# The simplest case of self-consistent (“Bold”) Diagrammatic MC

## [See also PRL, 99, 250201 (2007)]

Consider the following problem: Find f defined by the series:

$$f = a - au + au^2 - au^3 + \ldots + a(-1)^n u^n + \ldots = a\sum_{n=0}^{\infty}(-1)^n u^n = \frac{a}{1+u}$$

Do it my Monte Carlo assuming that the answer in blue is not known.
Consider this as "diagrams" characterized only by the diagram "order" , i.e.

$$\nu = n, \quad D_\nu = \left|au^n\right|, \quad A_\nu = \mathrm{sgn}[au^n(-1)^n], \quad f = \langle A \rangle = \sum_\nu A_\nu D_\nu$$

Monte carlo Algorithm: **one update** $n \leftrightarrow n \pm 1$ (decide at random + or -)

**Detailed balance equation**: $D_n \frac{1}{2} P_{n \to n\pm1} = D_{n\pm1} \frac{1}{2} P_{n\pm1 \to n}$

$$R = \frac{P_{n\to n+1}}{P_{n+1\to n}} = u, \qquad R = \frac{P_{n\to n-1}}{P_{n-1\to n}} = \frac{1}{u}$$ (reject if n=0)

**This is all!** $f^{(MC)} = f^{(MC)} + A_n, \quad Z = Z + \delta_{n,0}, \quad f = |a| \frac{f^{(MC)}}{Z}$

Convergence of the scheme: Same as iterations

$$f_0=a$$
$$f_{n+1}=a-uf_n$$

⇓

$$f_0=a$$
$$f_1=a-uf_0=a-ua$$
$$f_2=a-uf_1=a-ua+u^2a$$
$$\ldots\ldots\ldots$$
$$f=a-ua+u^2a-u^3a+u^4a- \cdots$$

**|u|>1, divergent series:**
**resummation techniques**

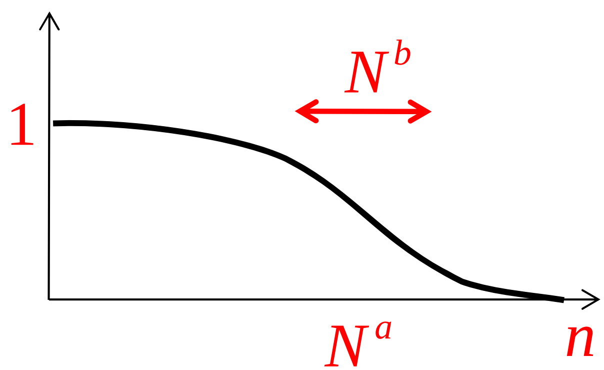
# Re-summation of divergent series with finite convergence radius.

**Example:** $A = \sum_{n=0}^{\infty} c_n = 3 - 9/2 + 9 - 81/4 + \ \ldots =$ **бред какой то**

**Define a function** $f_{n,N}$ **such that:**

$f_{n,N} \to 1 \ \text{ for } \ n << N$

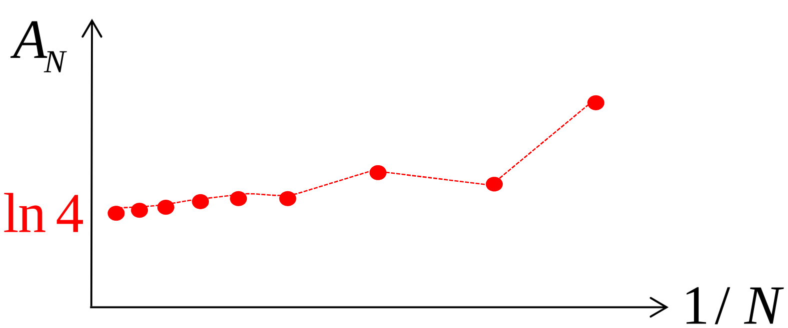
$f_{n,N} \to 0 \ \text{ for } \ n > N$




$f_{n,N} = e^{-n^2/N}$ **(Gauss)**

$f_{n,N} = e^{-\varepsilon n \ln(n)}$ **(Lindeloef)**

**Construct sums** $A_N = \sum_{n=0}^{\infty} c_n f_{n,N}$ **and extrapolate** $\lim_{N\to\infty} A_N$ **to get** $A$



Rewrite the problem identically as $f = a - uf$ and try to solve it by self-consistent ("**Bold**") **MC**

$$\nu = 0,1 \quad D_0 = a \qquad D_1 = |uf|$$

$$A_0 = \mathrm{sgn}[a] \quad A_1 = \mathrm{sgn}[uf] \qquad f = \langle A \rangle = \sum_{\nu = 0,1} A_\nu D_\nu$$

Algorithm:
if n=0 propose n=1
if n=1 propose n=0

$$R = \frac{P_{0\to 1}}{P_{1\to 0}} = \left| \frac{uf}{a} \right|, \qquad R = \frac{P_{1\to 0}}{P_{1\to 0}} = \left| \frac{a}{uf} \right|$$

Done!

$$f^{(MC)} = f^{(MC)} + A_n, \quad Z = Z + \delta_{n,0}\,, \qquad f = |a| \frac{f^{(MC)}}{Z}$$

In other words, you can simulate series based on unknown functions which are defined in terms of themselves by the same series --- important for Feynman diagrams

Convergence of the scheme: Similar to

$$f_{n+1} = a - u \langle f \rangle_n$$

$$\langle f \rangle_n = \sum_{i=1}^{n} \frac{f_i}{n}$$

i.e. “damped” iterations $$f_{n+1} = a - u \sum_{i=1}^{n} \frac{f_i}{n}$$

You can prove that it converges for any $u > -1$ , i.e. even for large **positive** values of $u$ .

## Write a simple program which mimicks a Monte Carlo calculation

$$f_1 = a$$

**do loop n=1, ….**

$$f_{n+1} = a - u\left(\frac{1}{n}\sum_{i=1}^{n} f_i\right)$$

**end do loop**

```
double precision:: a=1.0,  u=2.5
 double precision:: f_new, f_sum
 integer :: n=20,i

 f_sum = a

 do i = 1, n
 print*, i, f_new
    f_new = a – u * f_sum/i
    f_sum = f_sum + f_new
 enddo

 print*, f_new
 end
```

**Printout for**

$$a = 1, \quad u = 2.5$$

| | |
|---|---|
| 1 | 1. |
| 2 | -1.5 |
| 3 | 1.625 |
| 4 | 0.0625 |
| 5 | 0.2578125 |
| 6 | 0.27734375 |
| 7 | 0.282226562 |
| 8 | 0.283970424 |
| 9 | 0.284733364 |
| 10 | 0.285114833 |
| 11 | 0.285324642 |
| 12 | 0.285448619 |
| 13 | 0.285526105 |
| 14 | 0.285576769 |
| 15 | 0.285611148 |
| 16 | 0.285635214 |
| 17 | 0.285652511 |
| 18 | 0.285665229 |
| 19 | 0.285674768 |
| 20 | 0.285682048 |

**$f$=0.285714286=1/(1+2.5)**