The Theory and Practice of Cosmological Perturbations

Part IV – Inflationary Massive Fields

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Why messing up inflation with massive fields?

1. They exist (SM in dS, and additions form all UV completions)

2. They couple to inflaton (at least by G_N, also, reheating)

3. Distinguished signatures from non-G

4. The cosmos as a particle collider

5. Probing the evolution history of the primordial universe

Source of massive fields

- The inverse problem of η-problem
- Standard model uplifting
- Terms not constrained by symmetry
- Symmetry breaking
- String phenomenology

Source of massive fields

The η-problem: Inflation is UV sensitive.

A dim-6 operator $\frac{V(\phi)}{M_p^2}\phi^2$ gives a mass to inflaton: $\Delta m^2 \sim H^2$.

Why the inflaton is light?

The inverse problem: Given the inflaton be light

(tuning, shift symmetry, etc).

We do not want to tune again for additional "light" fields.

From $\frac{V(\phi)}{M_p^2}\sigma^2$, other "light" fields naturally have $m^2 \sim H^2$.

Source of massive fields

Standard Model uplifting:

Very roughly:

$T_{dS} \sim \frac{H}{2\pi}$ (Gibbons-Hawking temperature of de Sitter)

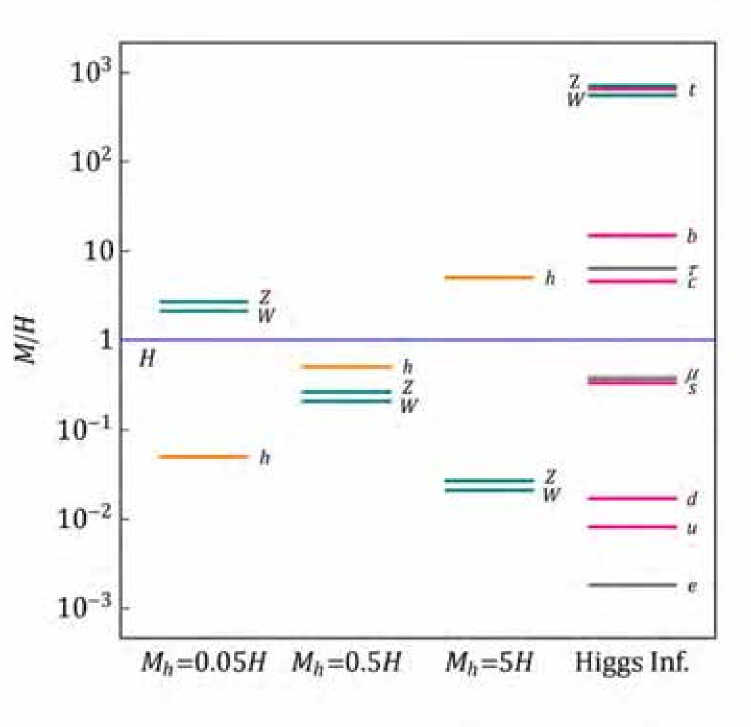
Higgs field: $\lambda\, h^4 \sim \lambda\, \langle h^2 \rangle h^2 \sim O\left(\sqrt{\lambda} H^2\right) h^2$ (Note: $\langle h^2 \rangle \sim \frac{H^2}{\sqrt{\lambda}}$)

Gauge fields (W, Z): $g h^2 A^2 \sim O\left(\frac{g H^2}{\sqrt{\lambda}}\right) A^2$

More details later (See also 1610.06597 & 1612.08122, Chen, YW & Xianyu)

Source of massive fields

Standard Model uplifting:



More details later (See also 1610.06597 & 1612.08122, Chen, YW & Xianyu)

Source of massive fields

- Terms not constrained by symmetry:

  $\xi \sigma^2 R \sim \xi H^2 R$

- Symmetry breaking

  e.g. supersymmetry breaking, at least Hubble scale

- String phenomenology: e.g. KK modes

  They may enter at equal or higher than H

Inflaton-isocurvaton coupling

The massive fields has a “decaying” amplitude.

$$\ddot{v}_k + 3H\dot{v}_k + \frac{k^2}{a^2}v_k + m^2 v_k = 0$$

$$v_{\mathbf{k}} = -ie^{i(\nu+\frac{1}{2})\frac{\pi}{2}}\frac{\sqrt{\pi}}{2}H(-\tau)^{3/2}H_\nu^{(1)}(-k\tau)$$

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LogLinearPlot[
 Evaluate[Re[e^(-i π ν/2) * Sqrt[π]/2 (x)^(3/2) HankelH1[ν, x]] /.
   {{ν → 3 / 2}, {ν → 1 / 2}, {ν → 0}}], {x, 0.001, 10},
 PlotLegends → {"ν=3/2 (m=0)", "ν=1/2 (m=√2 H)",
   "ν=0 (m=3H/2)"}]
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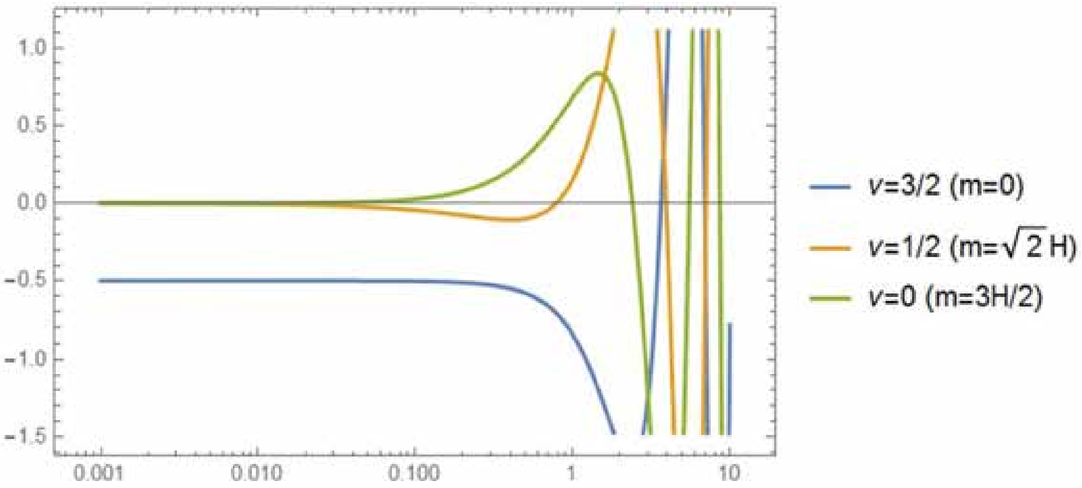


Inflaton-isocurvaton coupling

The massive fields has a “decaying” amplitude.

So to show up in observations, they need to couple to observables:

Inflaton, gravitational waves, or isocurvature

Coupling to inflaton is the most important:

- Inflaton fluctuation is the only confirmed one

  GW should exist but is smaller; Isocurvature may or may not exist

- Inflaton decays (reheating), so should exist non-trivial couplings

- Existence of a UV-sensitive dim-5 operator

Inflaton-isocurvaton coupling

Possible inflaton-isocurvaton couplings?

EFT: Lowest dimension with inflaton shift symmetry: $\frac{1}{\Lambda}(\partial\phi)^2\sigma$

Features of this term:

- Even if $\Lambda \sim M_p$, this term still gives quite "relevant" 2pt-vertex:

$$\dot{\phi} \sim M_p\sqrt{\dot{H}} \sim \sqrt{\epsilon}\, M_p H \quad \Rightarrow \quad \frac{1}{M_p}\, \dot{\phi}\delta\dot{\phi}\sigma \sim \sqrt{\epsilon}\, H\, \delta\dot{\phi}\sigma$$

- Physical meaning: turning of trajectory.

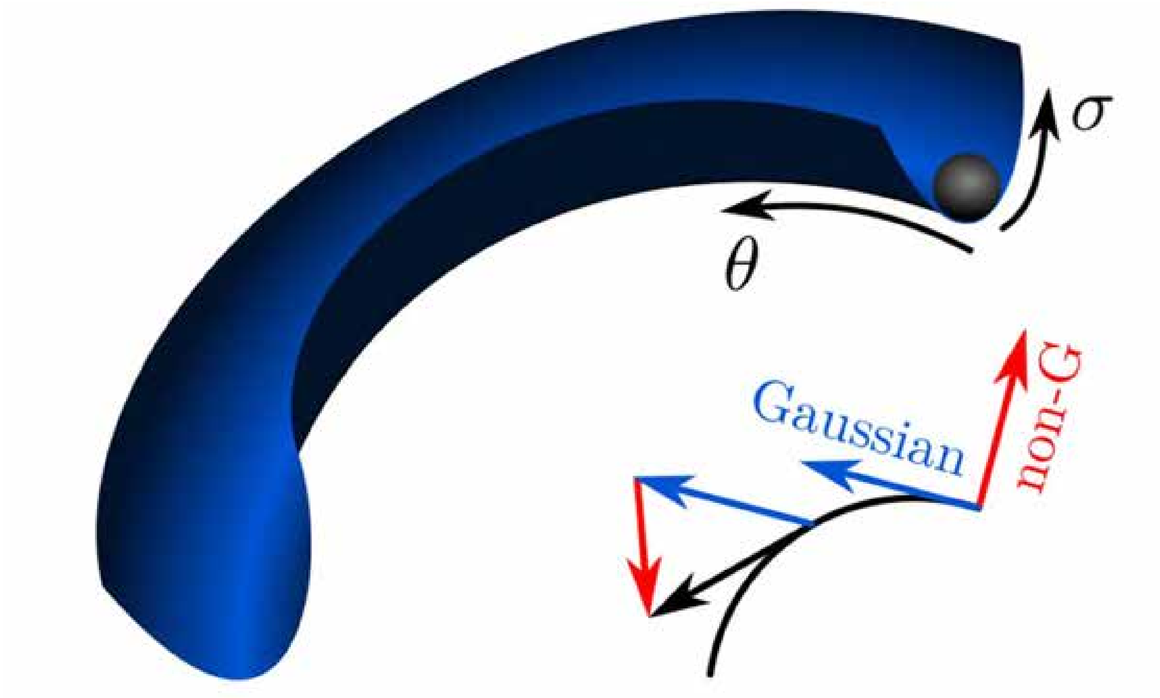
- Have associated 3pt-vertex: $(\partial\delta\phi)^2\sigma$ (may or may not dominate)

- Not a term in Einstein gravity with minimal Lagrangian of σ

Correction to power spectrum

Focus on the shift symmetry (will return to non-shift symmetry later).

Consider turning trajectory example:



Correction to power spectrum

Focus on the shift symmetry case. Consider turning trajectory example:

$$S_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\left(\widetilde{\mathcal{R}}+\sigma\right)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)\right]$$

$$\Delta\mathcal{L}_2 = 2a^3\dot{\theta}_0 \times (\mathcal{R}\dot{\delta\theta}) \times \delta\sigma$$

$$g_2 \equiv \frac{\Delta\mathcal{L}_2}{\mathcal{L}_2} = \begin{cases} \dot{\theta}/H, & \text{if } m \sim H \\ \dot{\theta}/m, & \text{if } m \gg H \\ \text{cutoff needed} & \text{if } m \ll H \end{cases}$$

$$P_\zeta = \frac{H^4}{4\pi^2 R^2 \dot{\theta}_0^2}(1+\delta) \qquad \delta \sim g_2^2$$

Typically cannot distinguished from redefinition of BG parameters.

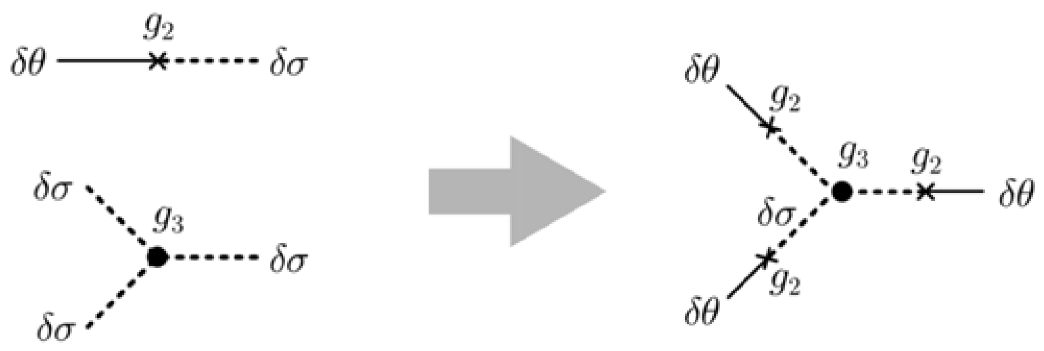
Non-G of QSFI

$$\Delta\mathcal{L}_3 = V'''\delta\sigma^3$$

$$g_3 \equiv \frac{\Delta\mathcal{L}_3}{\mathcal{L}_2} = \begin{cases} V'''/H, & \text{if } m \sim H \\ V'''H^2/m^3, & \text{if } m \gg H \\ \text{cutoff needed} & \text{if } m \ll H \end{cases}$$

c.f. The inflaton self-interaction:

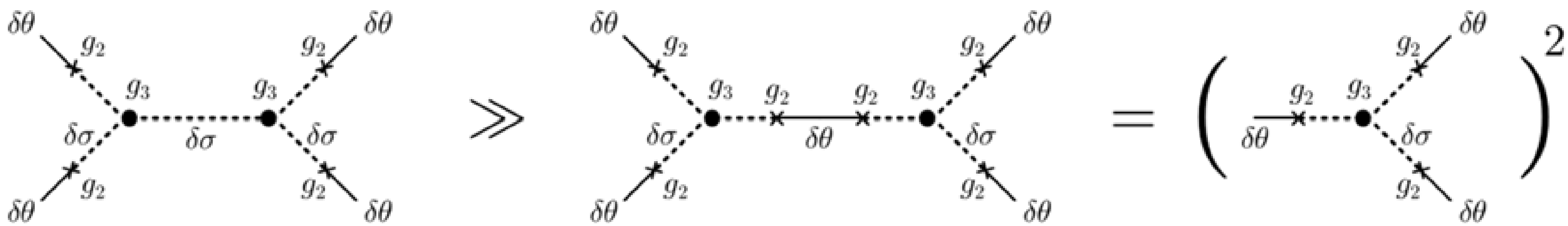
$$V'''_{\rm sr} \sim H\mathcal{O}(\epsilon^2)\sqrt{P_\zeta}$$



$$f_{NL} \sim \frac{g_3(g_2)^3}{\sqrt{P_\zeta}} \qquad g_{NL} \sim g_4(g_2)^4/P_\zeta\ , \qquad \tau_{NL} \sim (g_3)^2(g_2)^4/P_\zeta$$

Non-G of QSFI

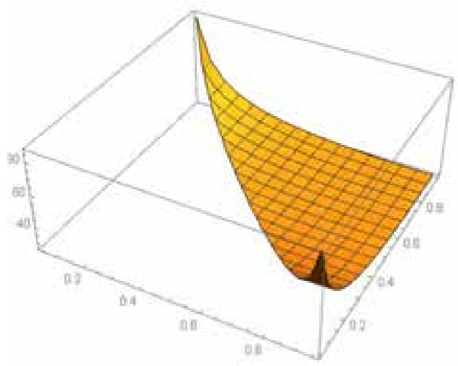
$\tau_{NL} \gg f_{NL}^2$

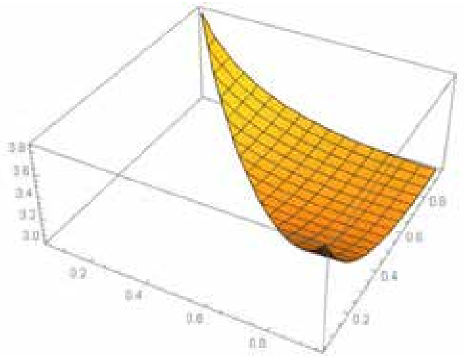


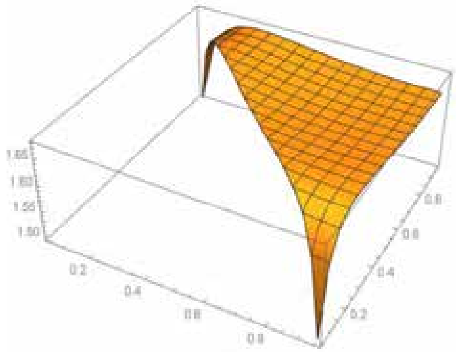
Shape of non-G

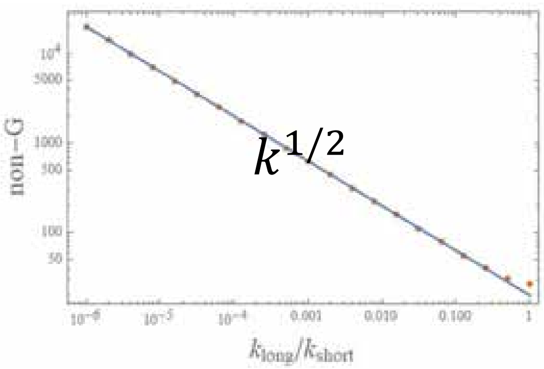
Real shapes for full shapes (upper) and squeezed limit (lower)

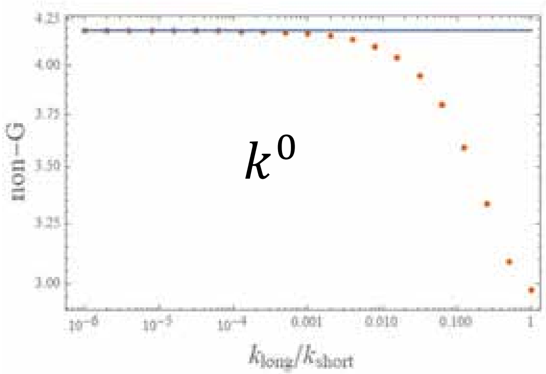
$\nu = 1$ | $\nu = 1/2$ | $\nu = 0$

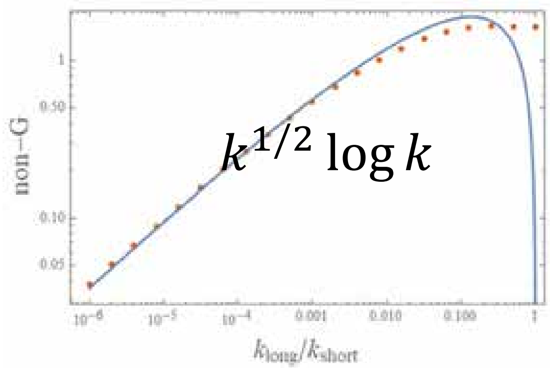










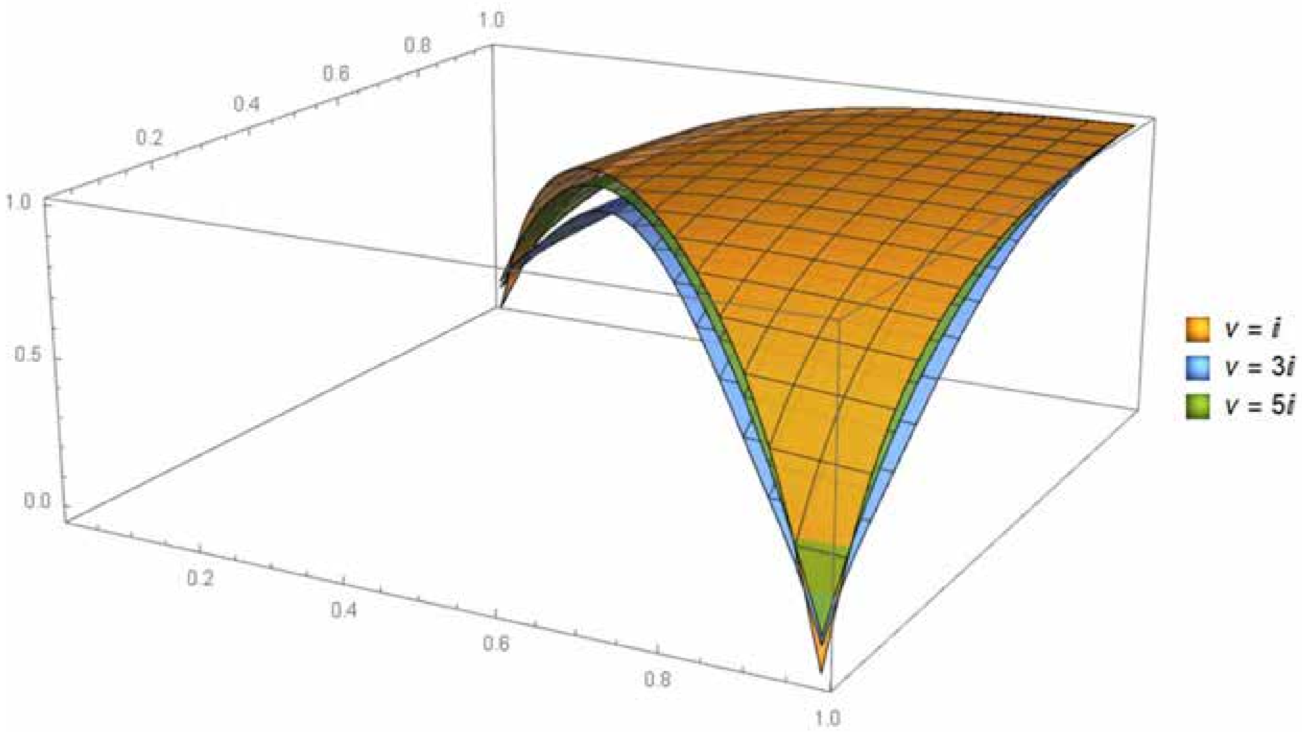


Shape of non-G

Imaginary shapes at the first sight:

they may be searched together as something

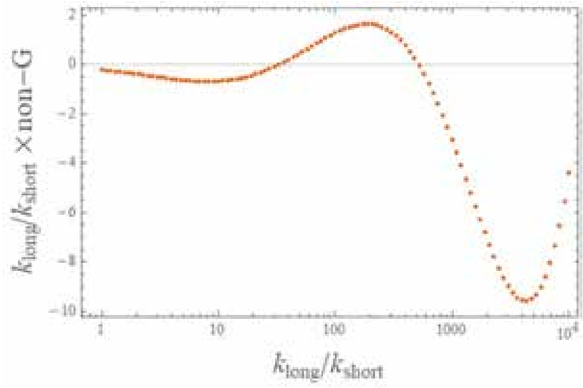
between local and equilateral shapes.



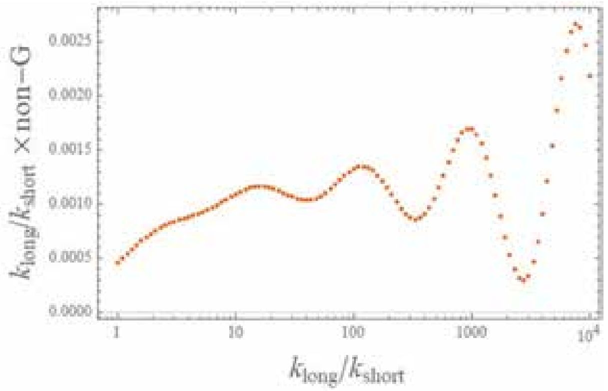
Shape of non-G

Oscillations in the imaginary shapes

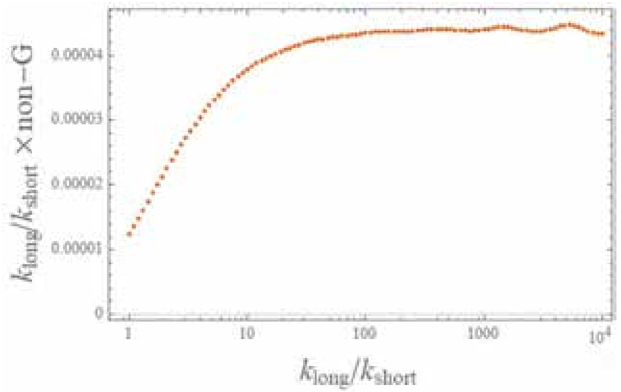
$\nu = i$



$\nu = 3i$



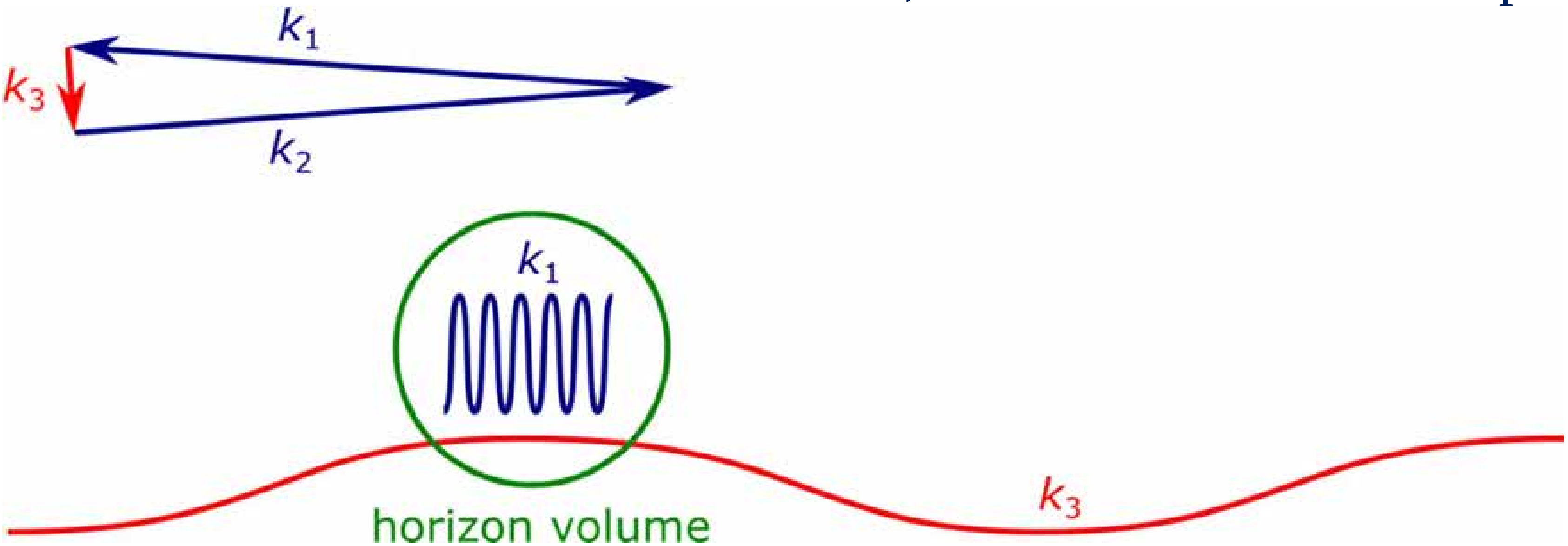
$\nu = 5i$



Shape of non-G

massive → time dependent phase $e^{imt} \sim (-\tau)^{im/H}$

curvature mode $\sim e^{ik\tau}$, at resonance record the phase



How can we make use of QSFI?

Massive fields oscillate according to physical time.

We measure scales according to Hubble-crossing of conformal time.

Observing massive oscillations → measuring $\tau(t)$, or $a(t)$

Direct measurement of evolution history of the primordial universe.

collider == search for new particles & interactions

search for new particles & interactions in cosmology == cosmological collider

Arkani-Hamed & Maldacena 1503.08043

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collider == search for new particles & interactions

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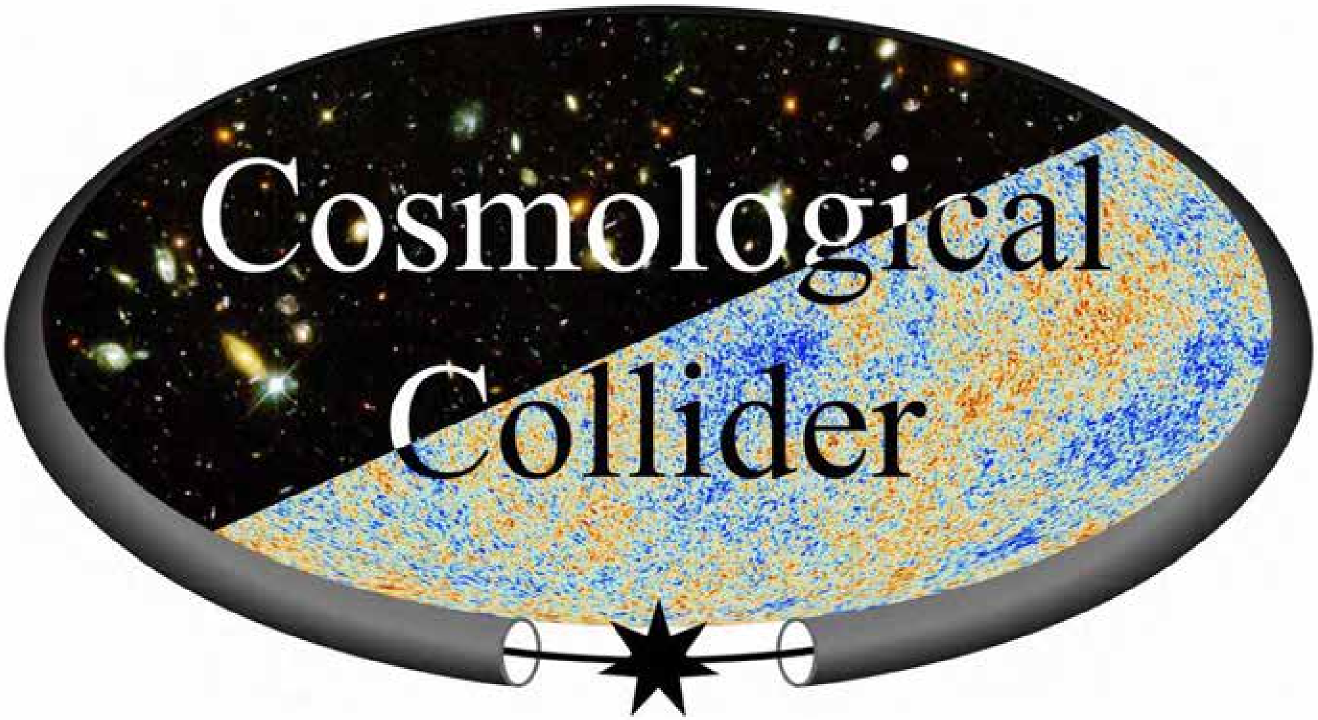
How can we make use of QSFI?



collider == search for new particles & interactions

search for new particles & interactions in cosmology == cosmological collider

Arkani-Hamed & Maldacena 1503.08043

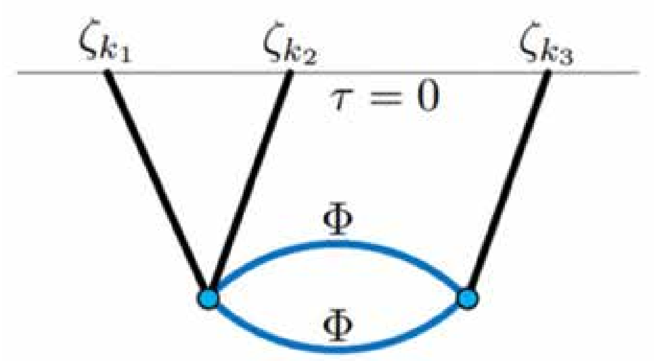
Cosmic Collider: SM background

What to do if particles are seen on the cosmic collider?

Question #1: Standard Model (SM) or Beyond Standard Model (BSM)?

Two possibilities of particles from SM:

- Non-Higgs inflation: inflaton has no SM charge → SM loops



- Higgs inflation: inflaton is the Higgs

Chen, YW, Xianyu 1610.06597, 1612.08122

Cosmic Collider: SM background

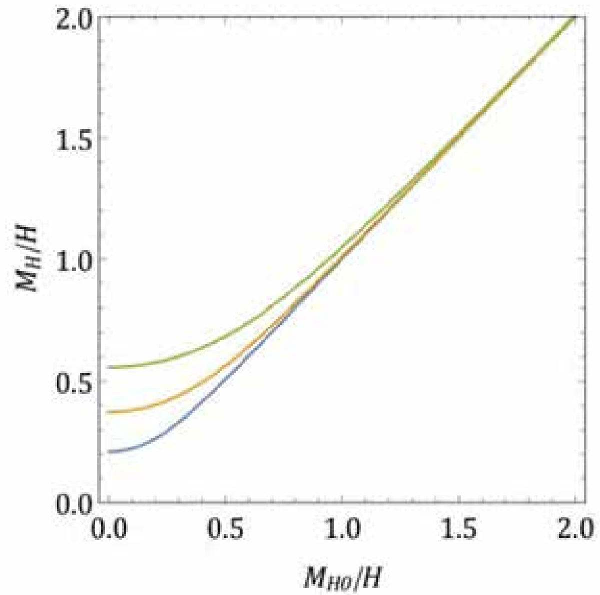
Non-Higgs inflation, the Higgs part:

May have tree-level mass from non-minimal coupling & coupling to inflaton

Loop-corrected mass is dominated by zero mode:

$$\langle h^2 \rangle \equiv \frac{\int d^{\mathcal{N}} h\, h^2 \exp[-V_D(m_0^2 h^2/2 + \lambda h^4/4)]}{\int d^{\mathcal{N}} h\, \exp[-V_D(m_0^2 h^2/2 + \lambda h^4/4)]}$$
$$= \frac{2}{\sqrt{V_D \lambda}} \frac{{}_1\widetilde{F}_1\left(\frac{\mathcal{N}+2}{4}; \frac{1}{2}; z^2\right) - z\, {}_1\widetilde{F}_1\left(\frac{\mathcal{N}+4}{4}; \frac{3}{2}; z^2\right)}{{}_1\widetilde{F}_1\left(\frac{\mathcal{N}}{4}; \frac{1}{2}; z^2\right) - z\, {}_1\widetilde{F}_1\left(\frac{\mathcal{N}+2}{4}; \frac{3}{2}; z^2\right)}$$

$$\langle h_i h_j \rangle = \delta_{ij} \frac{H^{D-2} Y_{\vec{0}}^2}{(m_{\text{eff}}/H)^2} = \delta_{ij} \frac{1}{V_D m_{\text{eff}}^2}$$

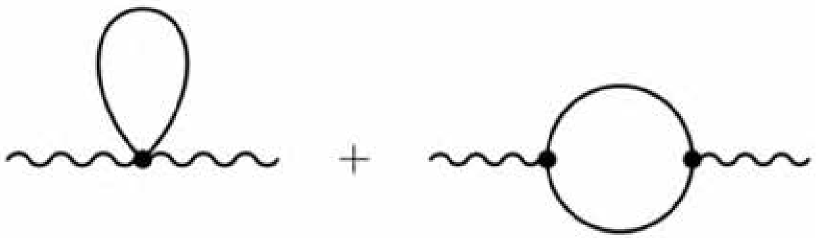


Cosmic Collider: SM background

$$g^2 = \frac{g_{\rm SM}^2}{1 + f_W(X_0, \phi_0)}$$
$$g'^2 = \frac{g'^2_{\rm SM}}{1 + f_B(X_0, \phi_0)}$$

Non-Higgs inflation, gauge bosons:

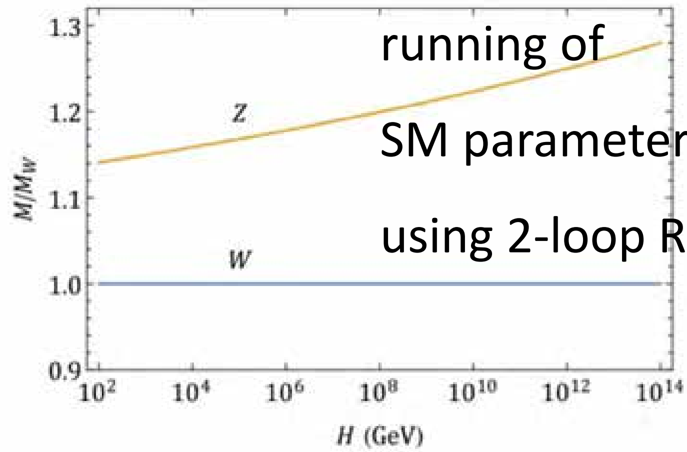
$$S \supset -\int \mathrm{d}^4x\sqrt{-g}\left[f_{DH}(X,\phi)|\mathrm{D}_\mu \mathbf{H}|^2 + \frac{1}{4}f_W(X,\phi)W^a_{\mu\nu}W^{\mu\nu a} + \frac{1}{4}f_B(X,\phi)B_{\mu\nu}B^{\mu\nu} + \cdots\right]$$



Mass from non-local part,

which has no UV divergence

$$M_W^2 = \frac{3g^2H^4}{8\pi^2 M_H^2}, \quad M_Z^2 = \frac{3g^2H^4}{8\pi^2 M_H^2 \cos^2\theta_W}$$



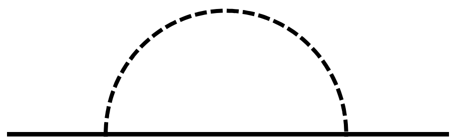
running of

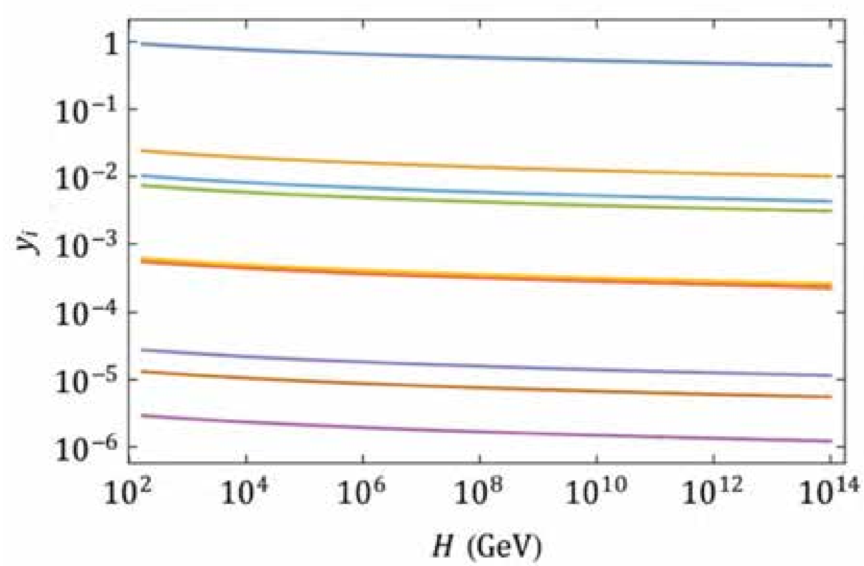
SM parameters

using 2-loop RG

Cosmic Collider: SM background

Non-Higgs inflation, fermions:





No long-wave-length loop correction → no mass correction

Cosmic Collider: SM background

Non-Higgs inflation, case of EWSB during inflation

if tree-level $M_{H0}^2 < 0$ from $\xi h^2 R$

$$v_h^2 = \frac{-4M_{H0}^2}{\lambda}$$

$$m_h^2 = \frac{1}{2}\lambda v_h^2 = -2M_{H0}^2$$

$$\Delta M_W^2 = \frac{g^2 v_h^2}{4}\,, \qquad \Delta M_Z^2 = \frac{\Delta M_W^2}{\cos^2\theta_W}$$

$$m_i = \frac{y_i|v_h|}{\sqrt{2}}$$

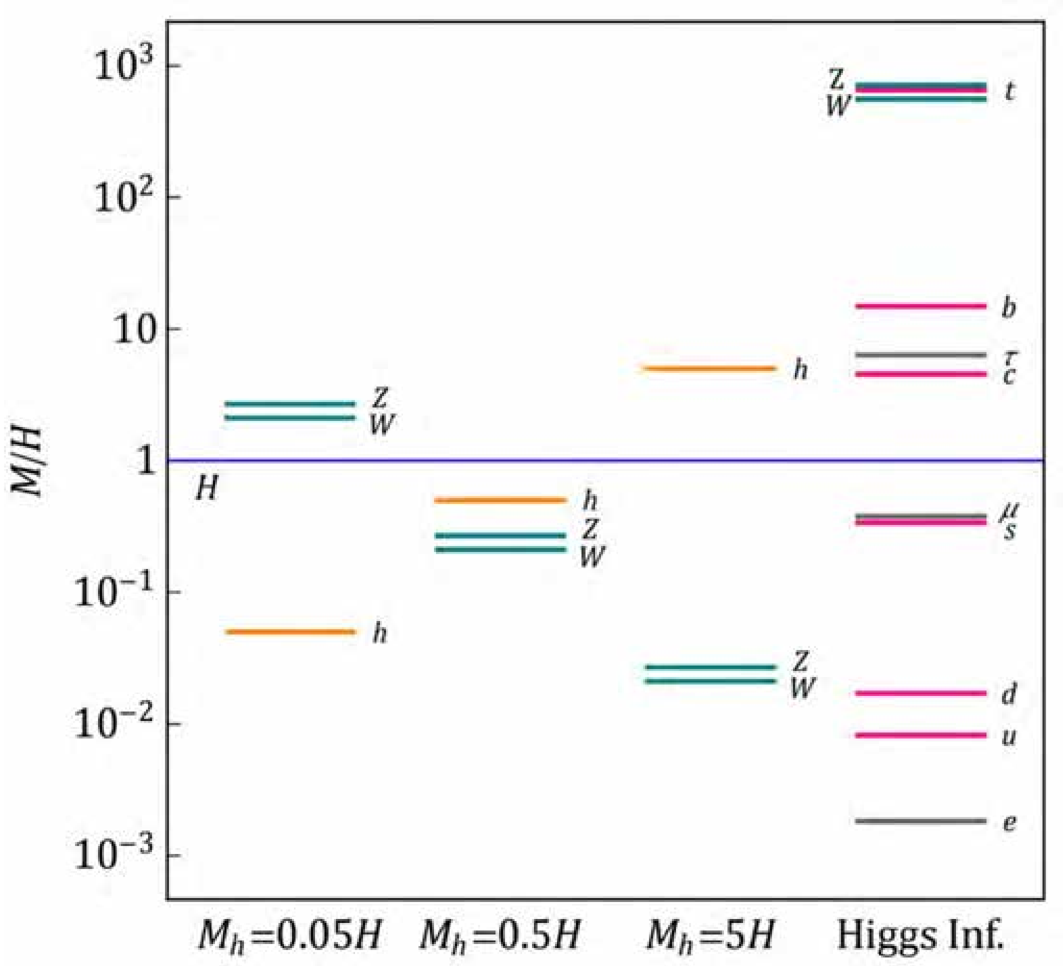
Cosmic Collider: SM background

Higgs inflation: Large Higgs VEV decides SM mass spectrum

Unfortunately, non-G of Higgs inflation SM sector is small, because

- Large SM-Higgs coupling → large mass → suppressed non-G
- Small SM-Higgs coupling → suppressed non-G

Cosmic Collider: SM background



Cosmic Collider: SM non-G

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle \equiv S(k_1,k_2,k_3)\frac{1}{(k_1k_2k_3)^2}P_\zeta^2(2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

$$S(k_L,k_S) \sim f_{NL}\left(\frac{k_L}{k_S}\right)^\gamma$$

$$S_H = \left[\frac{f'_H(X_0)}{1+f_{DH}(X_0)}\right]^2 \frac{\dot\phi_0^2}{2\pi^4}\left[C_H(\mu_h)\left(\frac{k_L}{2k_S}\right)^{2-2\mu_h} + (\mu_h\to-\mu_h)\right],$$

$$S_{DH} = \left[\frac{f'_{DH}(X_0)}{1+f_{DH}(X_0)}\right]^2 \frac{H^4\dot\phi_0^2}{8\pi^4}\left[C_{DH}(\mu_h)\left(\frac{k_L}{2k_S}\right)^{2-2\mu_h} + (\mu_h\to-\mu_h)\right],$$

$$S_\Psi = \left[\frac{f'_\Psi(X_0)}{1+f_\Psi(X_0)}\right]^2 \frac{H^4\dot\phi_0^2\mu_{1/2}^2}{2\pi^4}\left[C_\Psi(\mu_{1/2})\left(\frac{k_L}{k_S}\right)^{1+2i\mu_{1/2}} + \text{c.c.}\right],$$

$$S_A = \left[\frac{f'_A(X_0)}{1+f_A(X_0)}\right]^2 \frac{27H^8\dot\phi_0^2}{16\pi^4M_A^4}\left[C_A(\mu_1)\left(\frac{k_L}{2k_S}\right)^{2-2\mu_1} + (\mu_1\to-\mu_1)\right],$$

Cosmic Collider: Beyond SM

SM background is in general hard to observe:

- Has to appear in loops for non-Higgs inflation
- Coupling issue for Higgs inflation

Rich possibilities for BSM

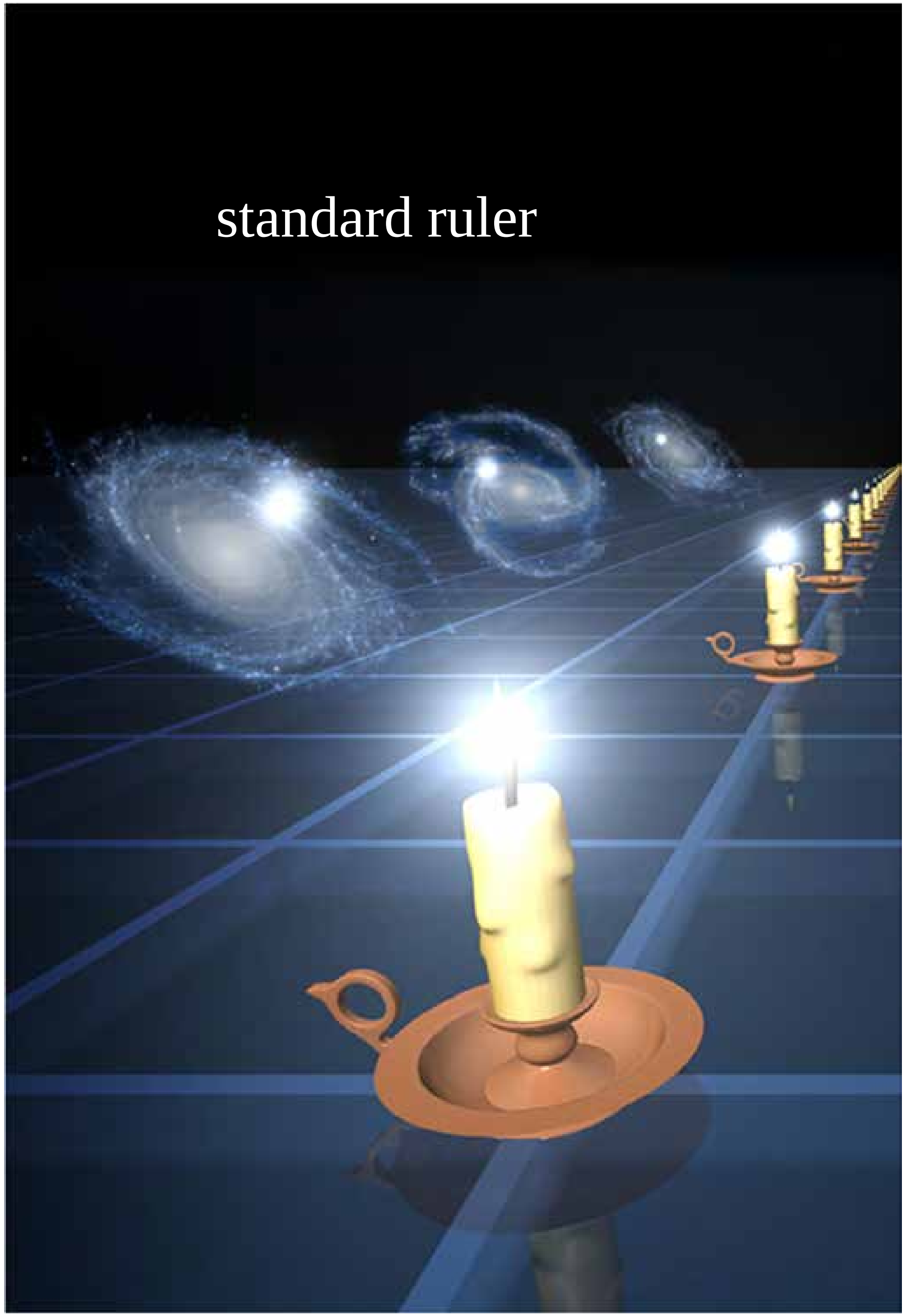
- Neutrino physics (on-going)
- Supersymmetries?
- High spin fields? Lee, Baumann, Pimentel 1607.03735
- …

Quantum Primordial Standard Clock

Chen, Namjoo, YW 1509.03930, 1601.06228, 1608.01299

Our present universe

standard candle



The primordial universe?

Quantum Primordial Standard Clock

What are the possible paradigms of the primordial universe?

Quantum Primordial Standard Clock

What are the possible paradigms of the primordial universe?

Only inflation?

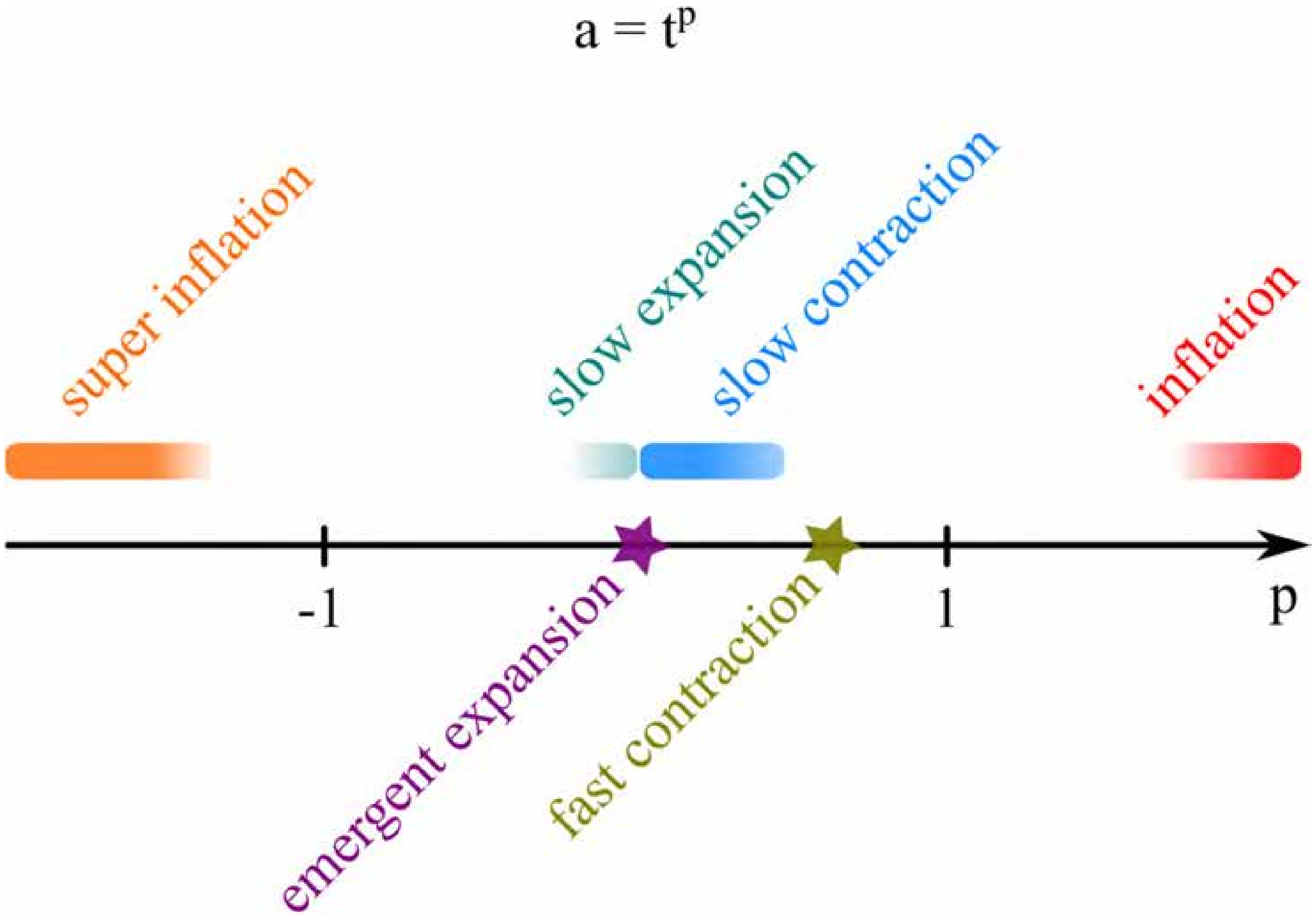
Quantum Primordial Standard Clock

What are the possible paradigms of the primordial universe?

Only inflation?

您可能学了一门假宇宙学

Quantum Primordial Standard Clock



Quantum Primordial Standard Clock

How to measure the evolution history of the primordial universe $a(t)$?

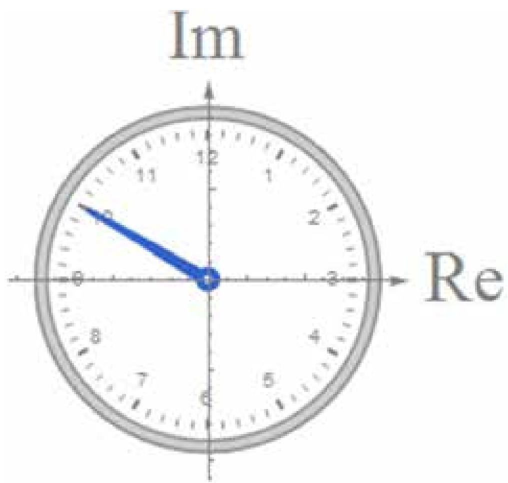
Model-independent ways:

- Primordial gravitational waves
- Quantum primordial standard clock

Quantum Primordial Standard Clock

$$|\Psi\rangle \xrightarrow{t} e^{i\omega t}|\Psi\rangle$$

$e^{i\#}$ =



Quantum Primordial Standard Clock

$$|\Psi\rangle \xrightarrow{t} e^{i\omega t}|\Psi\rangle$$

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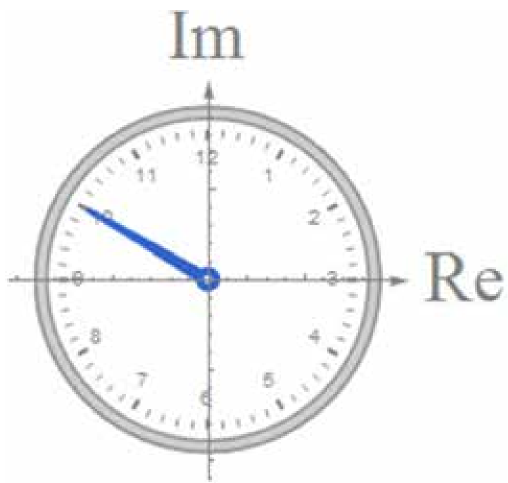
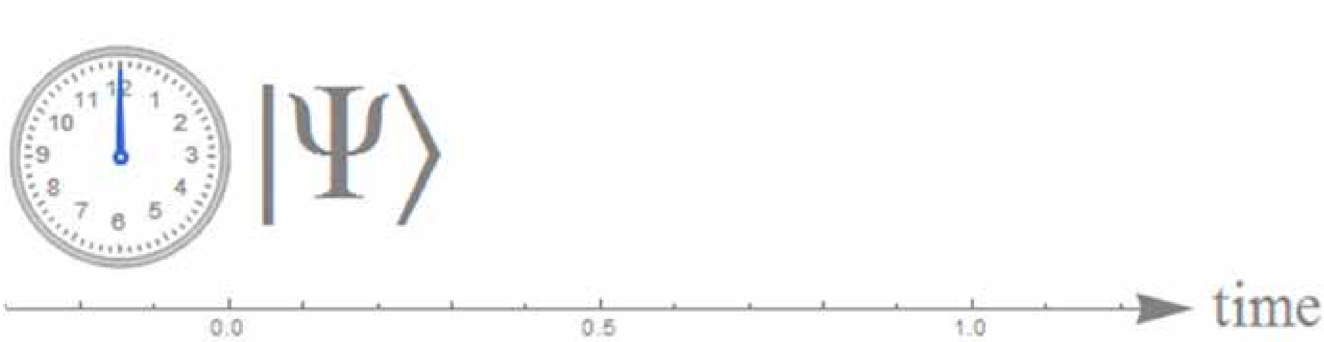
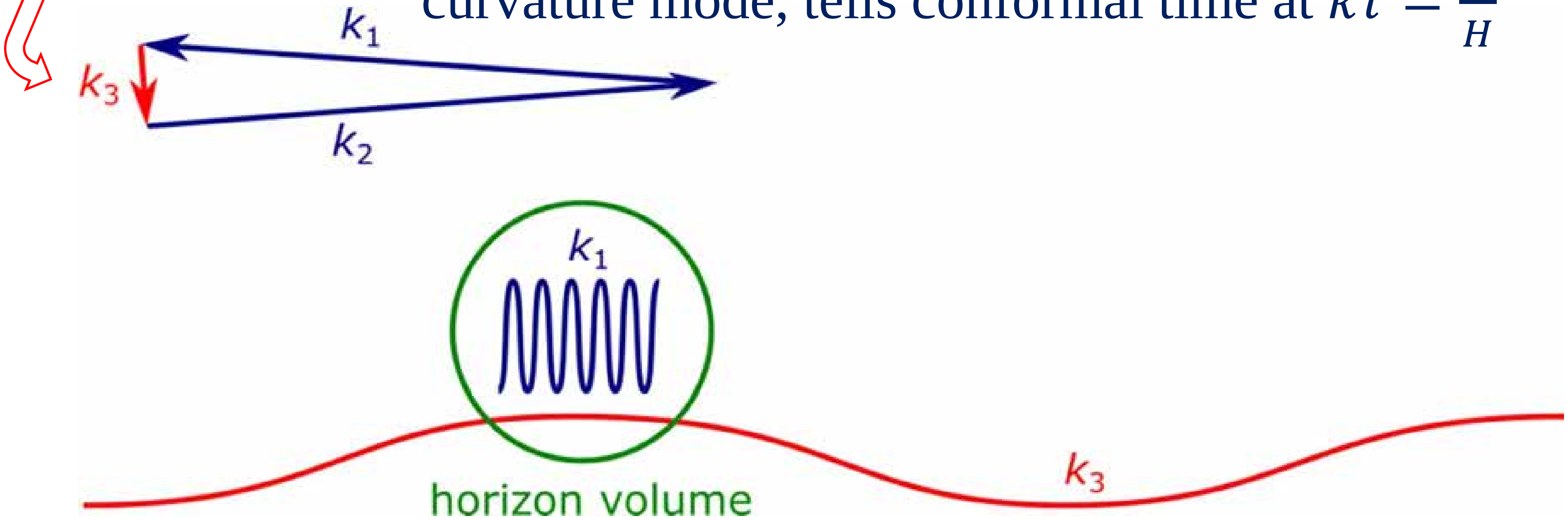




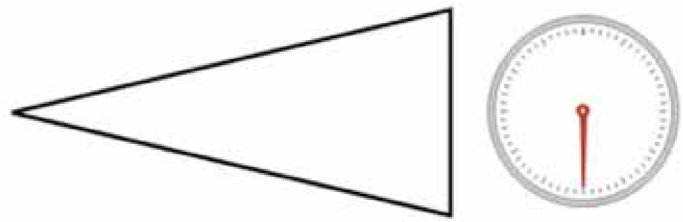
Quantum Primordial Standard Clock

massive → curvature, tells physical time

curvature mode, tells conformal time at $k\tau = \frac{M}{H}$



Quantum Primordial Standard Clock

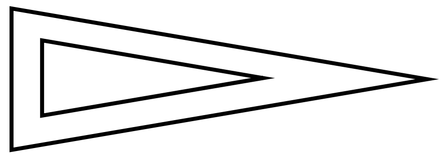


Correlation between the density fluctuation and a clock

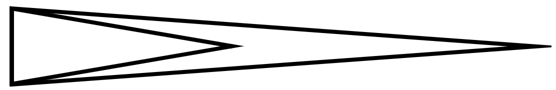
Quantum Primordial Standard Clock

Observational consequence:

scale-independent

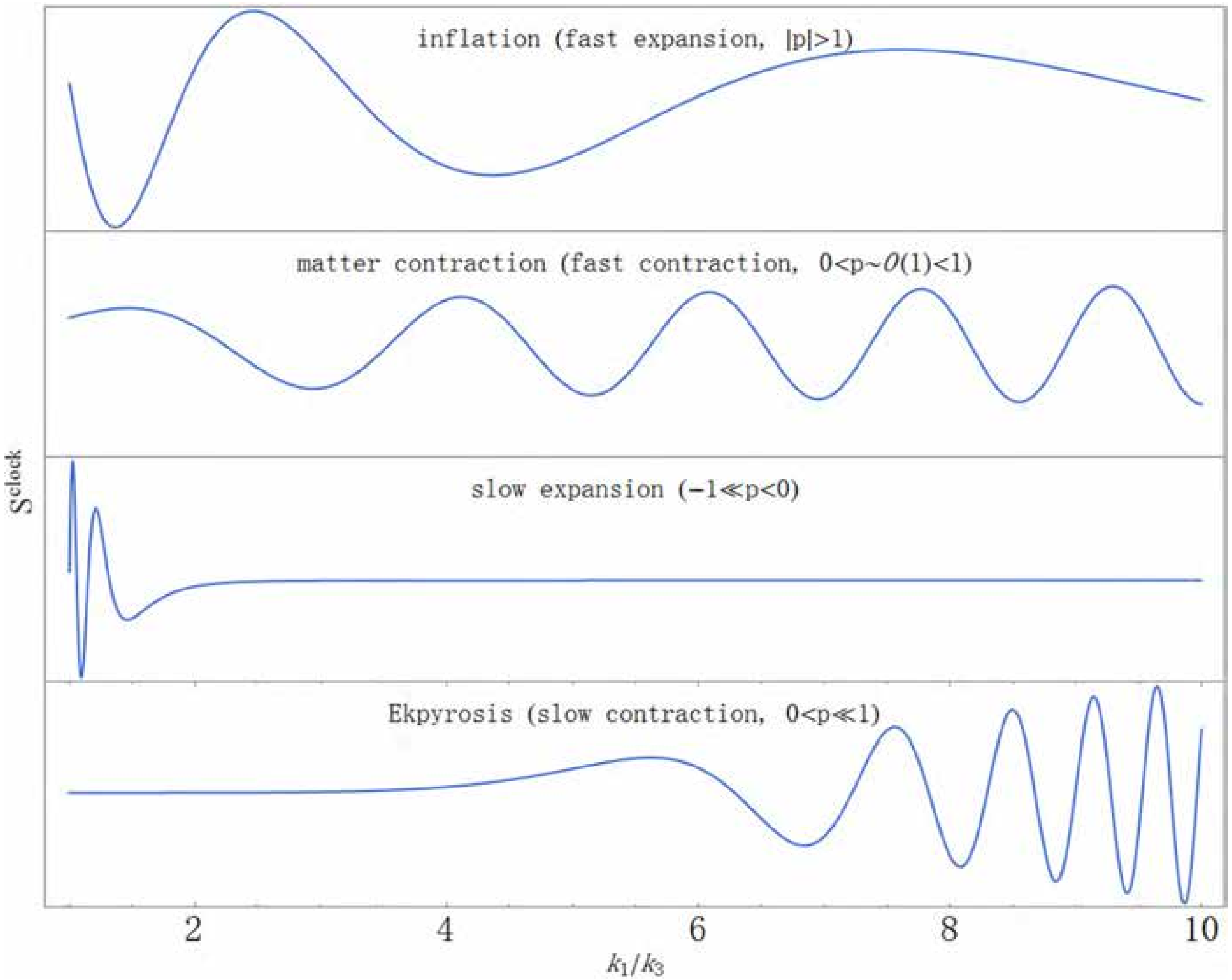


shape-dependent

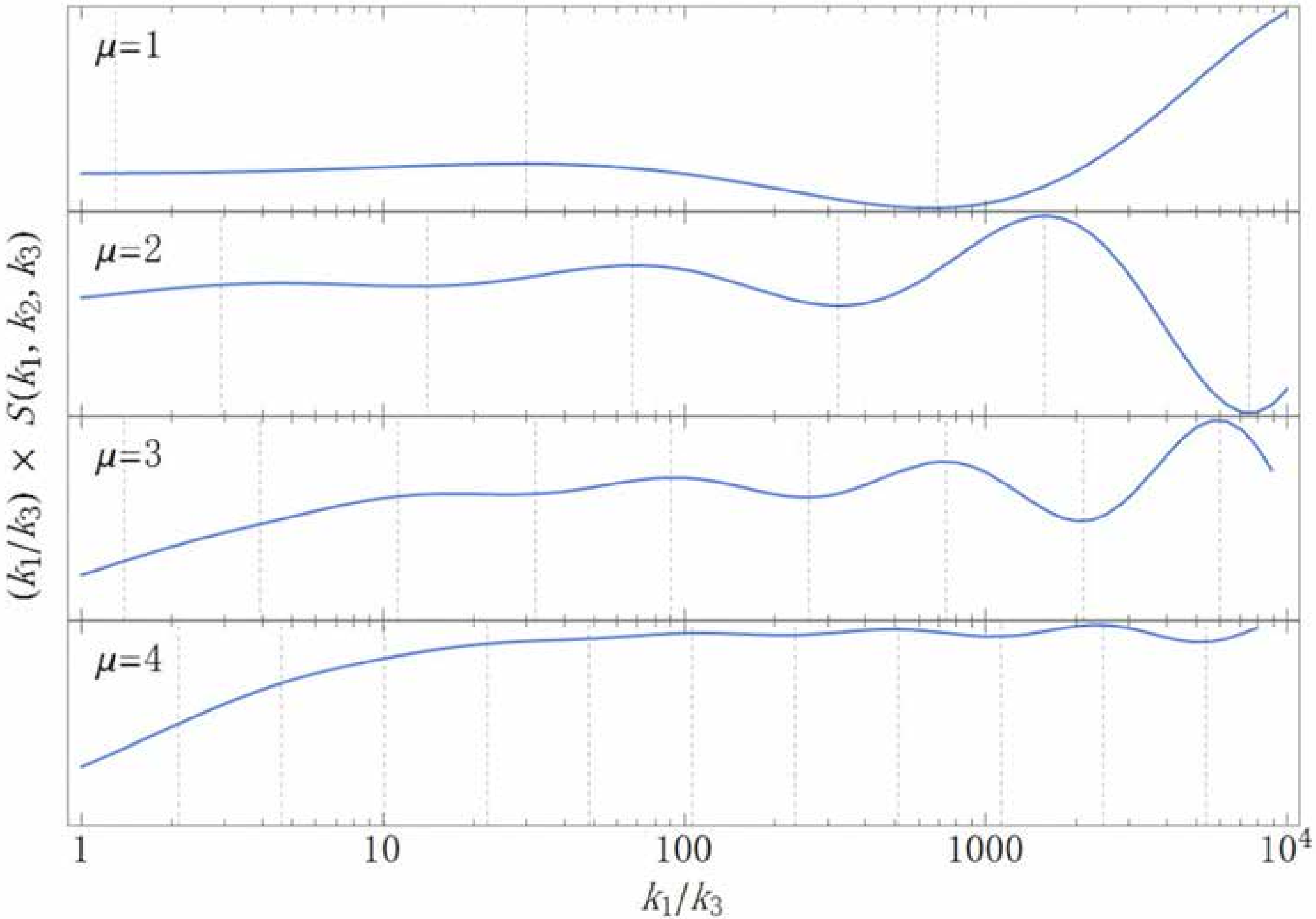


oscillations on non-Gaussianities

The clock part of the signal



Full signal for inflation



Compared to PGWs

Similarities to PGWs

- Existing physics
- Probe evolution history of universe

  Weakness of PGW in this aspect:

  a) Scale invariant power spectrum is easy to mimick

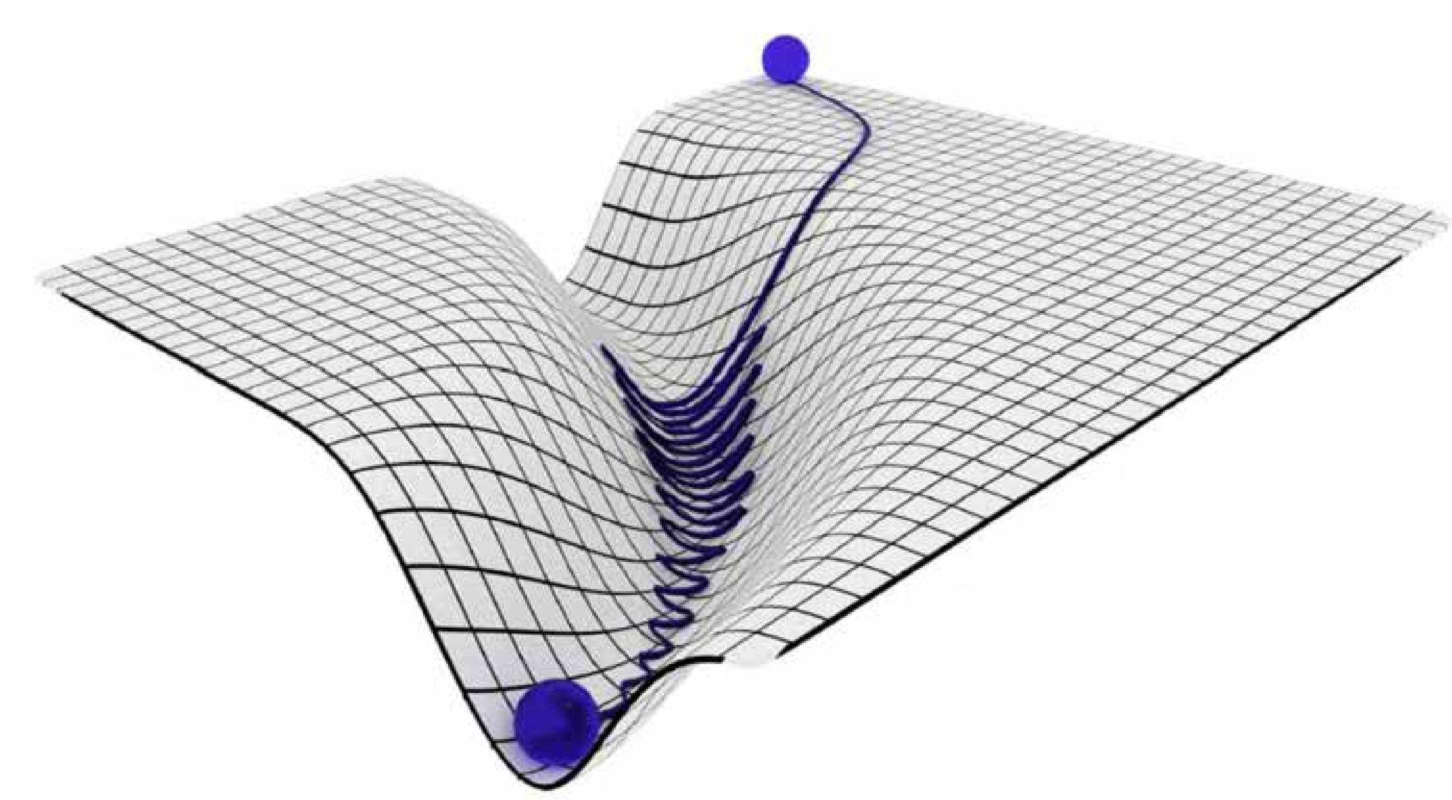
  b) Has assumptions for $P_T \sim \left(\frac{H}{M_p}\right)^2$ -- vacuum, expansion

- Tells important additional physics
- Very hard to probe

Sometimes, energy scale other than H matters.

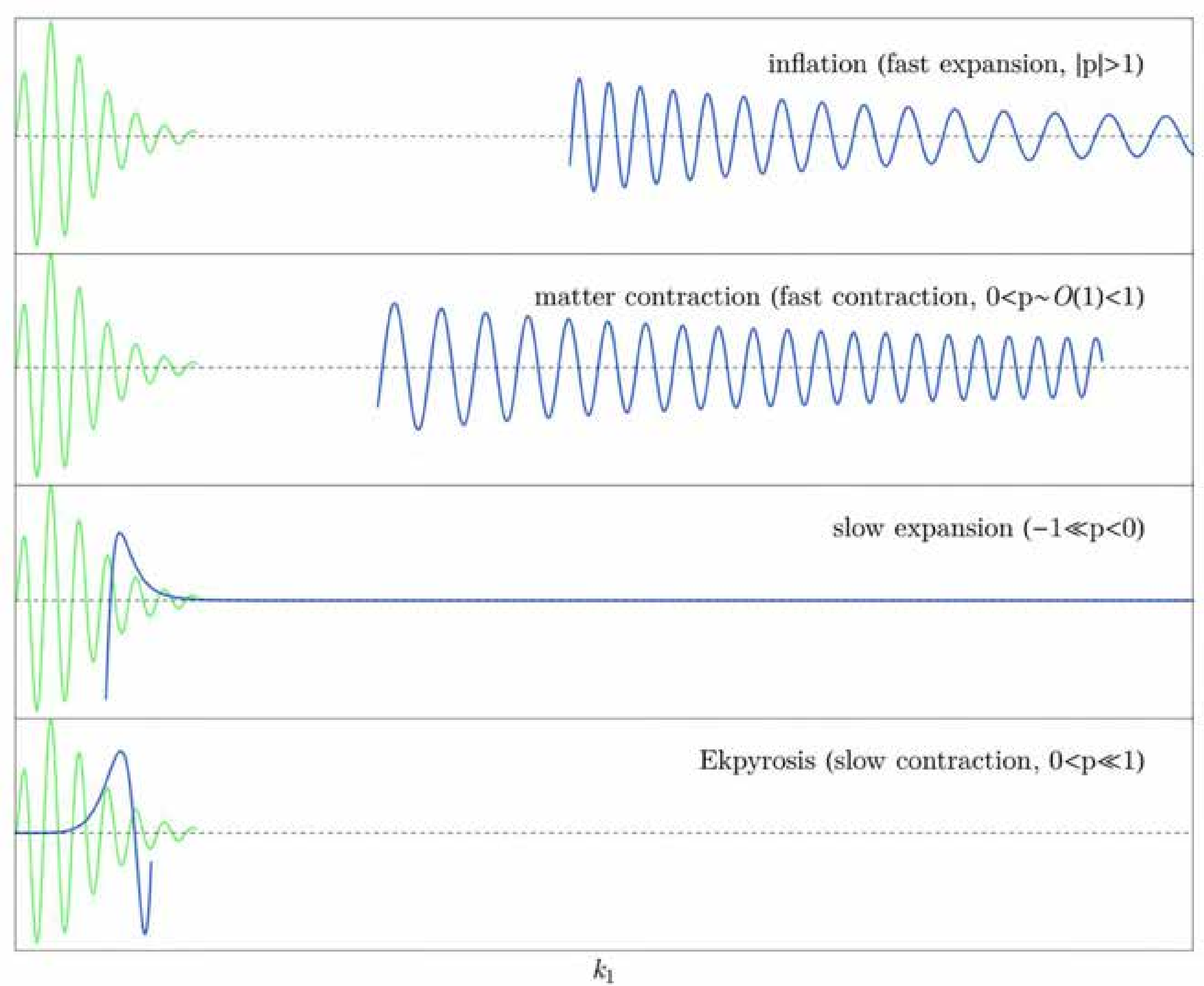
Let us consider two examples.

The classical standard clock



Chen 1104.1323, Chen, Namjoo & YW 1411.2349

The classical standard clock



The classical standard clock

Difference between the quantum clock and the classical clock

- Pros of the classical clock
  - Signal can be much larger
- Pros of the quantum clock
  - The existence does not depend on luck
  - More unique (no feature at power spectrum)
  - Depend less on shape of potential

Particle Production

Flauger, Mirbabayi, Senatore, Silverstein 1606.00513

Key idea: has $\cos(\phi)$ or $(\phi - \phi_0)$ in potential

Replace the Boltzmann factor from

$\exp(-E/H)$ into $\exp(-E/\sqrt{\dot{\phi}})$

Summary of This Lecture

By precision test of non-Gaussianity, we can

- Probe particle physics during inflation
- Probe the evolution history of primordial universe

This is extremely difficult to probe, but existing physics

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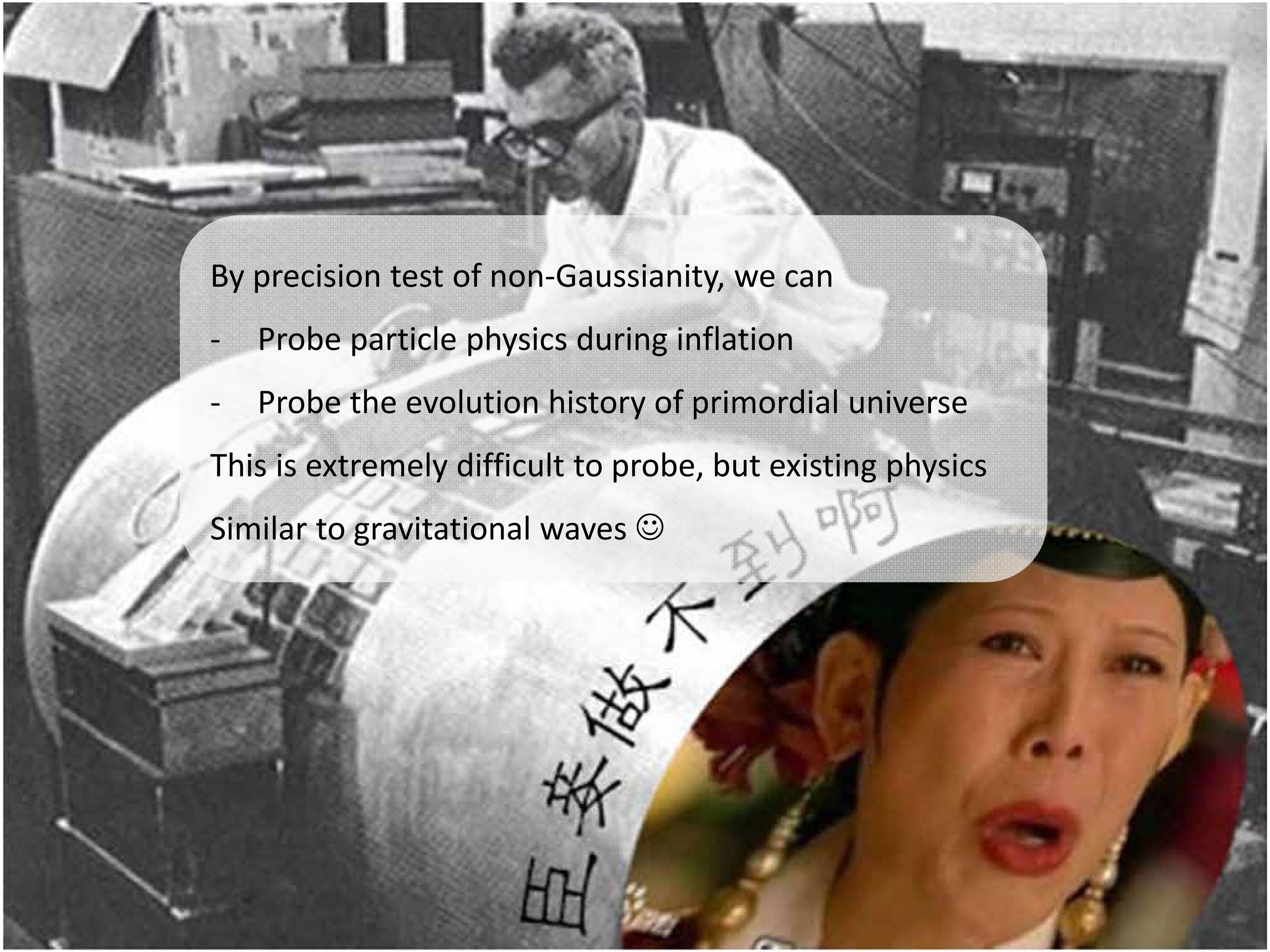

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臣妾做不到啊

Part I -- Linear Fluctuations

Part II – Computer Assisted Computation

Part III -- Nonlinear Fluctuations

Part IV – Inflationary Massive Fields

Thank you!