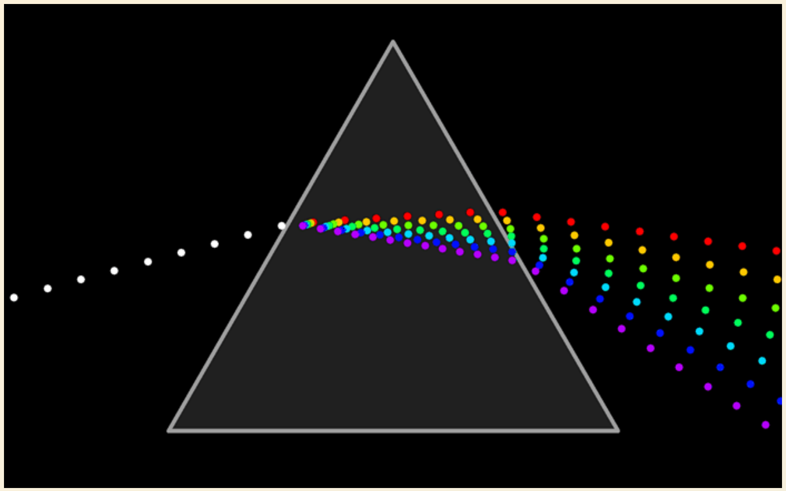
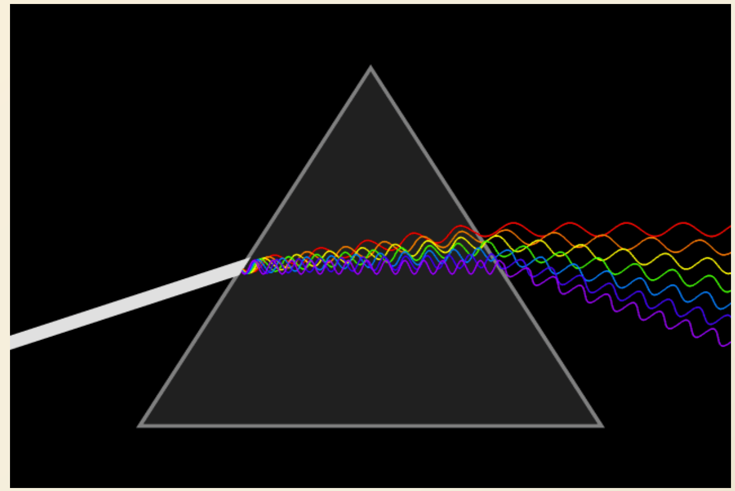
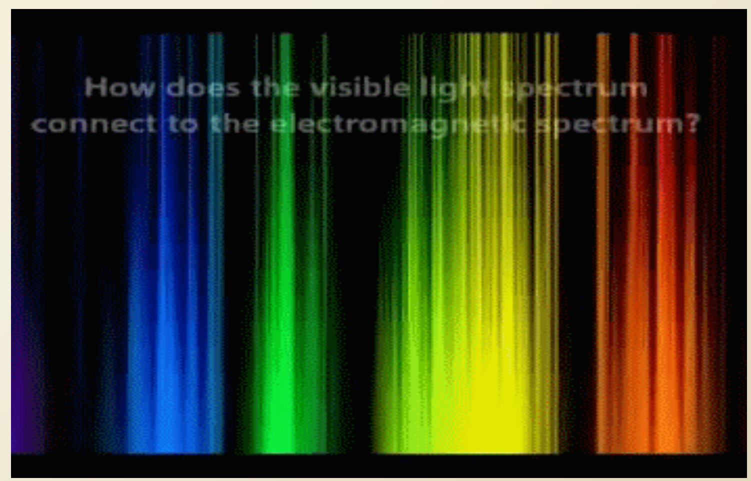
电磁波






How does the visible light spectrum
connect to the electromagnetic spectrum?

完备的Maxwell方程组

积分形式

$$
\begin{cases}
\oiint_S \vec{E}\cdot d\vec{S} = \dfrac{Q_{内}}{\varepsilon_0} \\
\oiint_S \vec{B}\cdot d\vec{S} = 0 \\
\oint_{\partial S} \vec{E}\cdot d\vec{l} = -\iint_S \dfrac{\partial \vec{B}}{\partial t}\cdot d\vec{S} \\
\oint_{\partial S} \vec{B}\cdot d\vec{l} = \mu_0\left(I_{内} + \underbrace{\varepsilon_0 \iint_S \dfrac{\partial \vec{E}}{\partial t}\cdot d\vec{S}}_{I_D}\right)
\end{cases}
$$

微分形式

$$
\begin{cases}
\nabla\cdot\vec{E} = \dfrac{\rho}{\varepsilon_0} \\
\nabla\cdot\vec{B} = 0 \\
\nabla\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\
\nabla\times\vec{B} = \mu_0\left(\vec{j} + \underbrace{\varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}}_{\vec{j}_D}\right)
\end{cases}
$$

完备的Maxwell方程组(物质中)

积分形式

$$\begin{cases} \oiint_S \vec{D}\cdot d\vec{S} = Q_{0内} \\ \oiint_S \vec{B}\cdot d\vec{S} = 0 \\ \oint_{\partial S} \vec{E}\cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} \\ \oint_{\partial S} \vec{H}\cdot d\vec{l} = I_{0内} + \iint_S \frac{\partial \vec{D}}{\partial t}\cdot d\vec{S} \end{cases}$$

微分形式

$$\begin{cases} \nabla\cdot\vec{D} = \rho_0 \\ \nabla\cdot\vec{B} = 0 \\ \nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla\times\vec{H} = \vec{j}_0 + \frac{\partial \vec{D}}{\partial t} \end{cases}$$

边值关系

$$\begin{cases} \hat{n}\cdot\left(\vec{D}_2 - \vec{D}_1\right) = \sigma_0 \\ \hat{n}\cdot\left(\vec{B}_2 - \vec{B}_1\right) = 0 \\ \hat{n}\times\left(\vec{E}_2 - \vec{E}_1\right) = 0 \\ \hat{n}\times\left(\vec{H}_2 - \vec{H}_1\right) = \vec{K}_0 \end{cases}$$

$$\varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t}$$

- **电磁性能方程**
  - 绝缘介质 $\vec{D} = \varepsilon_0\varepsilon_r\vec{E},\ \vec{B} = \mu_0\mu_r\vec{H}$
  - 导电介质 $\vec{D} = \varepsilon_0\varepsilon_r\vec{E},\ \vec{B} = \mu_0\mu_r\vec{H},\ \vec{j}_0 = \sigma\vec{E}$

自由空间的Maxwell方程组

$$\begin{cases} \nabla\cdot\vec{E}=0 \\ \nabla\cdot\vec{B}=0 \\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{B}=\mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t} \end{cases}$$

$\mu_0\varepsilon_0 = 1/c^2$

$$\begin{cases} \nabla^2\vec{E}-\dfrac{1}{c^2}\dfrac{\partial^2\vec{E}}{\partial t^2}=0 \\ \nabla^2\vec{B}-\dfrac{1}{c^2}\dfrac{\partial^2\vec{B}}{\partial t^2}=0 \end{cases}$$

where $c=\dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

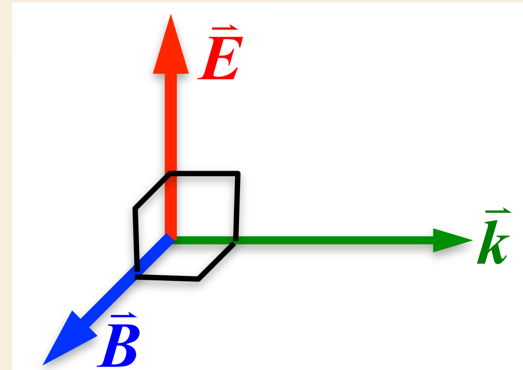
■ 单色平面波解

$$\begin{cases} \vec{E}=\vec{E}_0\exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right] \\ \vec{B}=\vec{B}_0\exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right] \end{cases}$$

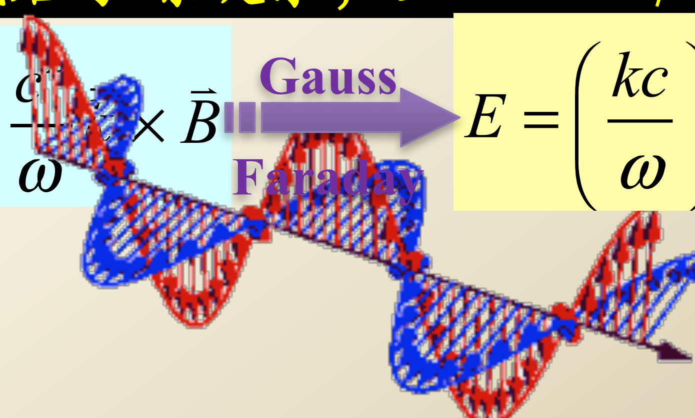
$$\partial/\partial t \quad \leftrightarrow \quad -i\omega$$
$$\nabla \quad \leftrightarrow \quad +i\vec{k}$$

自由空间中电磁波的性质

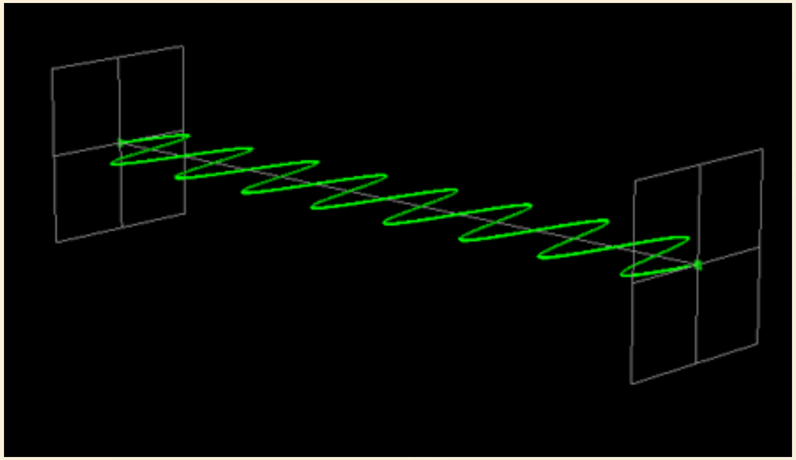
- **Gauss定律** $\nabla\cdot\vec{E}=0$ ⇒ $\vec{k}\cdot\vec{E}=0$
  - 电场与传播方向垂直(横电)
- **磁无源** $\nabla\cdot\vec{B}=0$ ⇒ $\vec{k}\cdot\vec{B}=0$
  - 磁场与传播方向垂直(横磁)

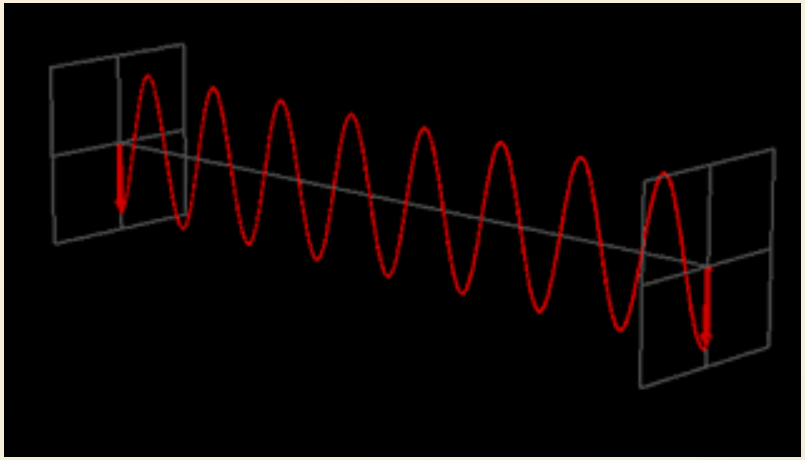


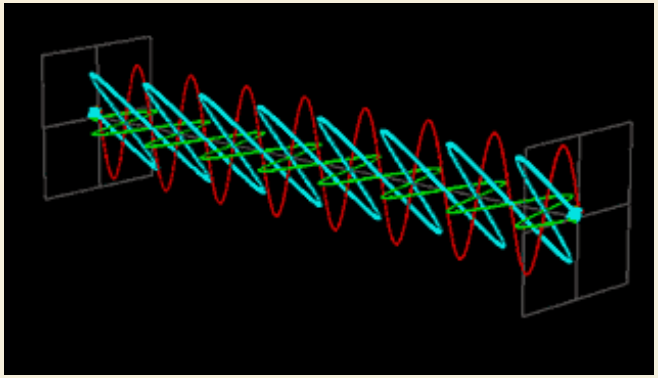
- **Faraday定律** $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$ ⇒ $\vec{B}=\frac{\vec{k}\times\vec{E}}{\omega}$
  - 电磁波为横波，(E、B、k)构成右手螺旋系，且 $B=kE/\omega$
- **A/M 定律** $\nabla\times\vec{B}=\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$ ⇒ $\vec{E}=-\frac{c^2}{\omega}\vec{k}\times\vec{B}$ ⇒ (Gauss, Faraday) $E=\left(\frac{kc}{\omega}\right)^2 E$
  - 相速度：$v_p=\omega/k=c$
  - 振幅关系：$E=cB$

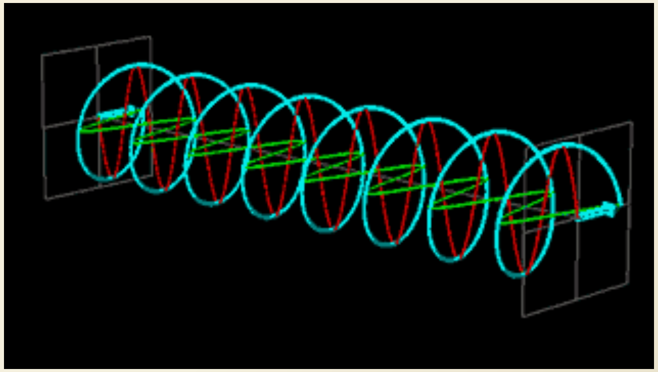


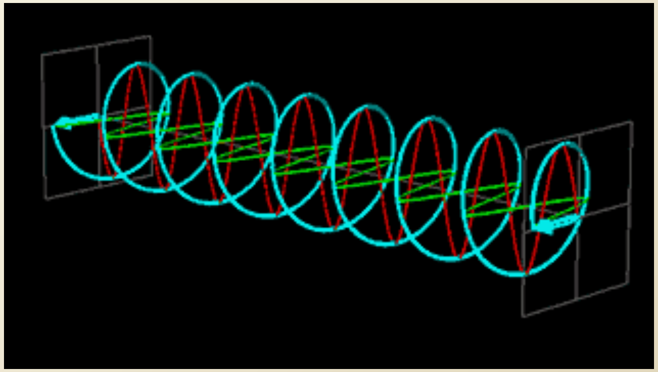
电磁波的偏振（Polarization）











无限均匀介质中的无源Maxwell方程组

$$\begin{cases} \nabla\cdot\vec{D}=\rho_0 \\ \nabla\cdot\vec{B}=0 \\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{H}=\vec{j}_0+\dfrac{\partial\vec{D}}{\partial t} \end{cases}$$

$\rho_0=0$

$\vec{j}_0=0$

$$\begin{cases} \nabla\cdot\vec{D}=0 \\ \nabla\cdot\vec{B}=0 \\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{H}=\dfrac{\partial\vec{D}}{\partial t} \end{cases}$$

对介质的考虑

介质中，电磁场方程**能否**写出波动方程的形式? 

如果可以，有无条件? **条件**是什么? 

$$c \to (\mu\varepsilon)^{-1/2} \quad ?$$

$$\nabla\times\vec{H} = \partial_t \vec{D} \Rightarrow \nabla\times\left(\vec{B}/\mu\right) = \partial_t\left(\varepsilon\vec{E}\right) \overset{?}{\Rightarrow} \nabla\times\vec{B} = \mu\varepsilon\partial_t \vec{E}$$

$$\nabla\cdot\vec{D} = 0 \Rightarrow \nabla\cdot\left(\varepsilon\vec{E}\right) = 0 \overset{?}{\Rightarrow} \nabla\cdot\vec{E} = 0$$

$$\nabla\times(\nabla\times\vec{E}) = -\frac{\partial}{\partial t}(\nabla\times\vec{B}) = -\frac{\partial}{\partial t}\left(\mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right) \overset{?}{=} -\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

✍ 均匀、稳定介质也不行!

$\vec{D}(t) = \varepsilon\vec{E}(t),\ \vec{B}(t) = \mu\vec{H}(t)$ 一般不成立

介质的色散性质

- **一般的介质具有色散性质**：介质对电磁场的相应性质与电磁场的变化频率有关

$$\vec{D}(\omega)=\varepsilon(\omega)\vec{E}(\omega),\ \vec{B}(\omega)=\mu(\omega)\vec{H}(\omega)$$

$$\Rightarrow\quad \vec{D}(t)\neq\varepsilon\vec{E}(t),\ \vec{B}(t)\neq\mu\vec{H}(t)$$

✍ 对一般介质中的电磁场，不满足波动方程

$$\omega\to\infty:\quad \varepsilon\to\varepsilon_0,\quad \mu\to\mu_0$$

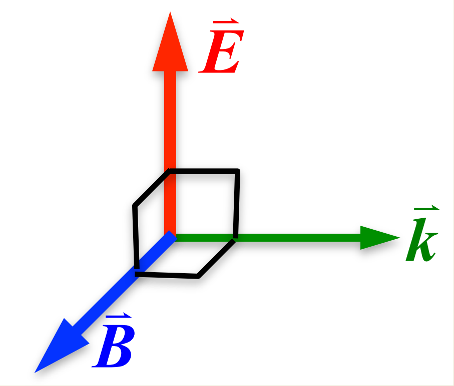
✍ 介质中的微观粒子(如电子)由于其惯性，来不及相应外场的变化

介质中的单色平面波

考察无源情形下绝缘介质中传播的单色平面波

$$\vec{E} = \vec{E}_0 \exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right] \quad \text{and} \quad \vec{B} = \vec{B}_0 \exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right]$$

- **Gauss定律** $\nabla\cdot\vec{D}=0$ → $\vec{k}\cdot\vec{E}=0$
- **磁无源** $\nabla\cdot\vec{B}=0$ → $\vec{k}\cdot\vec{B}=0$
- **Faraday定律** $\nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}$ → $\vec{B}=\dfrac{\vec{k}\times\vec{E}}{\omega}$



- **电磁波为横波，(E、B、k)构成右手螺旋系，且 $B=kE/\omega$**

- **A/M 定律** $\nabla\times\vec{H}=\dfrac{\partial\vec{D}}{\partial t}$ → $\vec{E}=-\dfrac{\vec{k}\times\vec{B}}{\omega\varepsilon\mu}$ —Gauss / Faraday→ $k^2=\omega^2\mu\varepsilon$

- 相速度：$v_p=\omega/k=1/\sqrt{\varepsilon\mu}$
- 振幅关系：$E=v_p B$

色散

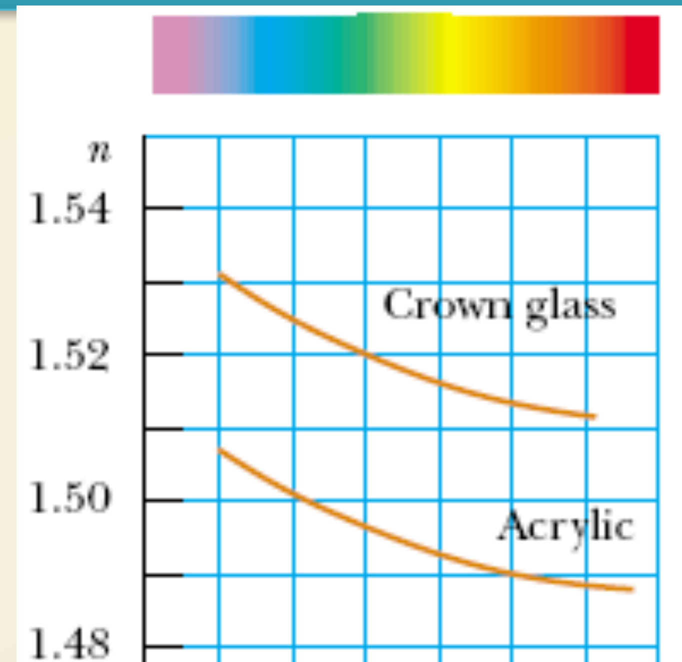
电磁波在介质中的速度为：

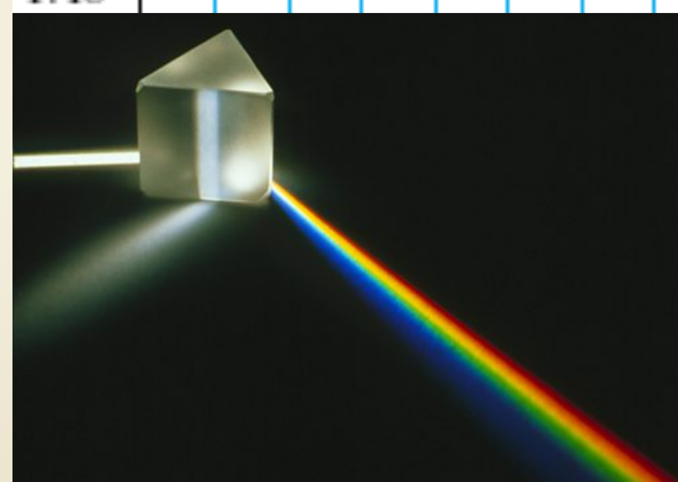
$$v=\frac{1}{\sqrt{\varepsilon\mu}}=\frac{c}{n}$$

- 折射率：$n=\sqrt{\varepsilon\mu/\varepsilon_0\mu_0}=\sqrt{\varepsilon_r\mu_r}$
  - 对于大多数介质$\mu_r\approx 1$，因而

$$n\approx\sqrt{\varepsilon_r}$$

  - 折射率通常与电磁波的频率有关，因而不同频率或者波长的光在同一介质中的速度不同，这种现象谓之“色散”





导体中的电荷密度

■ 导体内部的电磁场方程

$$\begin{cases} \nabla\times\vec{E}=-\partial_t\vec{B} \\ \nabla\times\vec{H}=\vec{j}_0+\partial_t\vec{D} \\ \nabla\cdot\vec{D}=\rho_0 \\ \nabla\cdot\vec{B}=0 \end{cases}$$

$$\partial\rho_0/\partial t=-\nabla\cdot\vec{j}_0$$

$$\vec{j}_0=\sigma\vec{E}$$

$$\frac{\partial\rho_0}{\partial t}=-\nabla\cdot\vec{j}_0=-\sigma\nabla\cdot\vec{E}=-\frac{\sigma}{\varepsilon}\rho_0$$

$$\rho_0(\vec{r},t)=\rho_0(\vec{r},0)\exp\left(-\frac{\sigma}{\varepsilon}t\right)$$

■ 导体内电荷密度随时间指数衰减，时间尺度为：

$$\tau=\varepsilon/\sigma\sim10^{-17}\ \text{s}$$

◆ f<<10^{17} Hz (λ>>3 nm)：良导体内部不存在自由电荷

◆ 假设 $\rho_0=0$，但是 $\vec{j}_0=\sigma\vec{E}$

导体中的单色平面波

考察导体中的单色平面波 $\vec{E}=\vec{E}_0 e^{i\phi},\ \vec{B}=\vec{B}_0 e^{i\phi}$ where $\phi=\vec{k}\cdot\vec{r}-\omega t$

- **Gauss定律** $\nabla\cdot\vec{E}=0 \Rightarrow \vec{k}\cdot\vec{E}=0$
- **磁无源** $\nabla\cdot\vec{B}=0 \Rightarrow \vec{k}\cdot\vec{B}=0$
- **Faraday定律** $\nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \Rightarrow \vec{B}=\dfrac{\vec{k}\times\vec{E}}{\omega}$
- **A/M 定律** $\nabla\times\vec{B}=\mu\sigma\vec{E}+\mu\varepsilon\partial_t\vec{E} \Rightarrow \vec{k}\times\vec{B}=-\left(\omega\mu\varepsilon+i\mu\sigma\right)\vec{E}$

Gauss / Faraday $\Rightarrow$

$$\vec{k}\cdot\vec{k}=k_x^2+k_y^2+k_z^2=\omega^2\mu\varepsilon+i\omega\mu\sigma$$

✍ k 必为复矢量： $\vec{k}=\vec{\beta}+i\vec{\alpha}$ $\Rightarrow$

$$\begin{cases}\beta^2-\alpha^2=\omega^2\mu\varepsilon\\ 2\vec{\alpha}\cdot\vec{\beta}=\omega\mu\sigma\end{cases}$$

导体中的单色平面波续

$$\begin{cases}\vec{E}(\vec{r},t)=\vec{E}_0 e^{-\vec{\alpha}\cdot\vec{r}} e^{i(\vec{\beta}\cdot\vec{r}-\omega t)} \\ \vec{B}(\vec{r},t)=\vec{B}_0 e^{-\vec{\alpha}\cdot\vec{r}} e^{i(\vec{\beta}\cdot\vec{r}-\omega t)}\end{cases},\quad \begin{cases}\beta^2-\alpha^2=\omega^2\mu\varepsilon \\ 2\vec{\alpha}\cdot\vec{\beta}=\omega\mu\sigma\end{cases}$$

波沿着 α 方向衰减，沿着 β 方向传播

- 如果 $\vec{\alpha}=\alpha\hat{z}$ and $\vec{\beta}=\beta\hat{z}$

$$\begin{cases}\beta^2-\alpha^2=\omega^2\mu\varepsilon \\ 2\alpha\beta=\omega\mu\sigma\end{cases} \Rightarrow \begin{cases}\alpha=\omega\sqrt{\dfrac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\dfrac{\sigma}{\varepsilon\omega}\right)^2}-1\right]^{1/2} \\ \beta=\omega\sqrt{\dfrac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\dfrac{\sigma}{\varepsilon\omega}\right)^2}+1\right]^{1/2}\end{cases}$$

导体中的传导电流与位移电流(不要求)

■ 传导电流与位移电流

$$j_d = \partial_t D = -i\omega\varepsilon E = -i\frac{\varepsilon\omega}{\sigma} j_0$$

$$\begin{cases} \alpha = \omega\sqrt{\dfrac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\dfrac{\sigma}{\varepsilon\omega}\right)^2}-1\right]^{1/2} \\ \beta = \omega\sqrt{\dfrac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\dfrac{\sigma}{\varepsilon\omega}\right)^2}+1\right]^{1/2} \end{cases} \qquad \frac{\sigma}{\varepsilon\omega} = \frac{1}{\omega\tau}$$

■ 如果 $\dfrac{\sigma}{\varepsilon\omega} << 1 \text{ or } j_0 << j_d$

$$\Rightarrow \alpha = \frac{1}{2}\sigma\sqrt{\frac{\mu}{\varepsilon}} = \frac{1}{2\tau}\sqrt{\mu\varepsilon}, \quad \beta = \omega\sqrt{\mu\varepsilon}$$

■ 只出现于不良导体，或导电性虽好但 EMW 频率极高

■ 如果 $\dfrac{\sigma}{\varepsilon\omega} >> 1 \text{ or } j_0 >> j_d$ $\Rightarrow$ $\alpha = \beta = \sqrt{\dfrac{\mu\sigma\omega}{2}}$

■ 金属导体总是属于此情形：$\tau \sim 10^{-17}$s

■ 当频率达到10^{17}s^{-1}时，宏观经典理论已经不能适用

不良导体(不要求)

$$\frac{\sigma}{\varepsilon\omega}=\frac{1}{\omega\tau}<<1 \Rightarrow \alpha=\frac{1}{2}\sigma\sqrt{\frac{\mu}{\varepsilon}}=\frac{1}{2\tau}\sqrt{\mu\varepsilon},\quad \beta=\omega\sqrt{\mu\varepsilon}$$

- 电磁波行为与绝缘介质相近 $\tilde{\varepsilon}=\varepsilon+i\frac{\sigma}{\omega}\simeq\varepsilon$
  - 相速度接近于绝缘介质中的速度 $v=\omega/\beta\approx c/n$
  - 衰减长度
    - 经过很多波长的距离，其振幅衰减为原来的1/e

$$\frac{d}{\lambda}=\frac{\beta}{2\pi\alpha}=\frac{\omega\tau}{\pi}>>1$$

    - 绝对值与频率无关，对于海水 d～1cm

$$d=\frac{1}{\alpha}=2v\tau$$

良导体(不要求)

$$\frac{\sigma}{\varepsilon\omega}=\frac{1}{\omega\tau}>>1 \Rightarrow \alpha=\beta=\sqrt{\frac{\mu\sigma\omega}{2}}$$

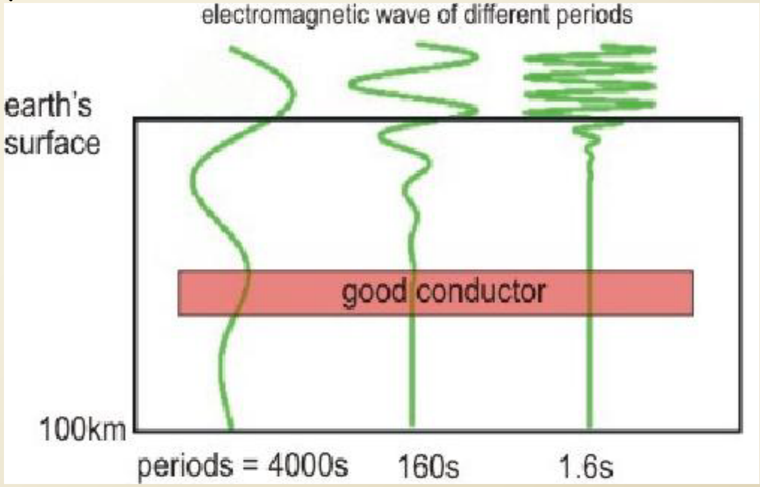
- 相速度 v 远小于绝缘介质中的速度 v_0

$$v=\omega/\beta\approx\sqrt{2\omega/\mu\sigma}=\sqrt{2\omega\tau/\mu\varepsilon}=v_0\sqrt{2\omega\tau}<<v_0$$

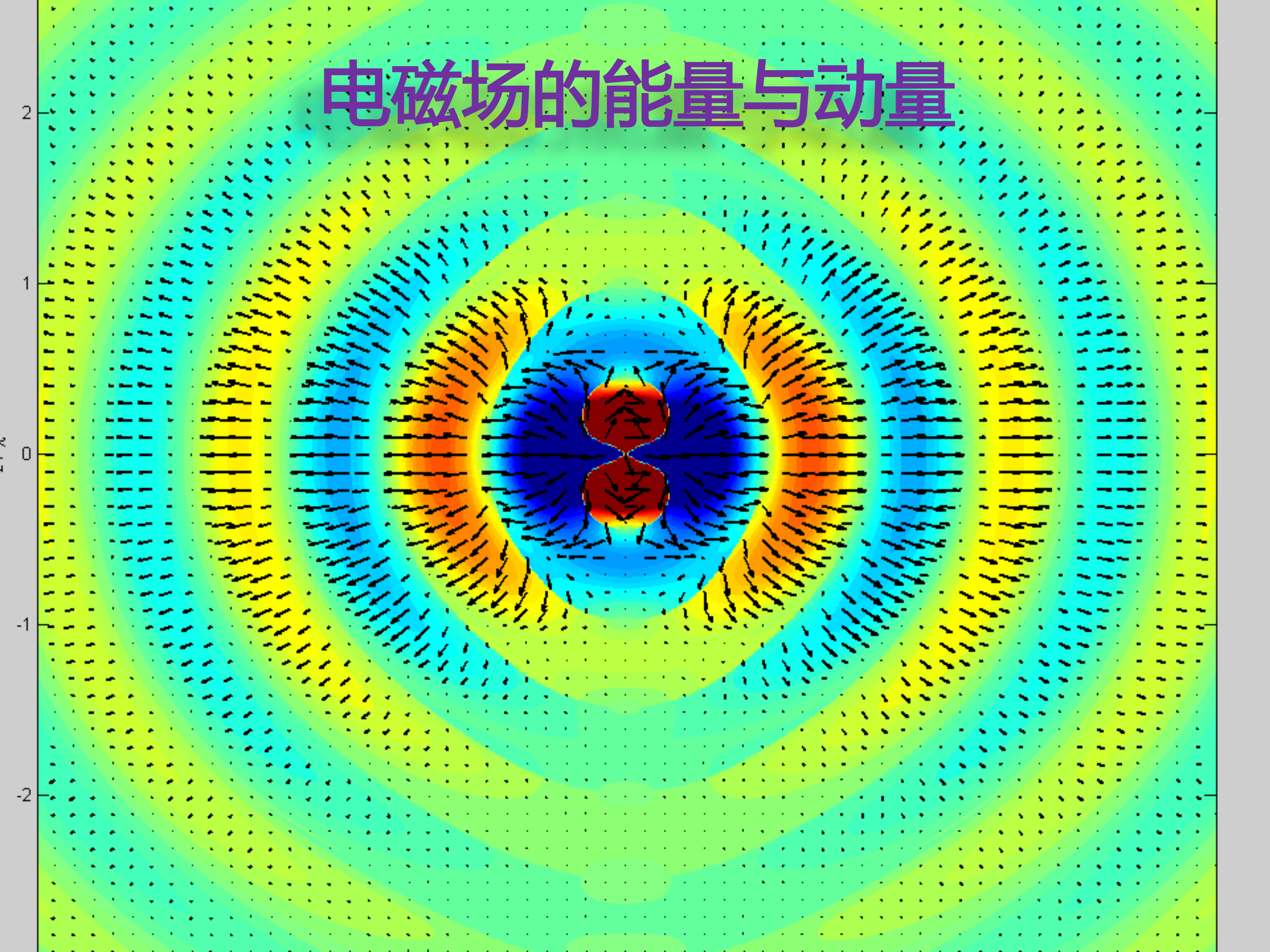
- 衰减长度为波长的 $1/2\pi$ $\quad d=1/\alpha=\lambda/2\pi$
  - 绝对值与频率有关，对于海水 d=1 km → $\omega\sim 0.5\ \mathrm{s}^{-1}$
- B 与 E $\quad \vec{k}=\vec{\beta}+i\vec{\alpha}=\sqrt{\mu\sigma\omega}\,e^{i\pi/4}\hat{n}$

$$\Rightarrow \vec{B}_0=\sqrt{\frac{\mu\sigma}{\omega}}e^{i\pi/4}\hat{n}\times\vec{E}_0$$

  - B与E之间有相位差 $\pi/4$



电磁场的能量与动量

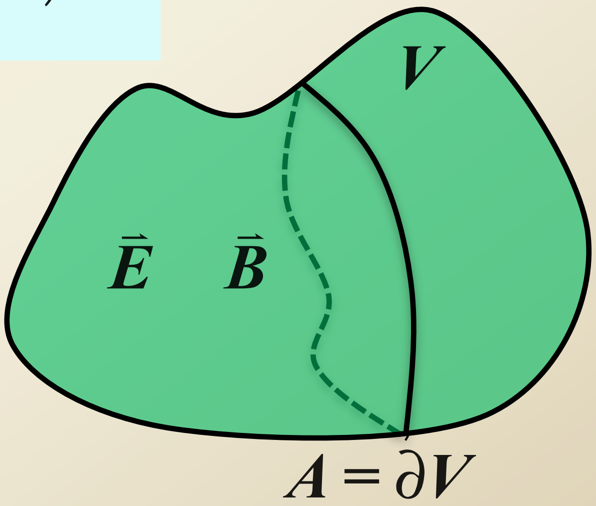


电磁场的能量

- **电磁场的能量**
  - 考察以闭曲面 A 为边界的区域 V，其中有 E 和 B

能量密度：$$w=\frac{1}{2}\varepsilon_0 E^2+\frac{1}{2\mu_0}B^2=\frac{1}{2}\varepsilon_0\left(E^2+c^2B^2\right)$$

总的能量：$$W=\int_V w\,dV=\frac{1}{2}\varepsilon_0\int_V\left(E^2+c^2B^2\right)dV$$



- **能量如何传播？**
  - 能量如何随时间变化?

$$\frac{dW}{dt}=\int_V\frac{\partial w}{\partial t}dV=\varepsilon_0\int_V\left(\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}+c^2\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}\right)dV$$

玻印廷(Poynting)矢量

- 能量随时间的变化 $$\frac{dW}{dt}=\varepsilon_0\int_V\left(\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}+c^2\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}\right)dV$$

- 真空中：$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$ and $\nabla\times\vec{B}=\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$

$$\Rightarrow\quad \frac{dW}{dt}=\varepsilon_0 c^2\int_V\left[\left(\nabla\times\vec{B}\right)\cdot\vec{E}-\left(\nabla\times\vec{E}\right)\cdot\vec{B}\right]dV$$

- 由于：$\nabla\cdot\left(\vec{E}\times\vec{B}\right)=\left(\nabla\times\vec{E}\right)\cdot\vec{B}-\left(\nabla\times\vec{B}\right)\cdot\vec{E}$

$$\Rightarrow\quad \frac{dW}{dt}=-\varepsilon_0 c^2\int_V\nabla\cdot\left(\vec{E}\times\vec{B}\right)dV=-\int_V\nabla\cdot\vec{S}\ dV$$

- 玻印廷矢量： $$\vec{S}\equiv\varepsilon_0 c^2\vec{E}\times\vec{B}=\frac{1}{\mu_0}\vec{E}\times\vec{B}$$

玻印廷矢量的含义

$$\frac{dW}{dt}=-\int_V \nabla\cdot\vec{S}\ dV$$

Guass定理 →

$$-\frac{dW}{dt}=\oint_{A=\partial V}\vec{S}\cdot d\vec{a}=\Phi_S(A)$$

区域 V 内电磁场能量随时间减少的速率

等于 穿过区域边界 A 的玻印廷矢量的通量

- 玻印廷矢量的物理解释：

$$\vec{S}\equiv\varepsilon_0 c^2\vec{E}\times\vec{B}=\frac{1}{\mu_0}\vec{E}\times\vec{B}$$

  - $\vec{S}$:指向电磁场能量流动的方向
  - $\vec{S}\cdot d\vec{a}$:穿过 $d\vec{a}$ 的功率(单位时间流过 $d\vec{a}$的电磁场能量)

- 电磁场能量守恒的微分形式

$$-\frac{dW}{dt}=\oint_{A=\partial V}\vec{S}\cdot d\vec{a}$$

Gauss定理 →

$$-\partial_t w=\nabla\cdot\vec{S}$$

喔~~~似曾相识？？？

应用一：单色平面波

$$\vec{E}=E_0\cos(kz-\omega t)\hat{x},\quad \vec{B}=B_0\cos(kz-\omega t)\hat{y}$$

- **玻印廷矢量**：$\vec{S}=\dfrac{1}{\mu_0}\vec{E}\times\vec{B}=c\varepsilon_0E_0^2\cos^2(kz-\omega t)\hat{k}$
- **能量密度**：$w=\dfrac{1}{2}\varepsilon_0\left(E^2+c^2B^2\right)=\varepsilon_0E_0^2\cos^2(kz-\omega t)$

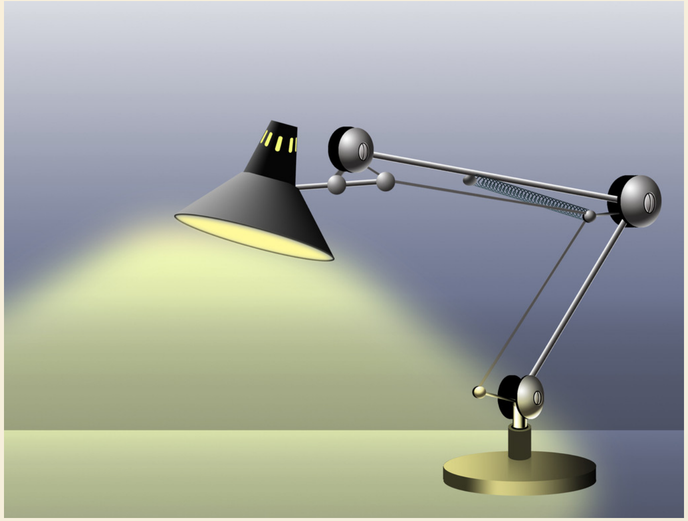
⇒ $\vec{S}=w\vec{c}=wc\hat{k}$ **能流密度**

- 类比 $\vec{j}=\rho\vec{v}$ ⇒ **S 描述能量流动**
- 通常电磁波的振荡极快(可见光～10^{14} Hz)
  →有意义的是其平均值

- **电磁波的平均能流密度 <S> 与强度 I**

$$\langle\vec{S}\rangle=\frac{1}{2}c\varepsilon_0E_0^2\hat{k}=\langle w\rangle c\hat{k},\qquad I=\left|\langle\vec{S}\rangle\right|=\frac{1}{2}c\varepsilon_0E_0^2$$

例：估算台灯 (60 W) 照于桌面上的光的电磁场的振幅。将台灯视为电磁辐射的点源，到桌面的距离为 30 cm；假设电能转化为可见光的效率为 5%。



- 距点源 r 处的电磁波强度为

$$I=\frac{\langle P\rangle}{4\pi r^2}=c\frac{1}{2}\varepsilon_0 E_0^2=\frac{E_0^2}{2\mu_0 c}$$

- 电磁场最大值

$$E_0=\sqrt{\frac{\mu_0 c\langle P\rangle}{2\pi r^2}}=\sqrt{\frac{(4\pi\times10^{-7})\times(3.0\times10^{8})\times(60\times5\%)}{2\pi(0.30)^2}}=45\ \text{V/m}$$

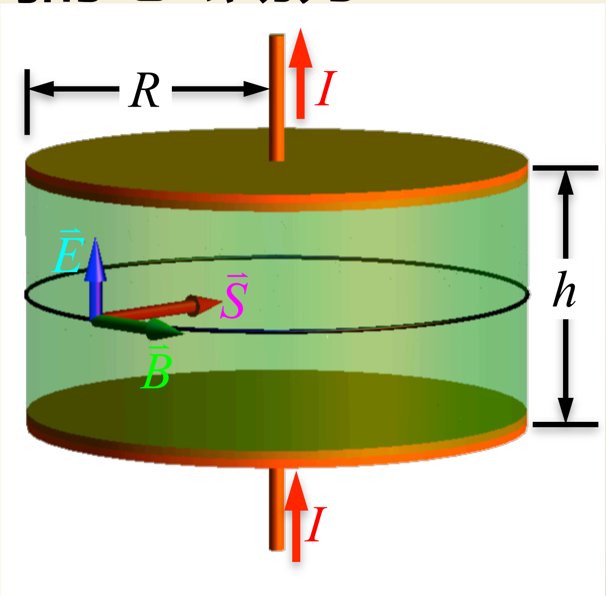
$$B_0=\frac{E_0}{c}=\frac{45}{3.0\times10^{8}}=1.5\times10^{-7}\ \text{T}$$

- 比地磁场(稳定、不随时间变化)小两个量级

应用二：电容器充电

- 设电荷随时间缓慢线性变化*Q*=*It*，极板间的电磁场为

$$\vec{E}=\frac{\sigma}{\varepsilon_0}\hat{z}=\frac{It}{\pi\varepsilon_0 R^2}\hat{z},\quad \vec{B}=\frac{\mu_0 Is}{2\pi R^2}\hat{\phi}$$



- 能流密度 $\vec{S}=\frac{1}{\mu_0}\vec{E}\times\vec{B}=-\frac{I^2 st}{2\pi^2\varepsilon_0 R^4}\hat{s}$

- 能量密度 $w=\frac{I^2t^2}{2\pi^2\varepsilon_0 R^4}+\frac{\mu_0 I^2 s^2}{8\pi^2 R^4}$

- 单位时间由圆柱(*s*=*R*)侧面流入电容器的能量

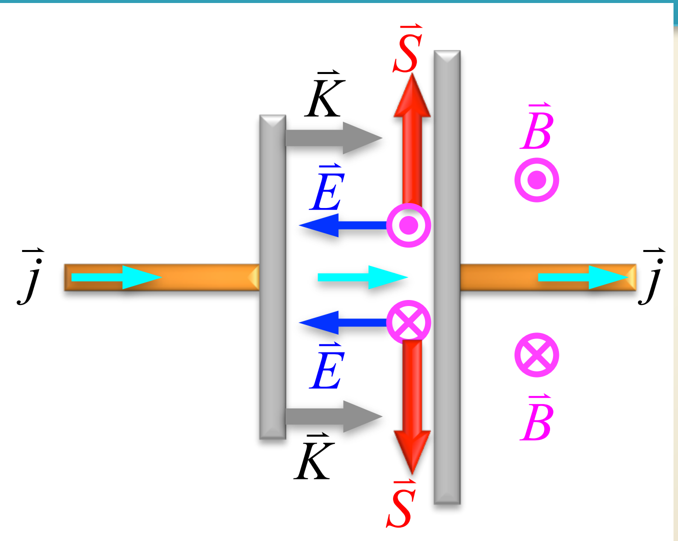
$$-\oiint \vec{S}\cdot d\vec{a}=S(R)\cdot 2\pi Rh=\frac{I^2ht}{\pi\varepsilon_0 R^2}$$

- 单位时间电容器能量的增加

$$\frac{dW}{dt}=\frac{dW_e}{dt}=\frac{dw_e}{dt}\cdot\pi R^2 h=\frac{I^2ht}{\pi\varepsilon_0 R^2}$$

相等

应用三：电路中的能量输运



- 电源内部
  - j 与 K 同向，j 与 E 反向
  - B 总是与电流 j 满足右手法则

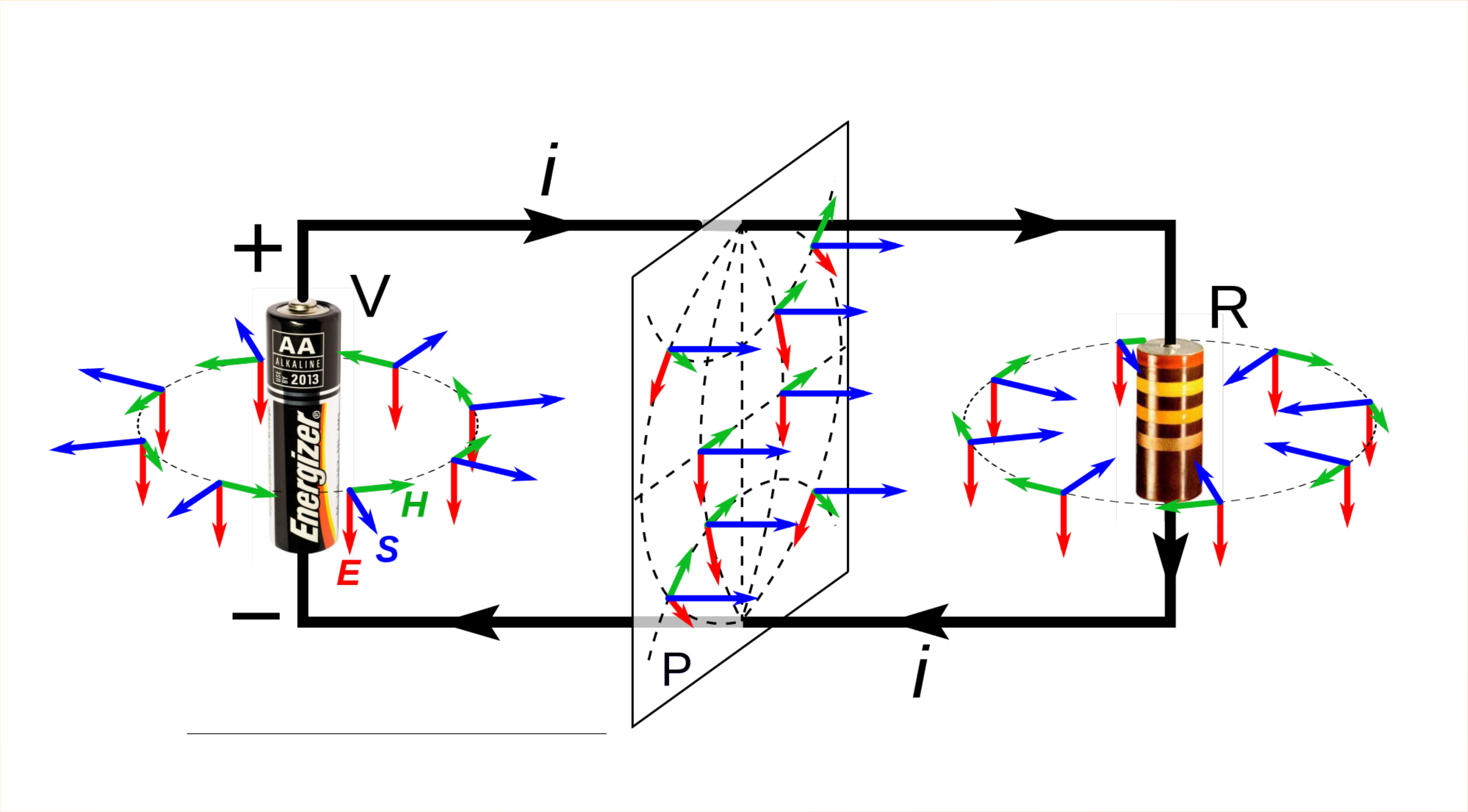
⇨ 电源内部 S 垂直于 j 向外

⇨ **电源向外部空间输出能量**

- 电源外部
  - 导线内：$E_{内}$与 j 同向，故 S 垂直于 j 指向导线内
  - 导线外：$E_{外}$一般有较大的法向分量，但是由于电场切向分量连续，故导线表面外侧的电场或多或少都有些切向分量。故 S 指向导线内部

⇨ **电源外部，电磁场能量向电路聚集**

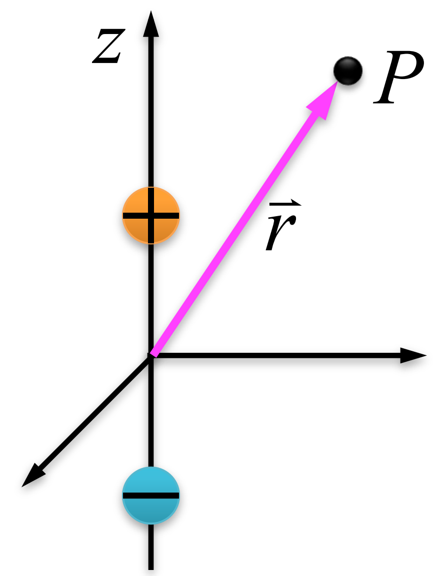
电路中的能量输运



应用四：电偶极辐射(不要求)

- 沿着 z 轴的振荡电偶极子发出的电磁辐射场(球坐标)

$$\begin{cases} \vec{E} = \dfrac{\mu_0 p_0 \omega^2}{4\pi} \sin\theta \dfrac{\cos(kr-\omega t)}{r} \hat{\theta} \\ \vec{B} = \dfrac{\mu_0 p_0 \omega^2}{4\pi c} \sin\theta \dfrac{\cos(kr-\omega t)}{r} \hat{\phi} \end{cases}$$



$$\vec{p} = \hat{z} p_0 \cos\omega t$$

成立条件：$r >> \lambda = \dfrac{2\pi}{k} >> l$

- 辐射沿着径向传播，且与角度有关

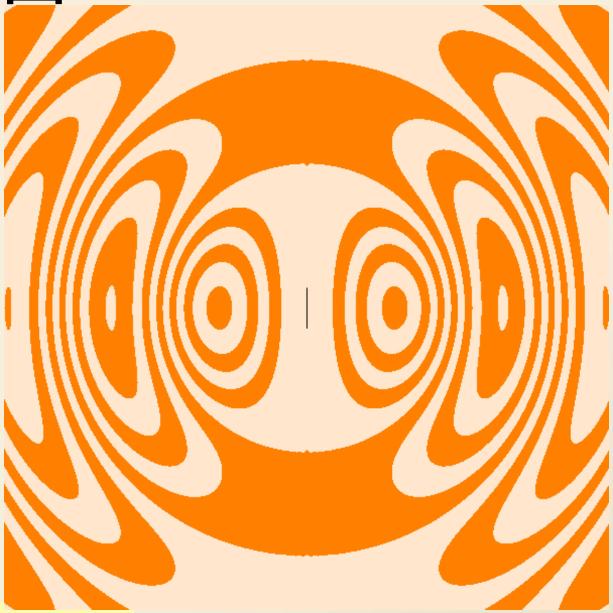
- 坡印廷矢量：

$$\vec{S} = \frac{\mu_0 p_0^2 \omega^4}{16\pi^2 c} \sin^2\theta \frac{\cos^2(kr-\omega t)}{r^2} \hat{r} \quad \Rightarrow \quad \langle \vec{S} \rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2\theta}{r^2} \hat{r}$$

应用四：电偶极辐射续

以电偶极子为中心画一半径为 $R>>l$ 的球面：

- 单位时间由该球面流出去的能量：



$$-\frac{d\langle W\rangle}{dt}=\oint\langle\vec{S}\rangle\cdot d\vec{a}$$

$$=\frac{\mu_0 p_0^2\omega^4}{32\pi^2 c}\oint\frac{\sin^2\theta}{r^2}\hat{r}\cdot d\vec{a}$$

由于 $d\vec{a}=\hat{r}R^2\sin\theta d\theta d\phi$

$$\Rightarrow\quad -\frac{d\langle W\rangle}{dt}=\frac{\mu_0 p_0^2\omega^4}{32\pi^2 cR^2}R^2\int_0^{\pi}\sin^3\theta\,d\theta\int_0^{2\pi}d\phi$$

由于 $\int_0^{\pi}\sin^3\theta\,d\theta=4/3$ $\quad\Rightarrow\quad -\dfrac{d\langle W\rangle}{dt}=\dfrac{\mu_0 p_0^2\omega^4}{12\pi c}$

穿过球面的功率与半径 R 无关，正比于 ω^4

【例】证明在给定的初始条件下，麦克斯韦方程组的解是唯一的。

【解】用反证法。假设存在两组不同的解(E’,B’)和(E”,B”)均满足麦克斯韦方程组，且具有相同的初始条件和边界条件。

t=0时，
$$\vec{E}'(r,0)=\vec{E}''(r,0),\ \vec{B}'(r,0)=\vec{B}''(r,0),$$

在边界时，
$$\vec{E}'(r,t)\Big|_S=\vec{E}''(r,t)\Big|_S,\ \vec{B}'(r,t)\Big|_S=\vec{B}''(r,t)\Big|_S,$$

令：
$$\vec{E}=\vec{E}'-\vec{E}'',\vec{B}=\vec{B}'-\vec{B}''$$

则，E和B满足的麦克斯韦方程为

$$\begin{cases}\nabla\cdot\vec{E}=0,\nabla\cdot\vec{B}=0\\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}\\ \nabla\times\vec{B}=\varepsilon_0\mu_0\dfrac{\partial\vec{E}}{\partial t}\end{cases}$$

并且，**E和B满足齐次初始条件和边界条件**

$$\vec{E}(r,0)=0,\ \vec{B}(r,0)=0,$$

$$\vec{E}(r,t)\Big|_S=0,\ \vec{B}'(r,t)\Big|_S=0,$$

对**(E,B)**的电磁场能量守恒定律为

$$\oint\!\!\!\oint_s \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)\cdot d\vec{S}+\frac{d}{dt}\iiint_V\left(\frac{1}{2}\varepsilon_0E^2+\frac{1}{2\mu_0}B^2\right)dV=-\iiint_V\vec{j}\cdot\vec{E}dV$$

=0　　　　$\vec{j}=0$　=0

$$\iiint_V\left(\frac{1}{2}\varepsilon_0E^2+\frac{1}{2\mu_0}B^2\right)dV=\text{常数}$$

常数=0

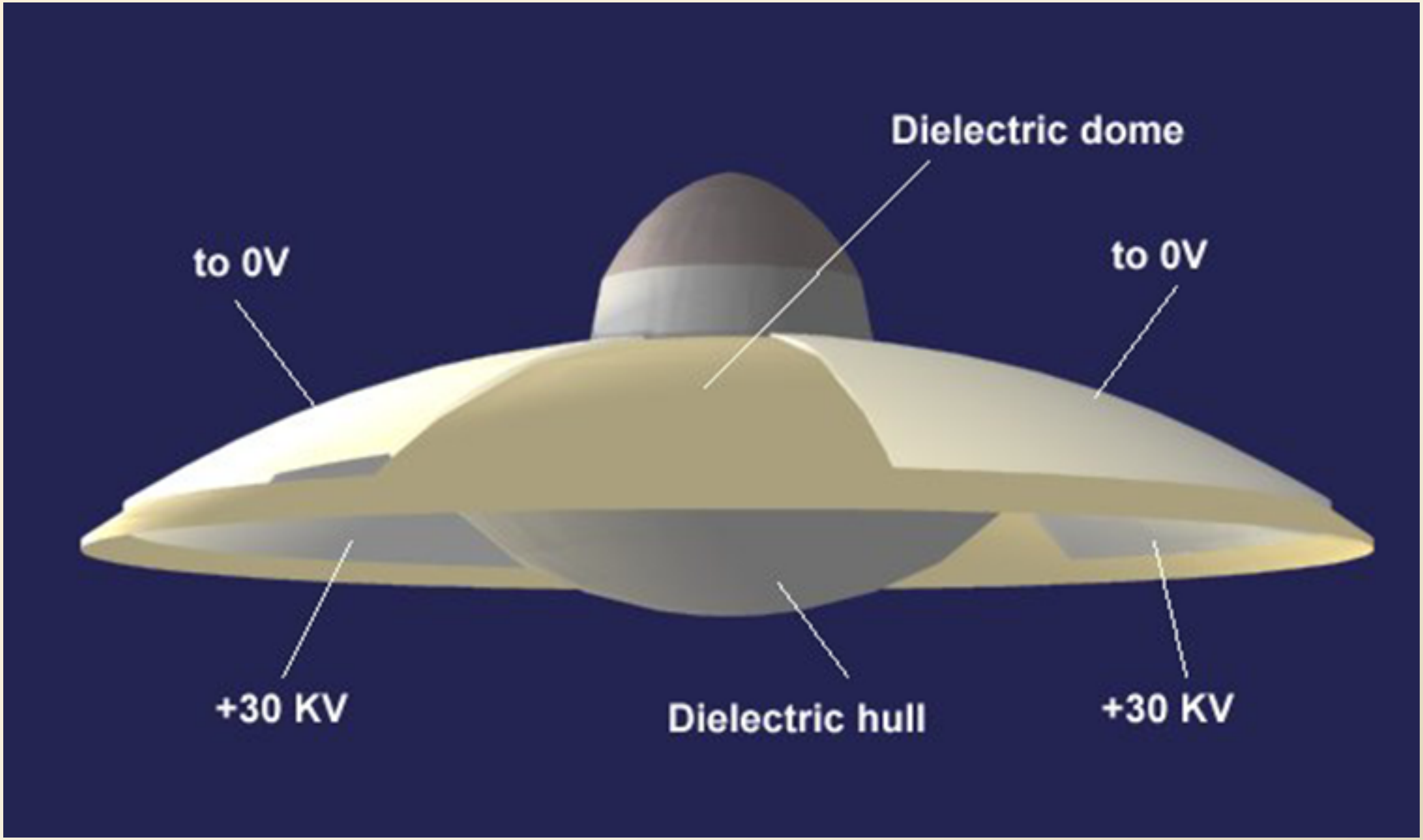
$$\frac{1}{2}\varepsilon_0E^2+\frac{1}{2\mu_0}B^2=0$$

$$\vec{E}(r,t)=0$$

$$\vec{B}(r,t)=0$$

$$\vec{E}'=\vec{E}''$$

$$\vec{B}'=\vec{B}''$$


Dielectric dome
to 0V
to 0V
+30 KV
Dielectric hull
+30 KV


JLN Labs

电磁场能量守恒的严格推导

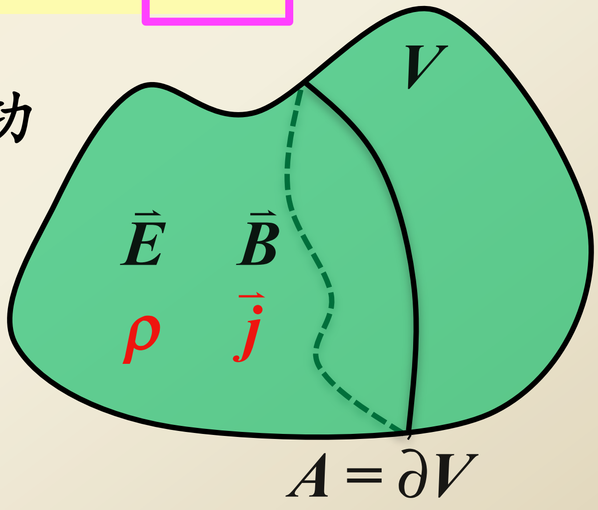
- 物质不仅仅包含电磁场，还有实物粒子
- 电磁场能量由边界 A 流出去会使得 V 内电磁场能量减少，电磁场对 V 内实物做了功，也会造成 V 内电磁场能量减少

- **功率密度**：单位时间对单位体积的实物粒子做的功

$$\vec{f}\cdot\vec{v}=\left(\rho\vec{E}+\vec{j}\times\vec{B}\right)\cdot\vec{v}=\rho\vec{E}\cdot\vec{v}=\rho\vec{v}\cdot\vec{E}=\boxed{\vec{E}\cdot\vec{j}}$$

- 电磁场单位时间对V内的实物做功

$$\boxed{\iiint_V \vec{E}\cdot\vec{j}\,dV}$$



电磁场能量守恒的严格推导续

$$\nabla\times\vec{B}=\mu_0\left(\vec{j}+\varepsilon_0\partial_t\vec{E}\right)$$

$$\vec{E}\cdot\vec{j}=\frac{1}{\mu_0}\vec{E}\cdot(\nabla\times\vec{B})-\varepsilon_0\vec{E}\cdot\partial_t\vec{E}$$

$$=-\frac{1}{\mu_0}\nabla\cdot(\vec{E}\times\vec{B})+\frac{1}{\mu_0}\vec{B}\cdot(\nabla\times\vec{E})-\varepsilon_0\vec{E}\cdot\partial_t\vec{E}$$

$$\nabla\times\vec{E}=-\partial_t\vec{B}$$

$$=-\frac{1}{\mu_0}\nabla\cdot(\vec{E}\times\vec{B})-\frac{1}{\mu_0}\vec{B}\cdot\partial_t\vec{B}-\varepsilon_0\vec{E}\cdot\partial_t\vec{E}$$

$$=-\nabla\cdot\left(\frac{1}{\mu_0}\vec{E}\times\vec{B}\right)-\partial_t\left(\frac{1}{2}\varepsilon_0E^2+\frac{1}{2\mu_0}B^2\right)$$

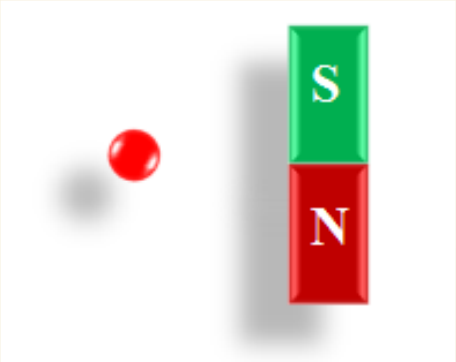
$$\vec{E}\cdot\partial_t\vec{E}=\partial_t\left(\frac{1}{2}E^2\right)$$

$$\vec{B}\cdot\partial_t\vec{B}=\partial_t\left(\frac{1}{2}B^2\right)$$

$$\nabla\cdot(\vec{E}\times\vec{B})=\vec{B}\cdot(\nabla\times\vec{E})-\vec{E}\cdot(\nabla\times\vec{B})$$

Poynting矢量佯谬

一个点电荷放置在条形磁铁旁边，空间将有电场和磁场，玻印廷矢量为



$$\vec{S}=\vec{E}\times\vec{H}$$

意味着有场的能量在传输？

$$\nabla\cdot\vec{S}\equiv 0$$

没有能量传输！

静电场为什么有玻印廷矢量？是否表示静电场与静磁场发生相互作用？

其实，玻印廷矢量是一个人为定义的矢量，真正有意义的是通过一个封闭曲面的数值。

$$\oiint_s \vec{S}\cdot d\vec{s}$$

对静电场

$$\oiint_s \vec{S}\cdot d\vec{s}=0$$

Poynting矢量对静场没有意义

电磁场的动量

- 由狭义相对论，粒子的相对论能量和动量分别为

$$\begin{cases} E = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} \\ \vec{p} = \dfrac{m\vec{v}}{\sqrt{1-v^2/c^2}} \end{cases} \Rightarrow \begin{cases} E = \sqrt{p^2c^2 + m^2c^4} \\ \dfrac{\vec{p}}{E} = \dfrac{\vec{v}}{c^2} \end{cases}$$

  - 无质量粒子(如光子)必然以光速运动，故有 $p = E/c$

- 电磁场的**动量密度** $\vec{g} \equiv \dfrac{1}{c^2}\vec{S} = \varepsilon_0 \vec{E} \times \vec{B}$

- 电磁场的角动量密度 $\vec{l} \equiv \vec{r} \times \vec{g}$

电磁场的动量：量纲分析

$$\vec{S} \equiv \varepsilon_0 c^2 \vec{E} \times \vec{B} = \frac{1}{\mu_0} \vec{E} \times \vec{B}, \quad \vec{g} = \frac{1}{c^2} \vec{S} = \varepsilon_0 \vec{E} \times \vec{B}$$

由于 $\left[\vec{E}\right] = \left[c\right]\left[\vec{B}\right]$ and $\left[w\right] = \left[\varepsilon_0\right]\left[E\right]^2$

$$\Rightarrow \left[\vec{S}\right] = [c][w] = \frac{\text{能量} \times \text{速度}}{\text{体积}} = \frac{\text{能量}}{\text{时间} \times \text{面积}} = \frac{\text{功率}}{\text{面积}}$$

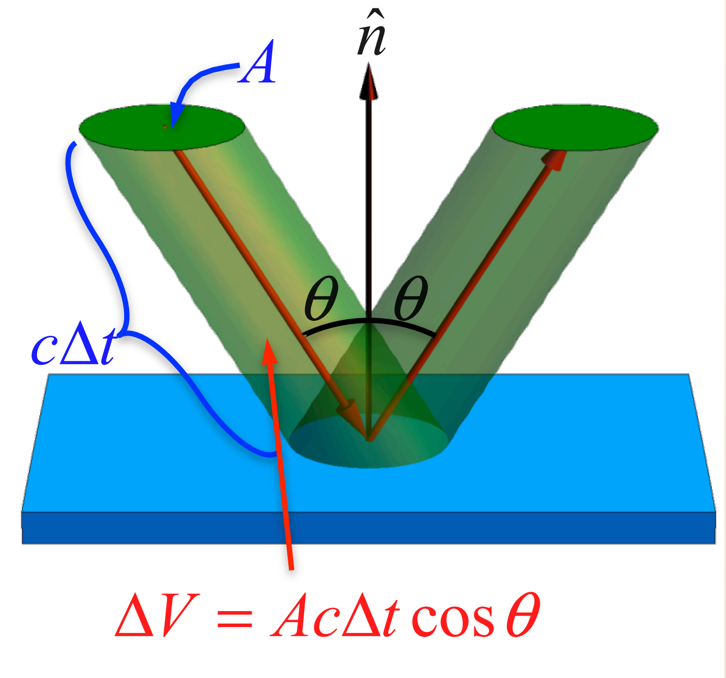
$$\Rightarrow \left[\frac{\vec{S}}{c^2}\right] = \frac{[w]}{[c]} = \frac{\text{能量}}{\text{体积} \times \text{速度}} = \frac{\text{动量}}{\text{体积}}$$

$$\Rightarrow \left[\frac{\vec{S}}{c}\right] = \frac{\text{动量} \times \text{速度}}{\text{体积}} = \frac{\text{动量}}{\text{时间} \times \text{面积}} = \frac{\text{力}}{\text{面积}} = \text{压强}$$

辐射压力

- Δt 内位于截面 A 内的光束法向动量的改变为

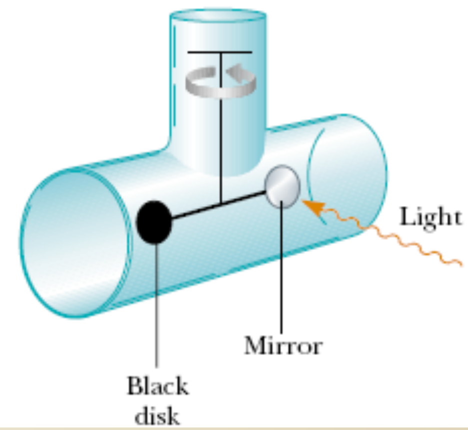
$$\hat{n}\cdot\left(\vec{g}_{反}-\vec{g}_{入}\right)\Delta V$$
$$=A\cdot c\Delta t\cdot\left(\frac{S_{入}}{c^2}+\frac{S_{反}}{c^2}\right)\cos^2\theta$$



- 考虑全反射 $\langle S_{反}\rangle=\langle S_{入}\rangle$

- **光压**：平均而言，物体单位面积受到的压力(压强)

$$p=\frac{2\langle S_{入}\rangle}{c}\cos^2\theta \Rightarrow p=2\langle w\rangle\cos^2\theta$$



例：激光笔功率 3 mW，照射到屏幕上的光斑直径 2 mm，屏幕反射系数为 70%。试确定屏幕上受到的光压。

- 平均的能流密度

$$\langle S\rangle=\frac{\langle P_{\text{laser}}\rangle}{A}=\frac{\langle P_{\text{laser}}\rangle}{\pi r^2}=\frac{3.0\times10^{-3}}{\pi\times(0.001)^2}=955\ \text{W/m}^2$$

- 光压

$$p=(1+0.7)\frac{\langle S\rangle}{c}=5.4\times10^{-6}\ \text{N/m}^2$$

例：当太阳光垂直照射到地面上时，每分钟射到地面每平方厘米上的能量为**1.94 cal**，**1 cal=4.1868 J**， 试求

(1)地面上太阳光的电场强度和磁场强度的振幅 $\boldsymbol{E_0}$ 和 $\boldsymbol{H_0}$。

(2)太阳光作用在整个地球上的力。

- **平均的能流密度** $$\langle S\rangle = \frac{(1.94\ \text{cal})\times(4.1868\ \text{J/cal})}{(60\ \text{s})\times(0.01^2\ \text{m}^2)} = 1.35\times10^3\ \text{W/m}^2$$
- **平均的能量密度** $$\langle w\rangle = \langle S\rangle / c = \frac{1}{2}\varepsilon_0 E_0^2$$
- **电磁场振幅**

$$E_0 = \sqrt{\frac{2\langle S\rangle}{c\varepsilon_0}} = \sqrt{\frac{2\times1354}{(3\times10^8)\times(8.85\times10^{-12})}} = 1.01\times10^3\ \text{V/m}$$

$$B_0 = \frac{E_0}{c} = 3.37\times10^{-6}\ \text{T} \quad \Longrightarrow \quad H_0 = \frac{B_0}{\mu_0} = 2.68\ \text{A/m}$$

■ 作用于面元da上的力的z分量

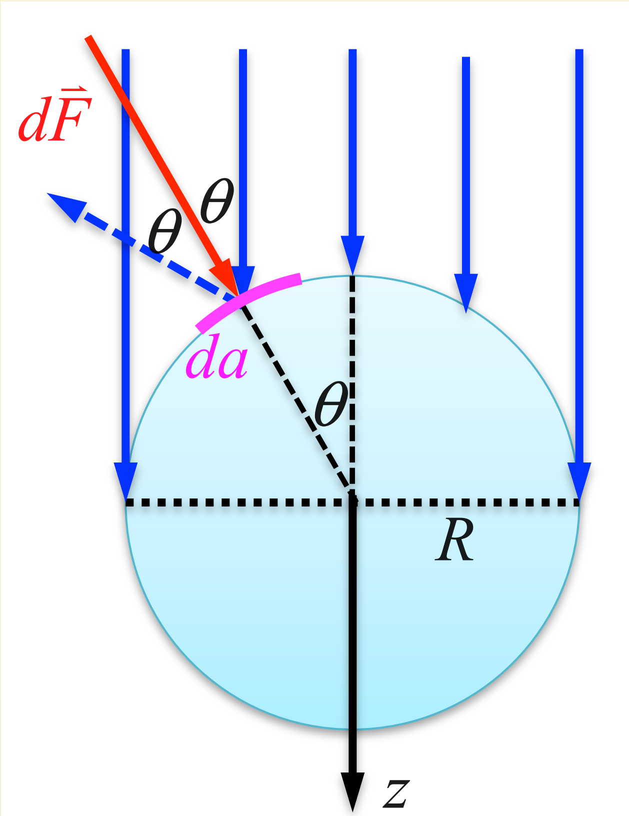
$$\left(d\vec{F}\right)_z=\left(pda\right)\cos\theta=\frac{2\langle S\rangle}{c}\cos^3\theta da$$

■ 作用于地球上的合力

$$F_z=\frac{2\langle S\rangle}{c}R^2\int_0^{\pi/2}\cos^3\theta\sin\theta d\theta\int_0^{2\pi}d\phi$$

$$\Rightarrow F_z=\frac{\langle S\rangle}{c}\pi R^2$$

$$\Rightarrow F_z=\frac{1.35\times10^3}{3\times10^8}\times\pi\times\left(6.4\times10^6\right)^2=5.8\times10^8\ \text{N}$$



■ 相对于太阳对地球的万有引力，这是微不足道的

$$F_{引}=G\frac{Mm}{r^2}=\left(6.67\times10^{-11}\right)\times\frac{\left(2\times10^{30}\right)\times\left(6\times10^{24}\right)}{\left(1.5\times10^{11}\right)^2}=3.6\times10^{22}\ \text{N}$$

电磁场的能量、动量和角动量

■ 能量	能量密度 $w=\frac{1}{2}\vec{D}\cdot\vec{E}+\frac{1}{2}\vec{B}\cdot\vec{H}$	能流密度 (Poynting矢量) $\vec{S}=\vec{E}\times\vec{H}$
■ 动量	动量密度 $\vec{g}=\vec{D}\times\vec{B}$	动量流密度 $\overleftrightarrow{T}=w\overleftrightarrow{I}-\vec{D}\vec{E}-\vec{B}\vec{H}$
■ 角动量	角动量密度 $\vec{l}=\vec{r}\times\vec{g}$	角动量流密度 $\overleftrightarrow{R}=-\overleftrightarrow{T}\times\vec{r}$

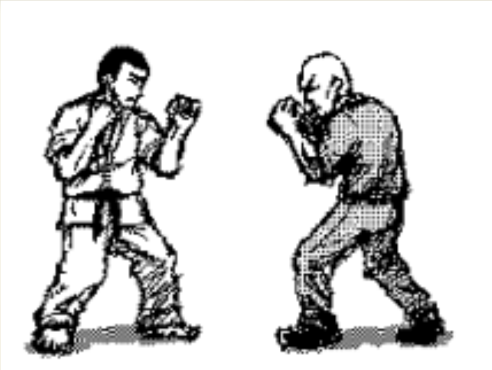
$$\begin{cases} \nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \vec{j}_0 + \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

场是电磁学的核心

- 静电场的自作用能为零！
- 电子产生的场不能对自己产生作用。

Feynman, 1945

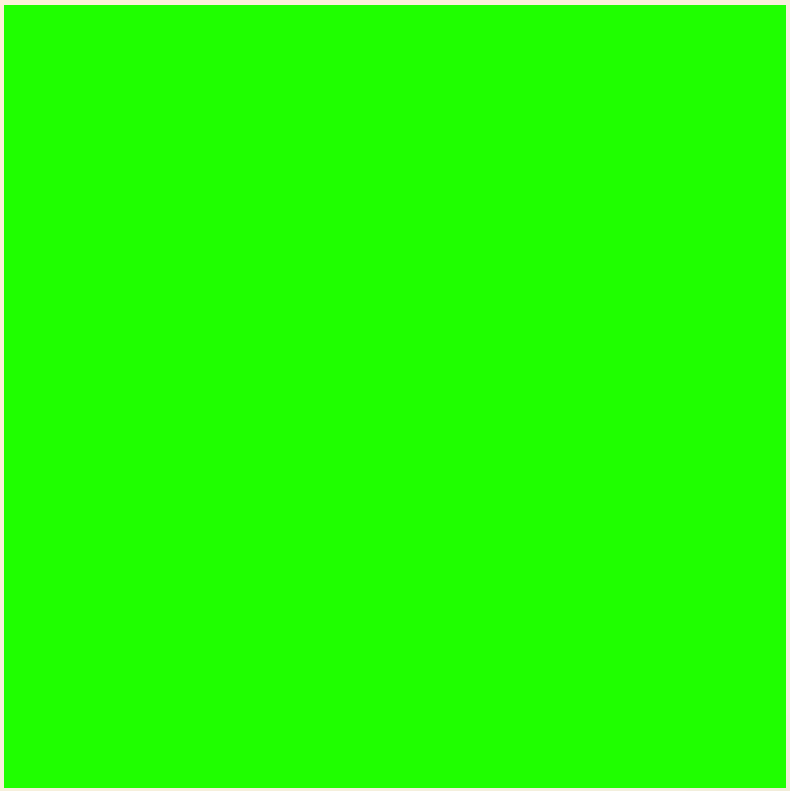
事物的“价值”只有通过同其它物体的“相互作用”才能得到体现！



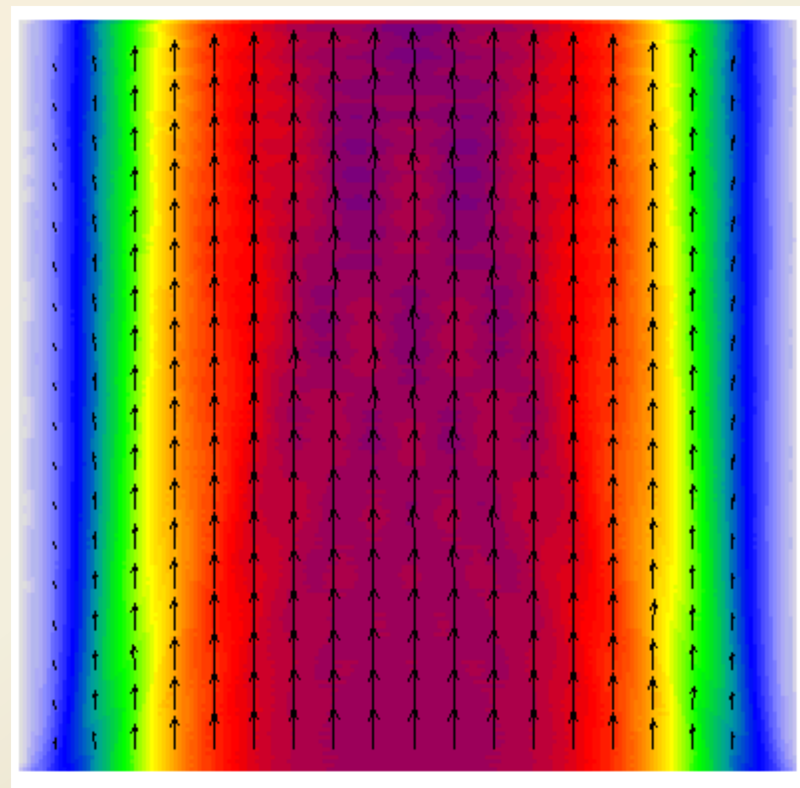
EM in Vacuum

This animation shows the propagation of an electromagnetic wave in a vacuum (refractive index and dielectric constant both 1).

Electric field animation

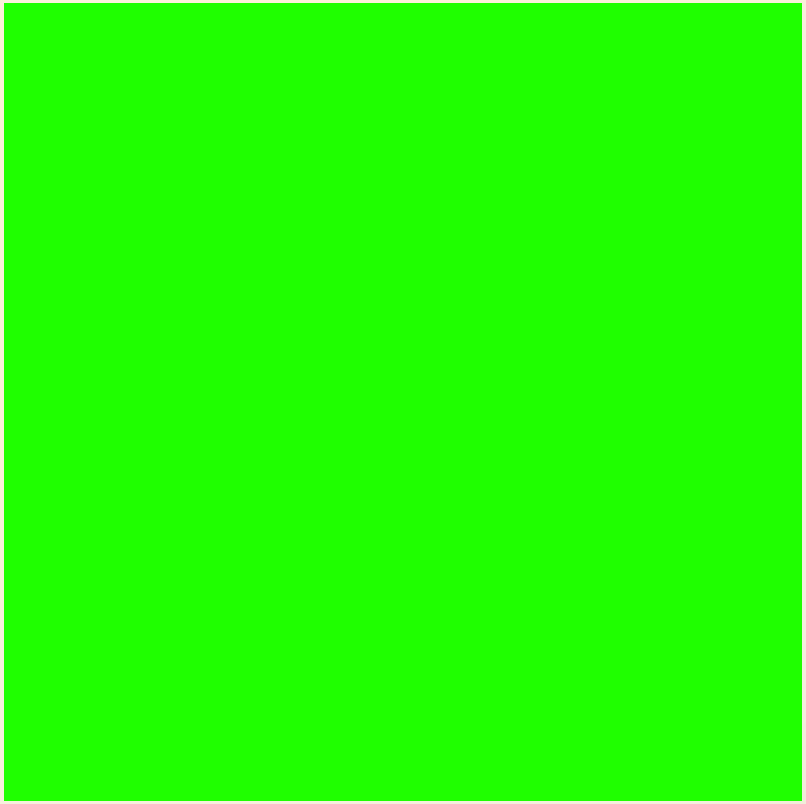


Time-averaged Poynting vector

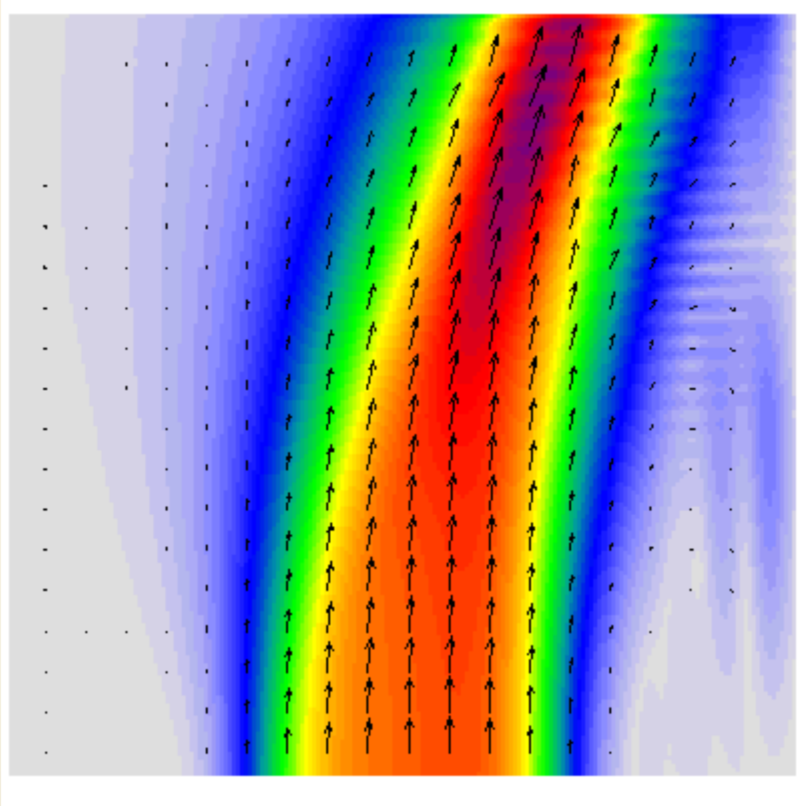


梯度折射率
Refractive index gradient

Electric field animation



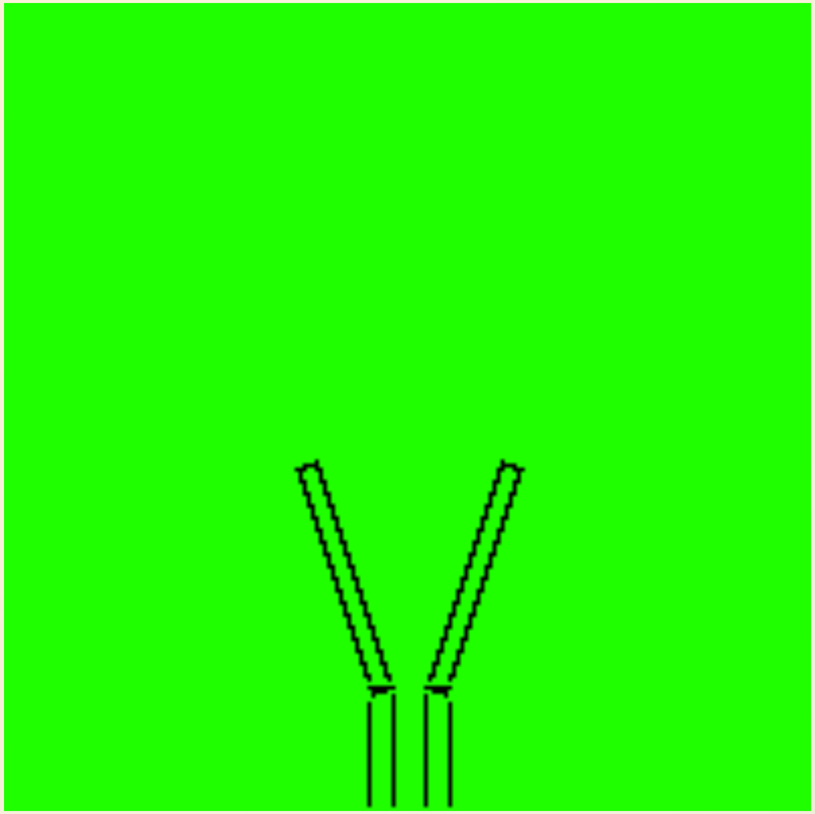
Time-averaged Poynting vector



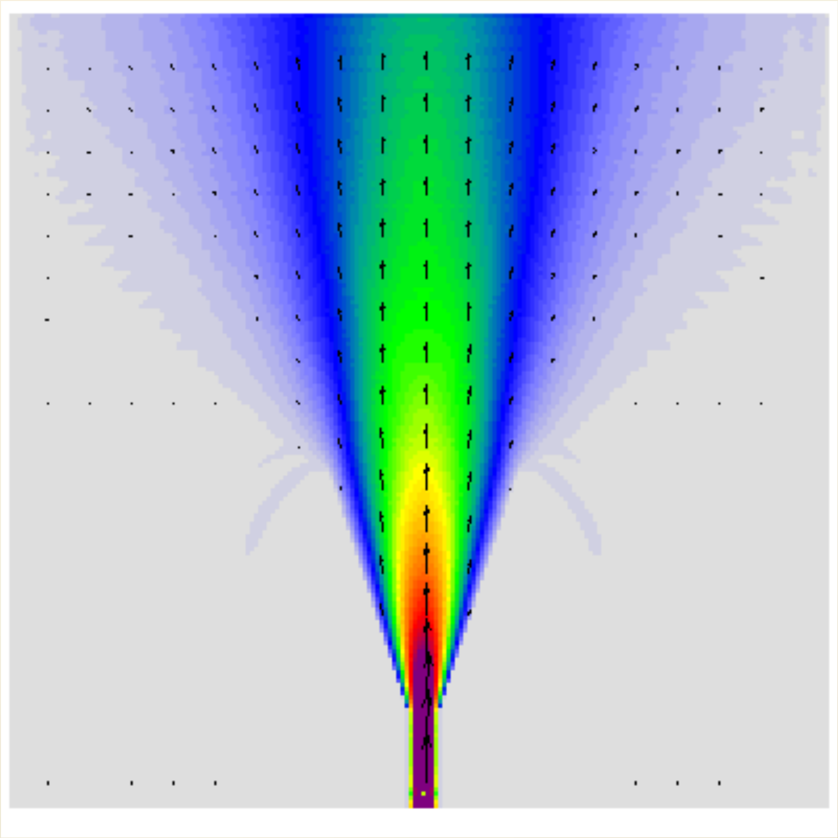
喇叭天线
Horn antenna

A *horn antenna* is commonly used as the feed for a dish antenna, and simply consists of a flaring at the end of a waveguide that results in a reasonably confined beam

Electric field animation



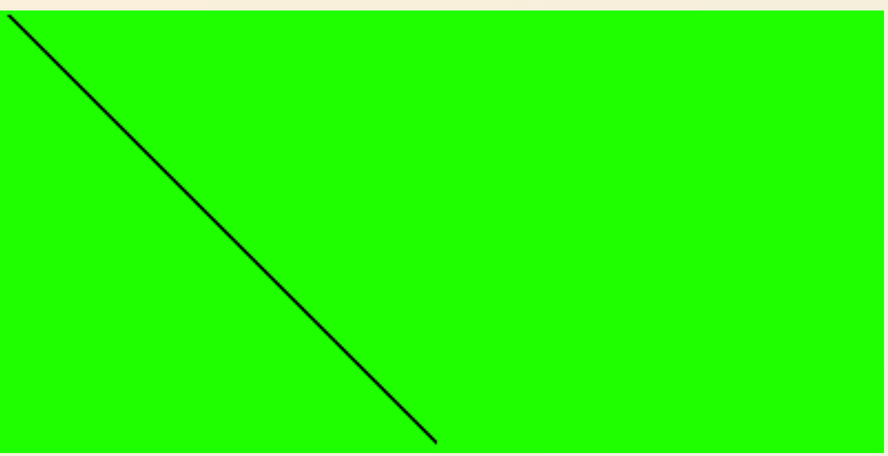
Time-averaged Poynting vector



折射率突变
Sharp change in refractive index

A sharp change in refractive index results in specular reflection from the surface and refraction into the medium according to *Snell's law,* with a reduction of the wave speed. The animation shows two panels: the left shows the complete wave field, while the right shows just the component that is scattered (calculated simply by subtracting the equivalent field in a vacuum). The bending of the light explains why the **bottom of a pool of water appears is nearer than it really is**. This kind of reflection is occasionally seen by clear-air radars as **Fresnel scattering from sharp horizontal gradients in atmospheric refractive index**.

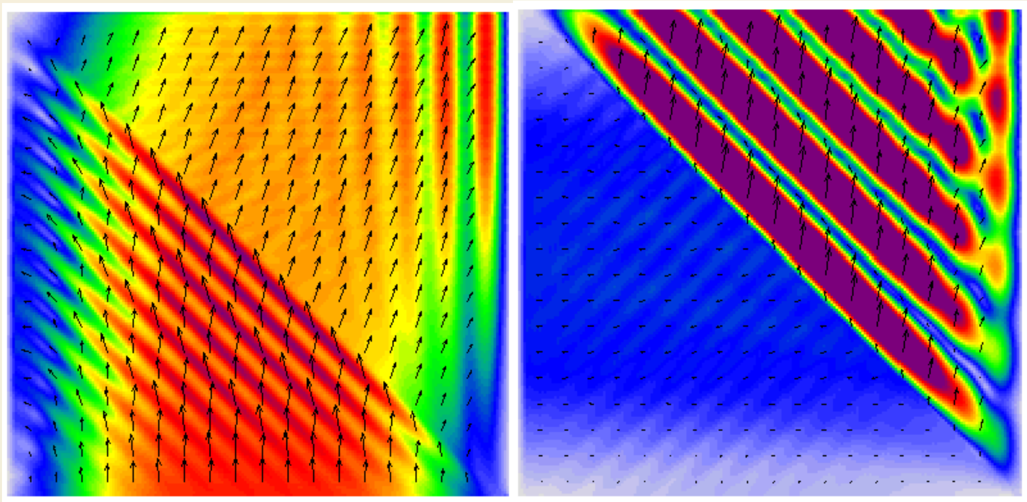
Electric field animation



Total field
(incident plus scattered)

Scattered field
(total minus incident)

Average Poynting vector



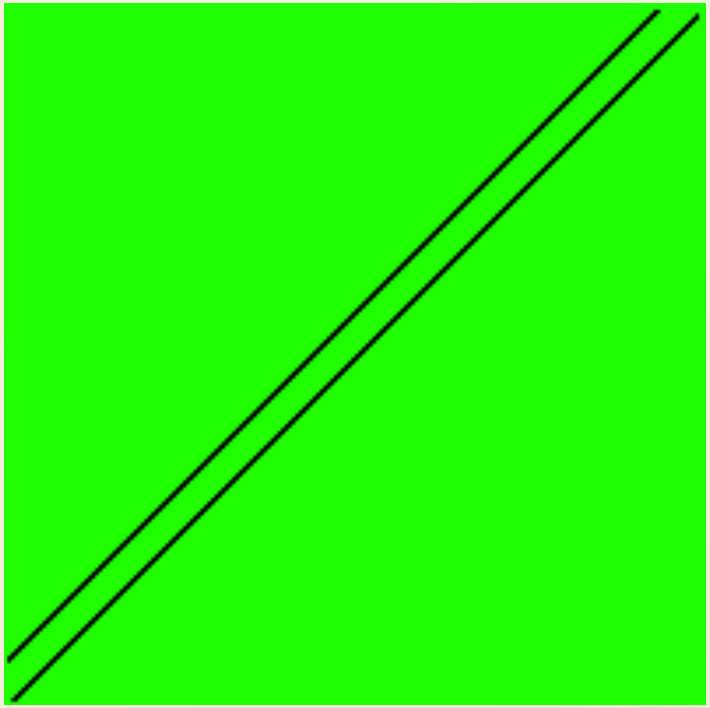
Total field
(incident plus scattered)

Scattered field
(total minus incident)

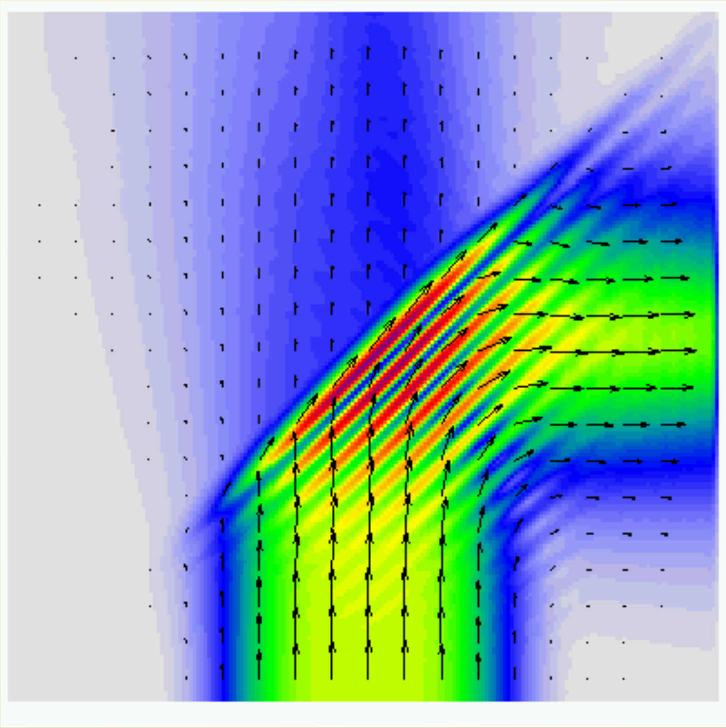
全反射
Total internal reflection

Consider the same set-up as with total internal reflection, but introducing a new medium a distance of order a wavelength away from the original interface, so before the *evanescent wave* has decayed away. In this case, there is still some propagation across the gap, leading to *evanescent wave coupling*.

Electric field animation



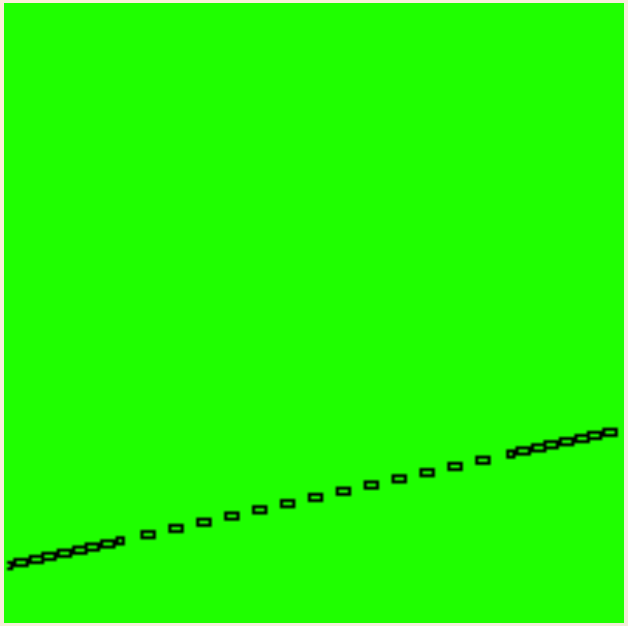
Average Poynting vector



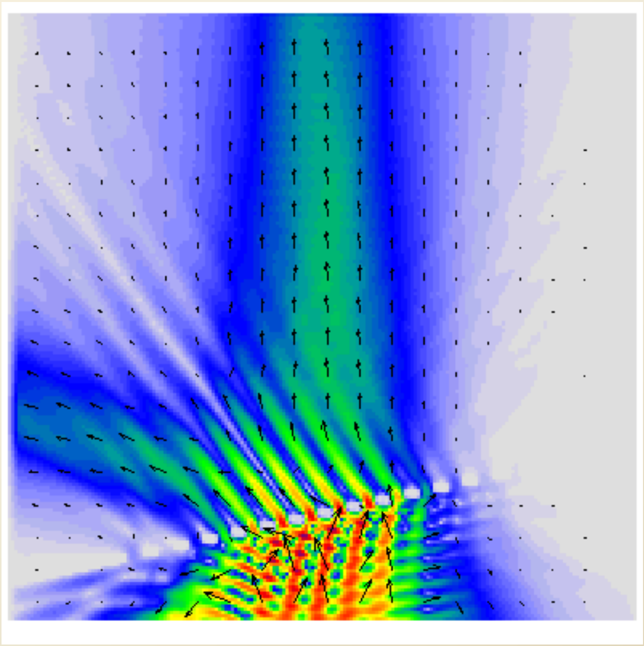
衍射光栅
Diffraction grating(low frequency)

A *diffraction grating* diffracts light by an angle that depends on its wavelength. It consists of a series of equally spaced groves of a similar spacing as the wavelength of the radiation. The simplest way to understand how it works is using *Huygens' Principle;* each grove acts like a separate source of electromagnetic waves, but the phase of each is offset from that of its neighbours to a degree that depends on the angle of the grating with respect to the incident ratiation.

Electric field animation



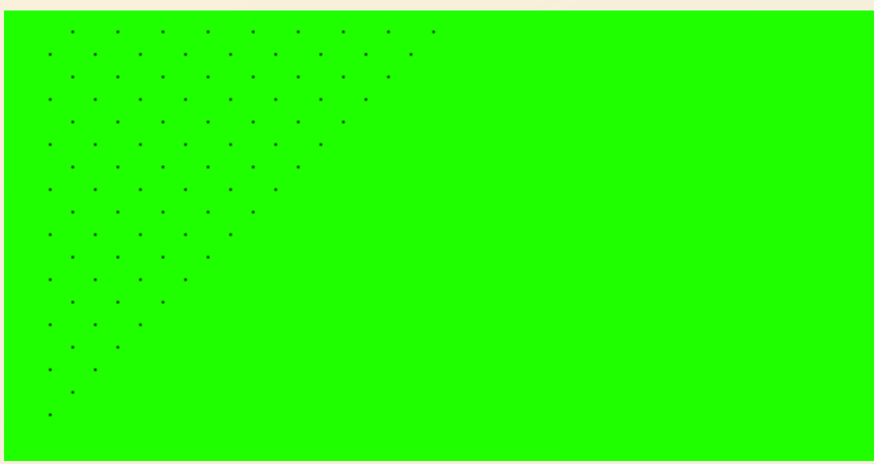
Average Poynting vector



布喇格散射
Bragg scattering

Bragg scattering is used to probe the structure of crystaline solids with X-rays. Each atom in a lattice behaves similarly to a Rayleigh scatterer under the influence of the incident wave, and certain wavelengths result in constructive interference in particular directions. This enables the structure of the material to be inferred. See what happens if the frequency is increased.

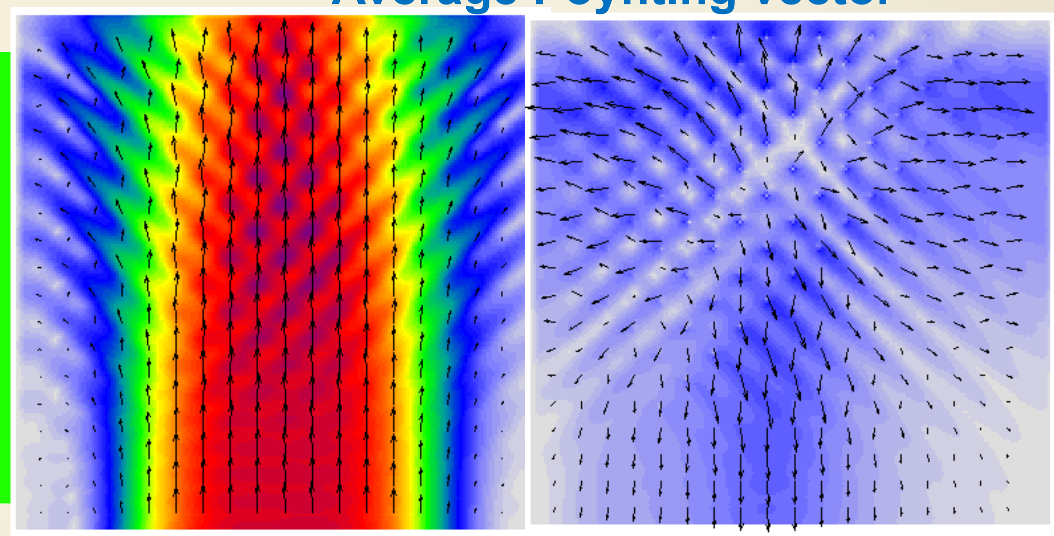
Electric field animation



Total field (incident plus scattered)

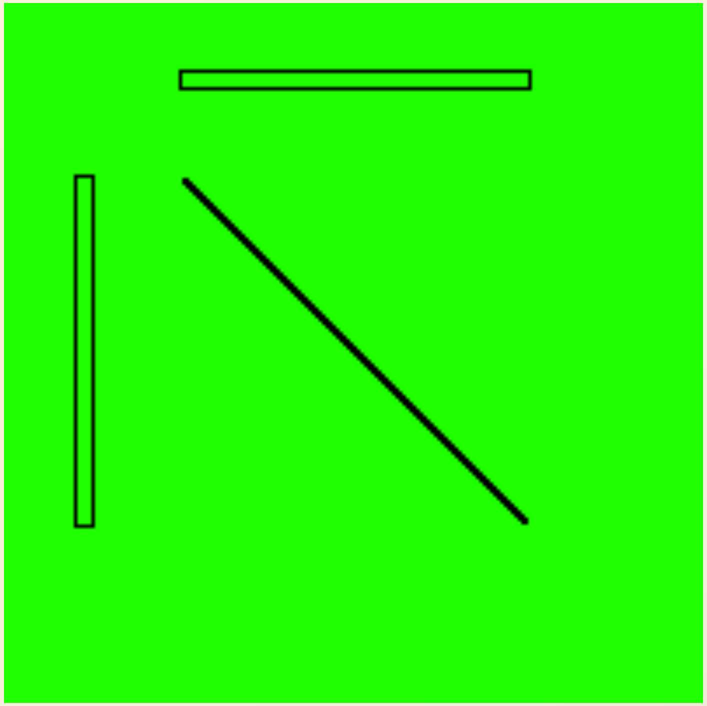
Scattered field (total minus incident)

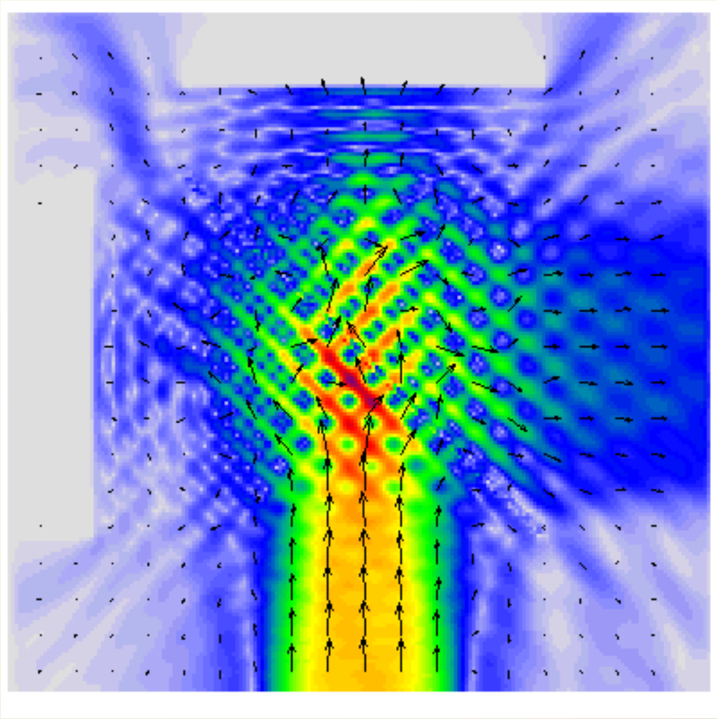
Average Poynting vector



迈克尔逊干涉仪
Michelson interferometer(equal path lengths)

A *Michelson interferometer* was used in the famous *Michelson-Morley* experiment to demonstrate the non-existence of the *luminiferous aether,* a hypothetical medium in which light was believed to travel. In the configuration to the left, coherent light is incident on a 45° half-silvered mirror (in this case simply a thin bar with a refractive index chosen such that half the light is transmitted and half is reflected). The transmitted and reflected beams travel equal paths to ordinary mirrors (in this case simply bars with a very high refractive index), and are reflected back and recombine at the 45° mirror. This results in constructive interference for the outgoing beam to the right.

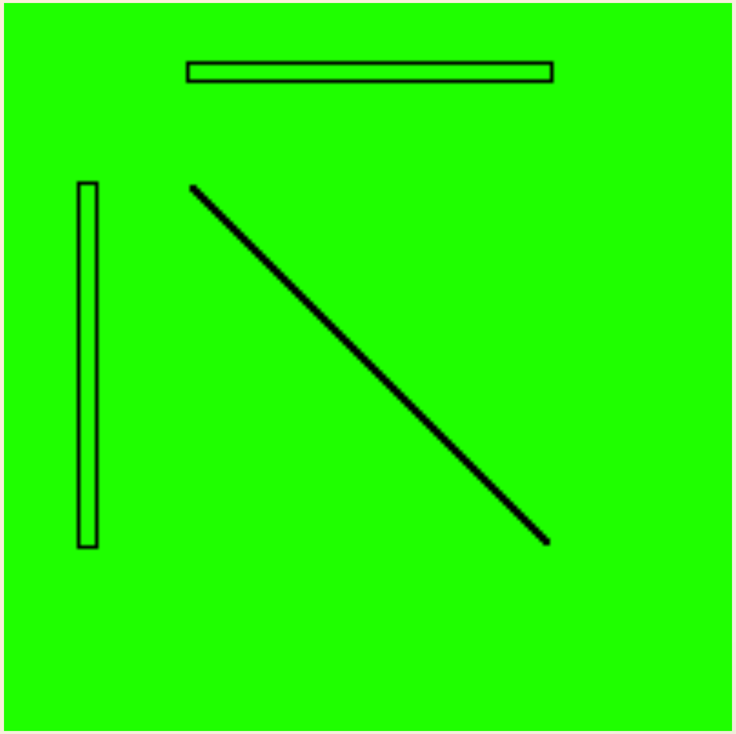




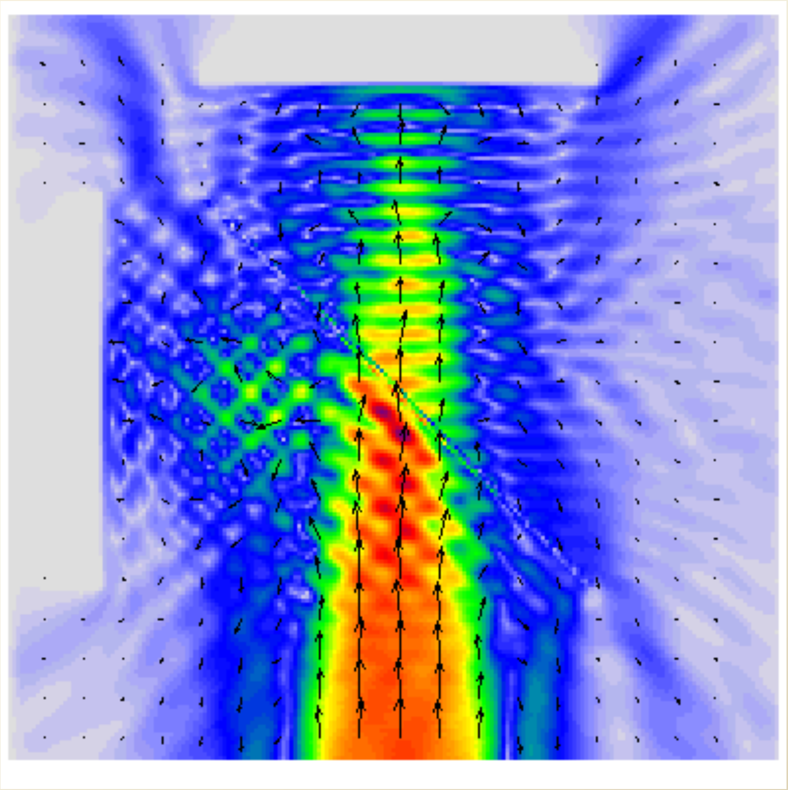
Michelson interferometer, (unequal path lengths)

A *Fourier transform spectrometer* consists simply of a *Michelson interferometer,* but with with one of the two mirrors movable. When one of the path lengths is changed by a quarter of a wavelength (as in this example), the radiation from the two paths destructively interferes in the rightward direction. Instead, the light constructively interferes in the downward direction (unlike in the previous case). The frequency spectrum of incoming light may be obtained by measuring the intensity of the outgoing radiation to the right as a function of the path length, then performing a Fourier Transform on it.

Electric field animation



Average Poynting vector

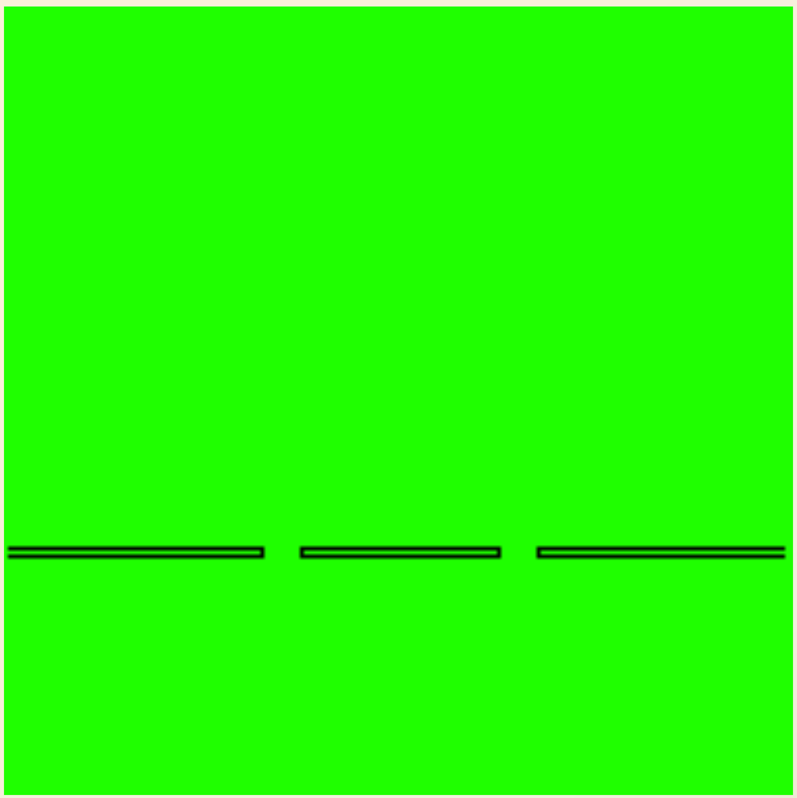


杨氏双缝干涉

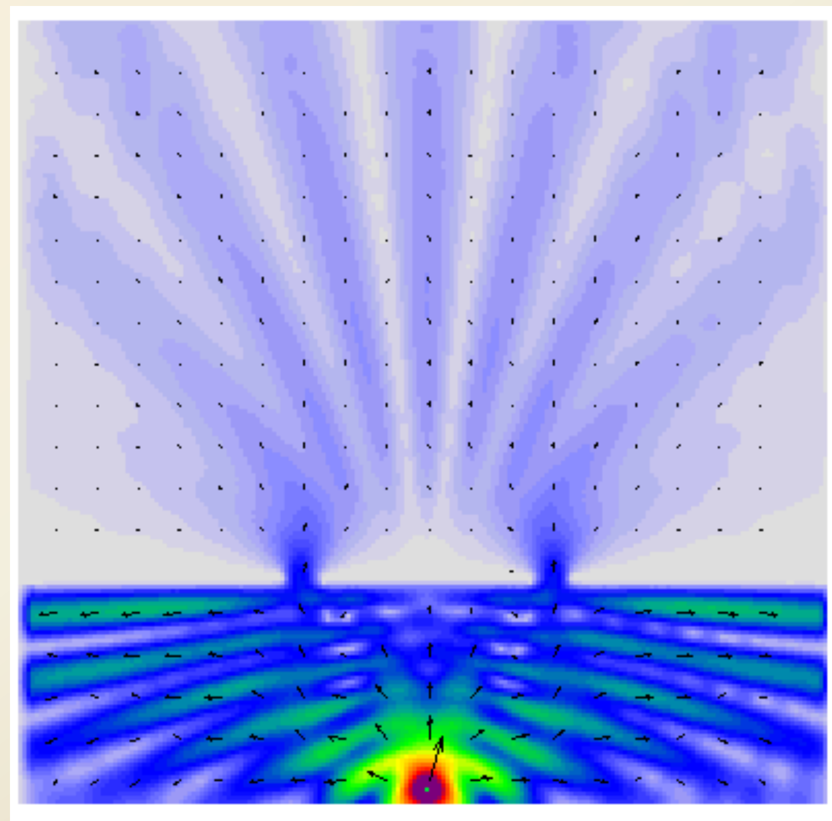
Young's two-slit experiment

This animation illustrates *Young's double-slit experiment* demonstrating the wave nature of light. An oscillating dipole illuminates two slits in a metal barrier, which themselves act as coherent oscillators that produce an interference pattern at the top of the domain.

Electric field animation



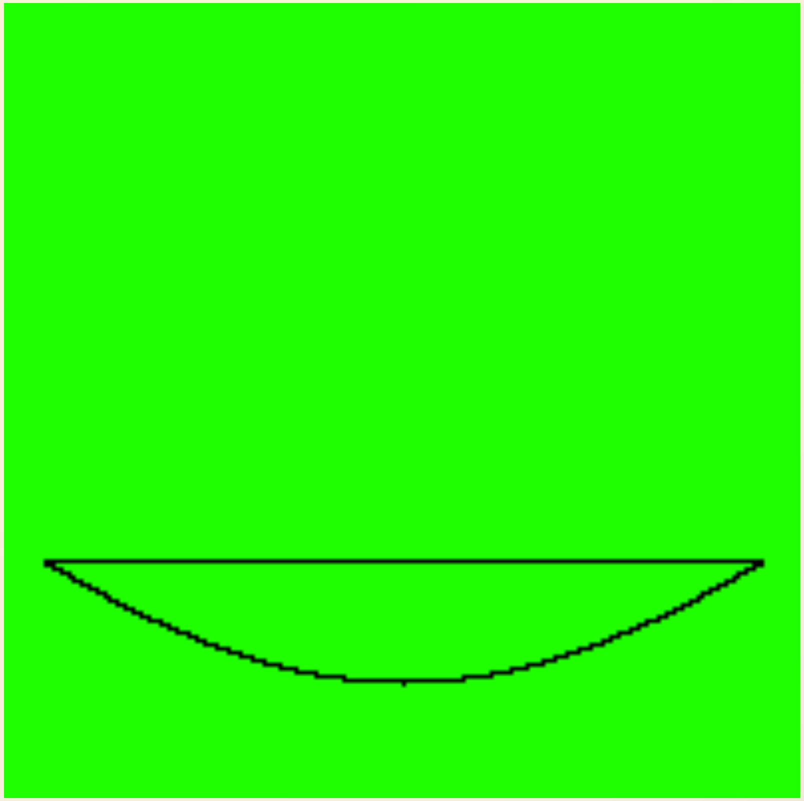
Average Poynting vector



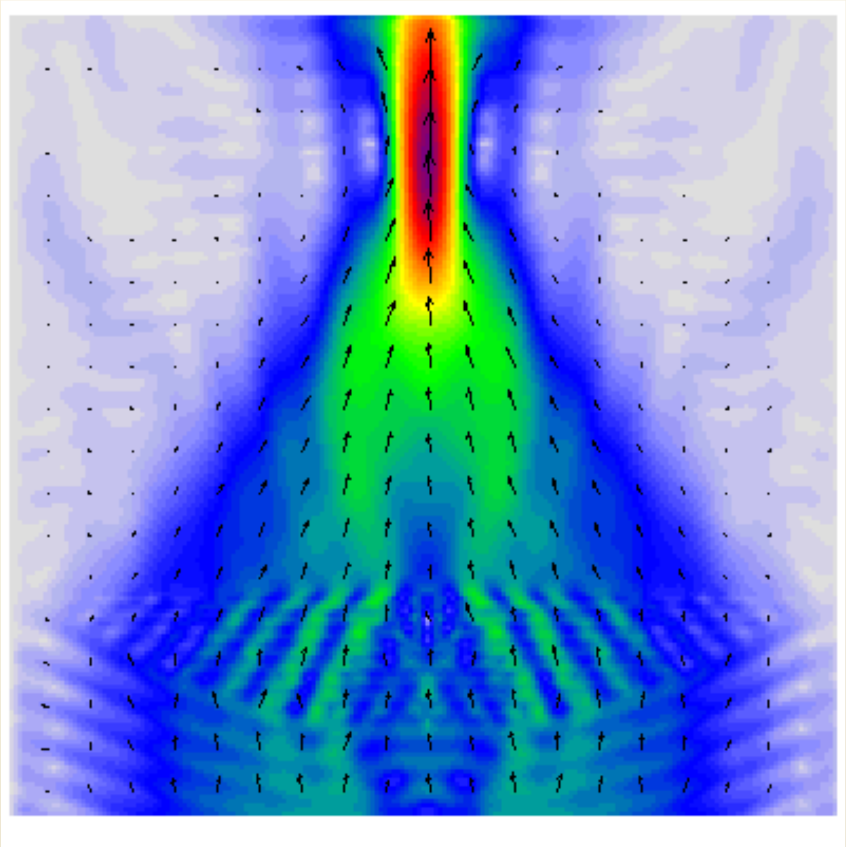
凸透镜
Convex lens

This animation illustrates the focussing effect of a convex lens. Note that the radiation cannot be focussed to a smaller scale than the wavelength. There are also internal reflections within the lens itself.

Electric field animation



Time-averaged Poynting vector

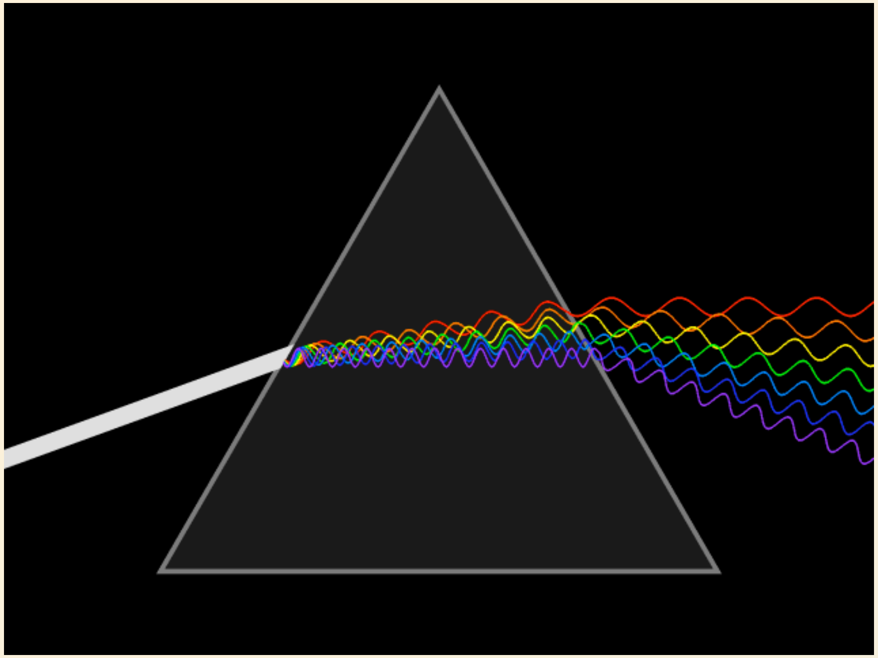


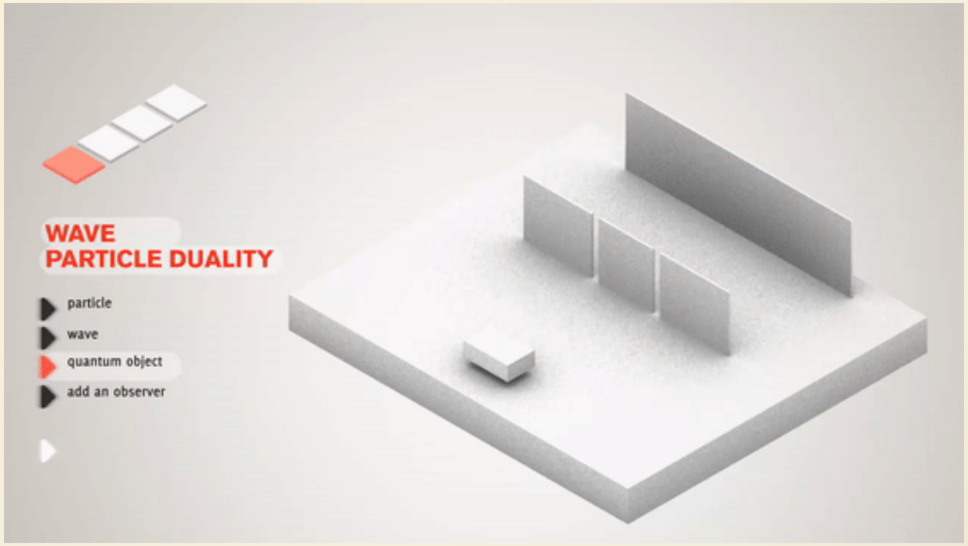
经典电磁理论的局限

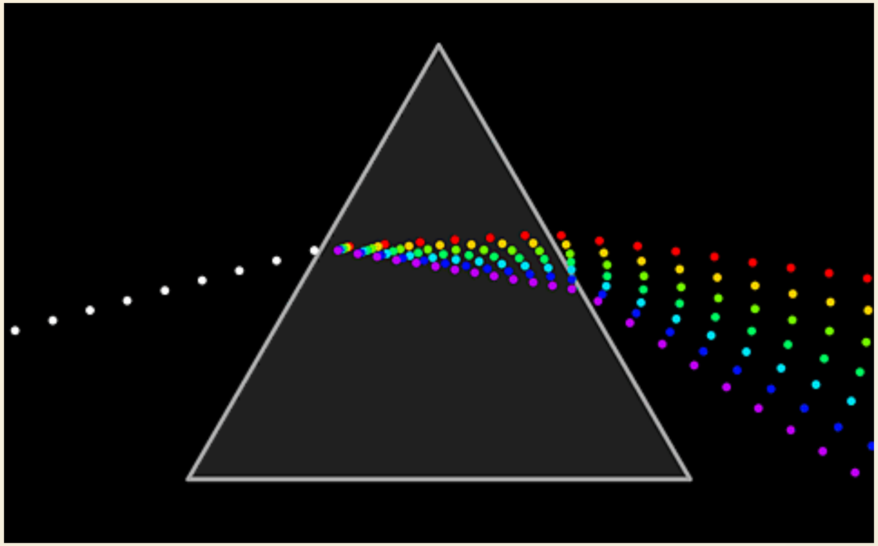
- 真空中电磁波的速度为 $c = 1/\sqrt{\mu_0\varepsilon_0}$
  - 让16岁的Einstein倍感困惑的问题：如果一个人以光速运动，电磁波看来是什么样子？静止的电磁波？

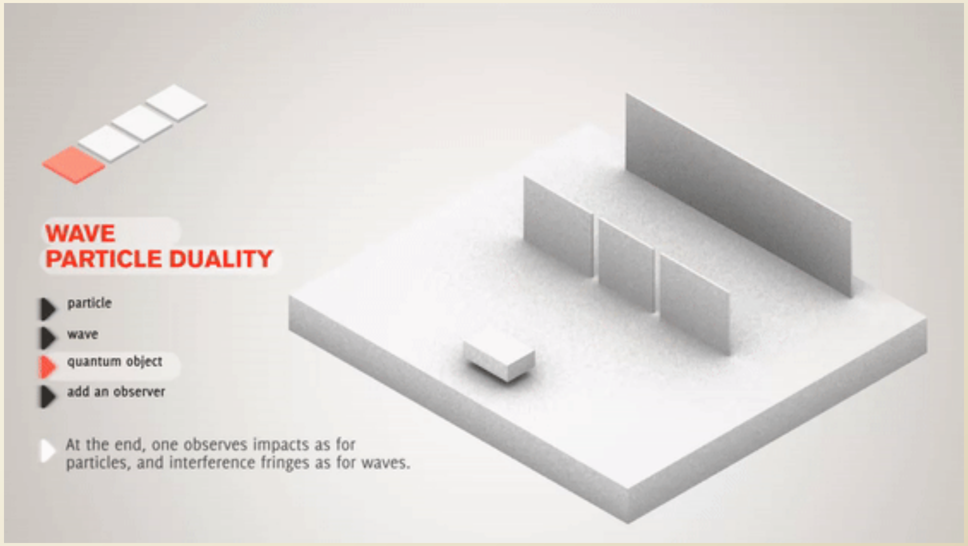
  但是……没有哪个人抓到过一把静止的光
  - 20世纪第一次物理革命：狭义相对论
- 加速运动的电荷必然会由于辐射而损失能量
  - 如同行星绕太阳运动一样，电子绕着原子核转动？
    - ➔ 电子速度(大小或/和方向)必然变化
    - ➔ 电子必然会由于辐射而不断地损失能量
    - ➔ 电子将很快 (10^{-11}秒) 地失去动能而落到原子核上
    - ➔ 原子从而我们整个的世界不可能稳定存在

  但是……你、我现在还能安然无恙地坐在教室里
  - 20世纪第二次物理革命：量子力学




WAVE
PARTICLE DUALITY
particle
wave
quantum object
add an observer




WAVE
PARTICLE DUALITY
particle
wave
quantum object
add an observer
At the end, one observes impacts as for particles, and interference fringes as for waves.

在美国国家科学促进会的讲话 (1883年8月15日)

—亨利·奥古斯特·罗兰　美国第一任物理学会会长

“…… 美国科学只存在未来，并没有今天和过去。在我这个位置上的人应该思考的问题是：我们必须做什么才能够创造出美国的物理学；而不是把电报、电灯及其它令人方便的产品称之为“科学”。我并不是低估这些时尚产品的价值，整个世界的进步需要依靠它们；成功发明这些高技术产品的人应该受到世界的尊重。但是，虽然一位厨师发明了餐桌上的一道美味佳肴，使世人享受到口福，但是我们不会尊称他为“化学家”……”

“……我常常被问及这样的问题：纯科学和应用科学究竟哪个对世界更重要？为了应用科学，纯科学本身必须存在！假如我们停止纯科学的进步而只留意科学的应用，我们很快就会退化成为中国人那样！多少代以来中国人没有什么进步，因为他们只满足于科学的应用，却从来没有追问过他们祖先所做事情中的原理。这些原理就构成了纯科学！中国人很早就知道火药的应用，如果他们能用正确的方法来探索“火药应用”的原理，中国人就能够在实现众多应用的同时还能够发展出化学，甚至物理学！因为只满足于火药爆炸的事实和应用，而没有寻根问底，中国人已经远远落后于世界进步；以至于我们现在只将世界上所有众多民族中这个最古老、人口最多的民族视为野蛮人！……”


感谢同学们的支持！