

证明下列公式

$$\mathbf{A} \cdot \mathbf{B} = \text{标量} = A_x B_x + A_y B_y + A_z B_z$$

$$\mathbf{A} \times \mathbf{B} = \text{矢量}$$

$$(\mathbf{A} \times \mathbf{B})_z = A_x B_y - A_y B_x$$

$$(\mathbf{A} \times \mathbf{B})_x = A_y B_z - A_z B_y$$

$$(\mathbf{A} \times \mathbf{B})_y = A_z B_x - A_x B_z$$

$$\mathbf{A} \times \mathbf{A} = 0$$

$$\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) = 0$$

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

从微分学方面得来的下列两个等式：

第 2 章 矢

$$(a) \nabla \cdot (\nabla T) = \nabla^2 T = \text{标量场};$$

$$(b) \nabla \times (\nabla T) = 0;$$

$$(c) \nabla(\nabla \cdot \mathbf{h}) = \text{矢量场};$$

$$(d) \nabla \cdot (\nabla \times \mathbf{h}) = 0;$$

$$(e) \nabla \times (\nabla \times \mathbf{h}) = \nabla(\nabla \cdot \mathbf{h}) - \nabla^2 \mathbf{h};$$

$$(f) (\nabla \cdot \nabla) \mathbf{h} = \nabla^2 \mathbf{h} = \text{矢量场}.$$

注意到, 我们从未试图发明一个新的矢量算符 $(\nabla \times \nabla)$ 。你看这是为