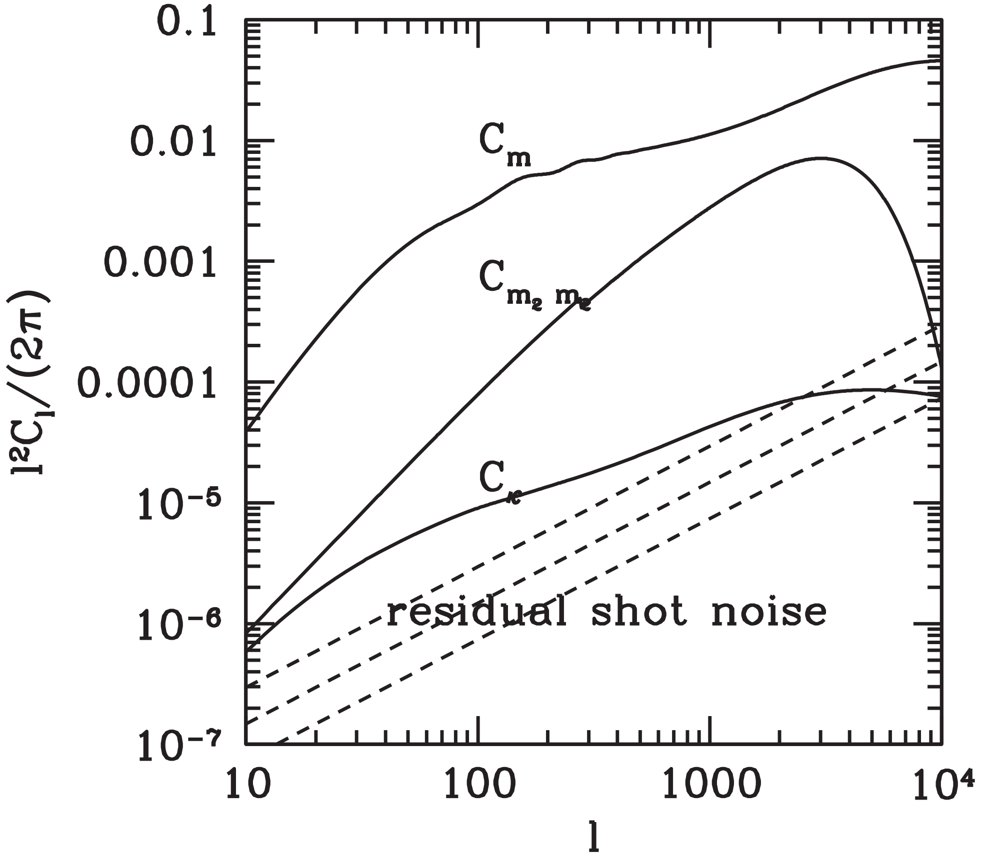
noise

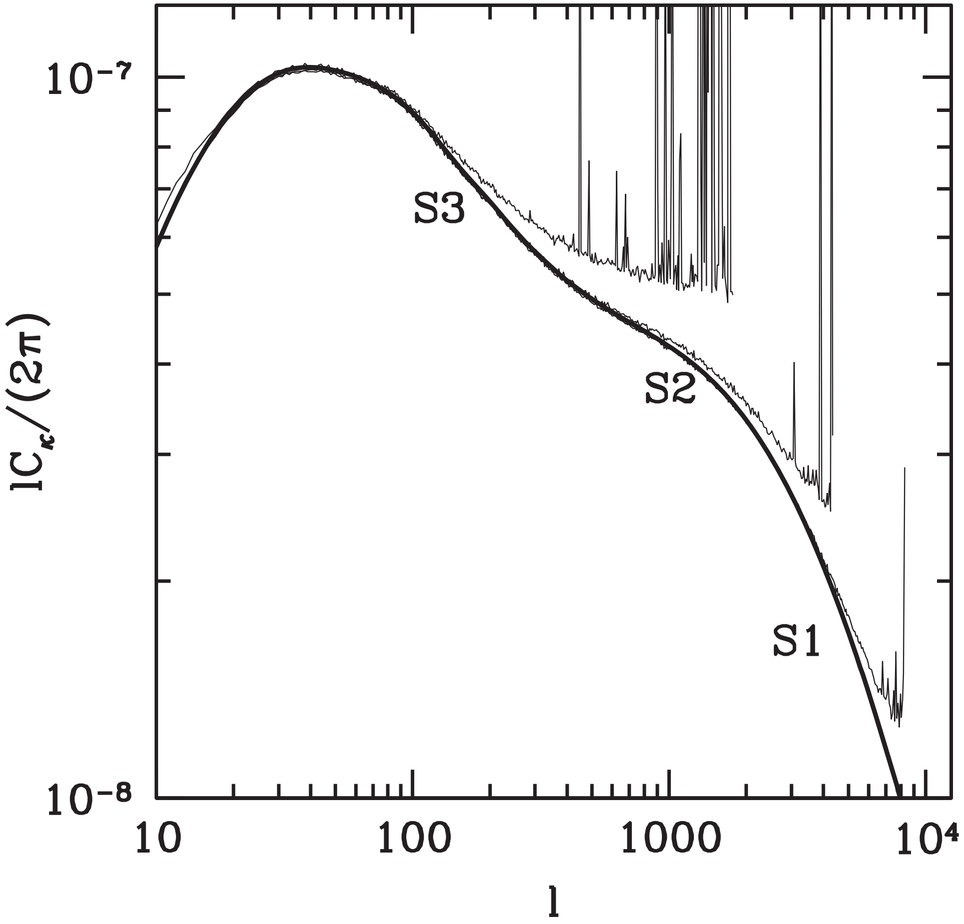
- δ_g and cosmic magnification have different flux dependences. The key to measure cosmic magnification by counting galaxies.
- ZPJ & Pen, 2005; Yang & ZPJ, 2011; Yang+, 2015; **Yang+2017 (The ABS method)**



ZPJ & Pen 2005, PRL

ABS eliminates O(1000) contaminations, recovers the input lensing power spectrum, without assumptions of contaminations





Yang+2017 (The ABS method)