Finite-temperature phase transition in a class of 4-state Potts antiferromagnets

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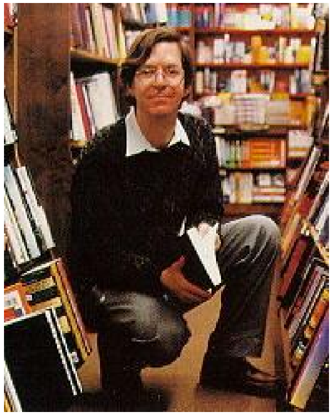

Alan D. Sokal

References

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Partial order and finite-temperature phase transitions in Potts models on irregular lattices,
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3. Yuan Huang, Youjin Deng (advisor),
Phase Diagram of the Ashkin-Teller model on the Union-Jack lattice,
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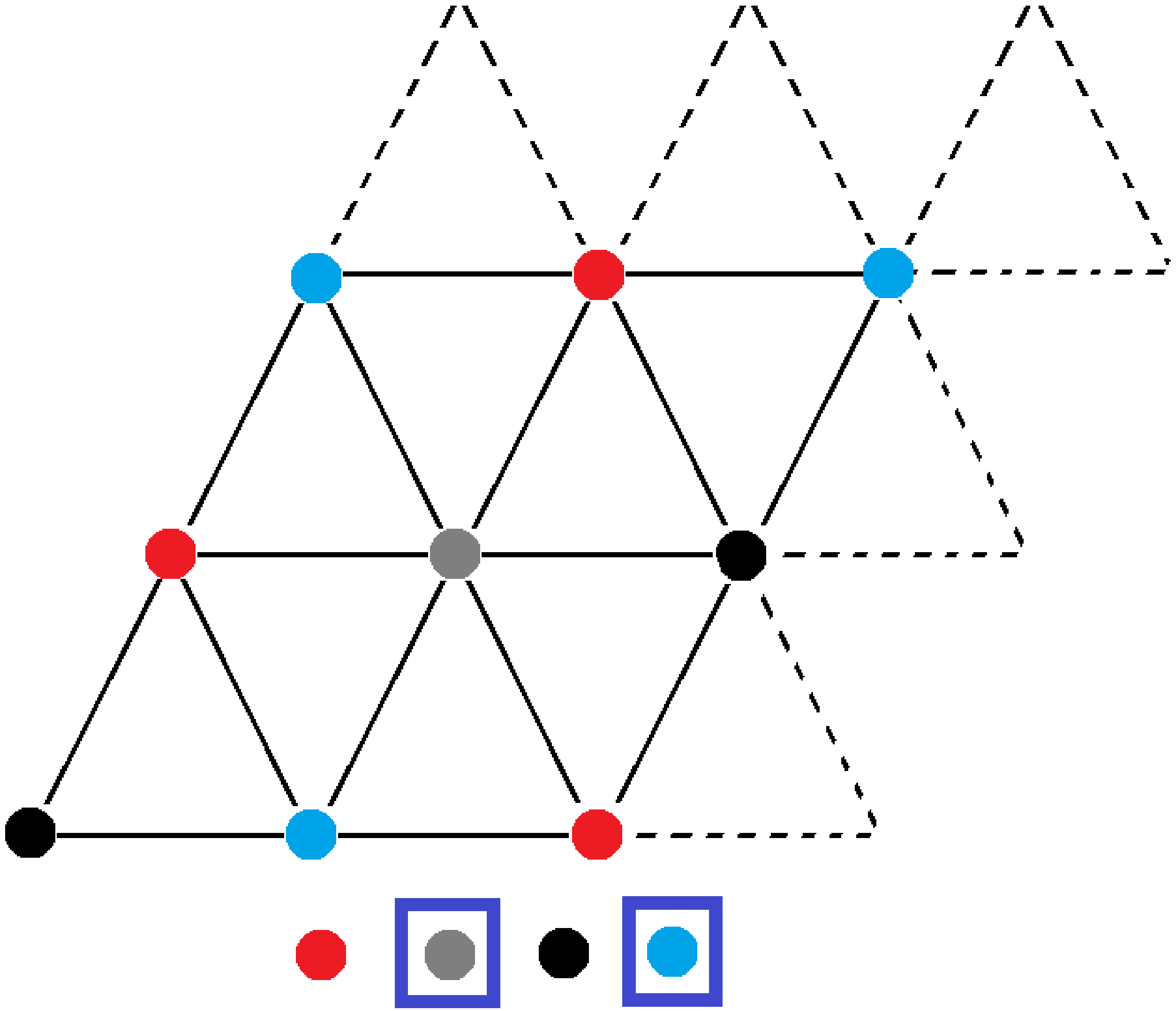
Finite-temperature phase transition in a class of 4-state Potts antiferromagnets,
arXiv:1108.1743, accepted by Phys. Rev. Lett. Sept. 02, 2011

Motivation

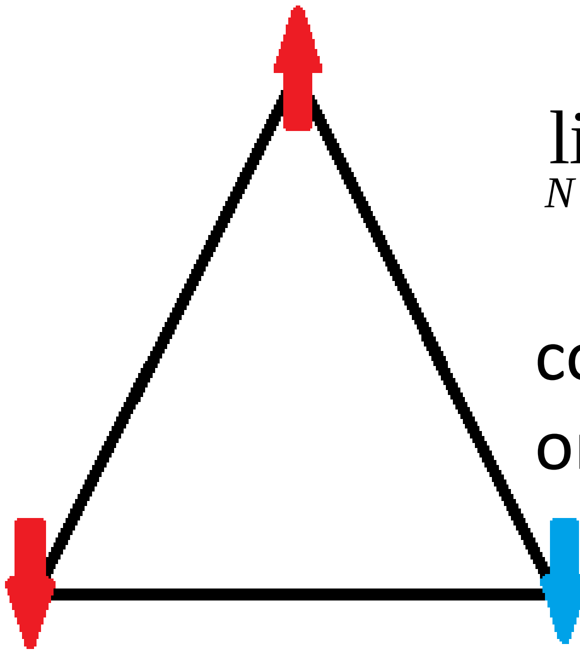
- Systems with non-zero entropy density could have long-range order

- Four-color theorem

four-color theorem on triangular lattice



T=0 AF Ising on triangular lattice



ground-state entropy
(Wannier, 1950)

$$\lim_{N\to\infty}\frac{S(0)}{N}\approx 0.338314$$

correlation function
on sublattices

$$\xi(r)\propto r^{-1/2}$$

Questions

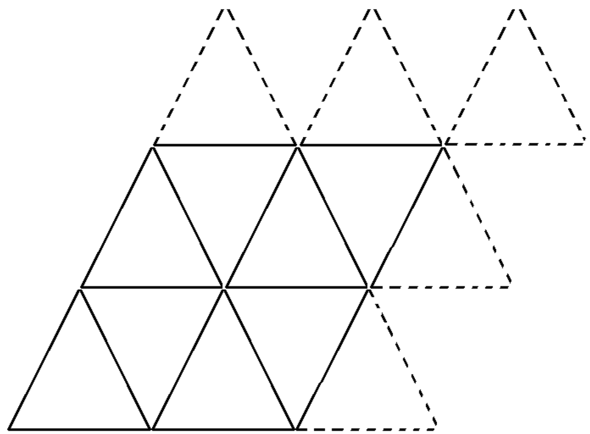
We considered q-state Potts antiferromagnet

$$H = J\sum_{<ij>}\delta_{\sigma_i,\sigma_j} \qquad (\ J>0, \quad \sigma_i = 1,2,\cdots,q \quad \forall i\ )$$

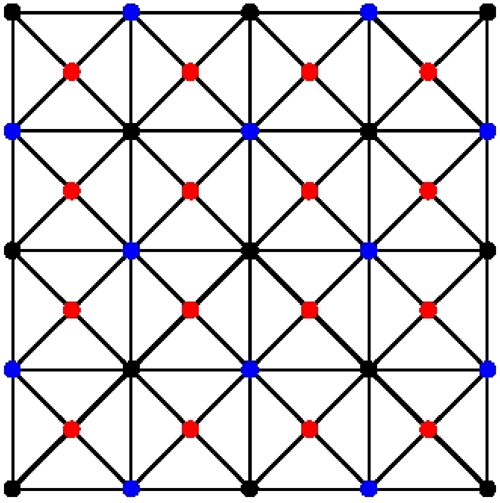
on 2D Eulerian (all vertices have even degree) plane triangulations (all faces are triangles).

- Is there a phase transition at finite-temperature, of what order?
- What is the nature of the low-temperature phase(s)?
- If there is a critical point, what are the critical exponents and the universality classes?

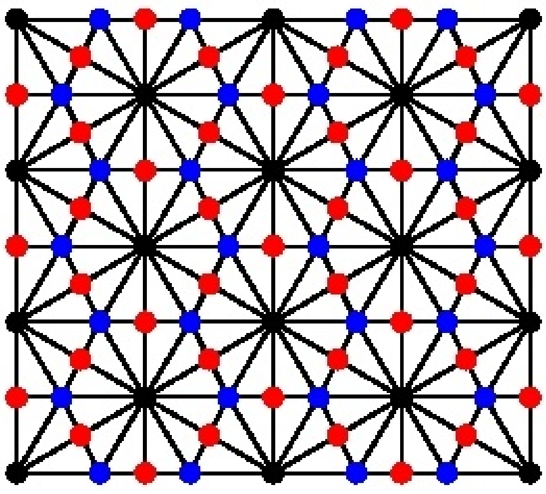
Lattices



triangular lattice

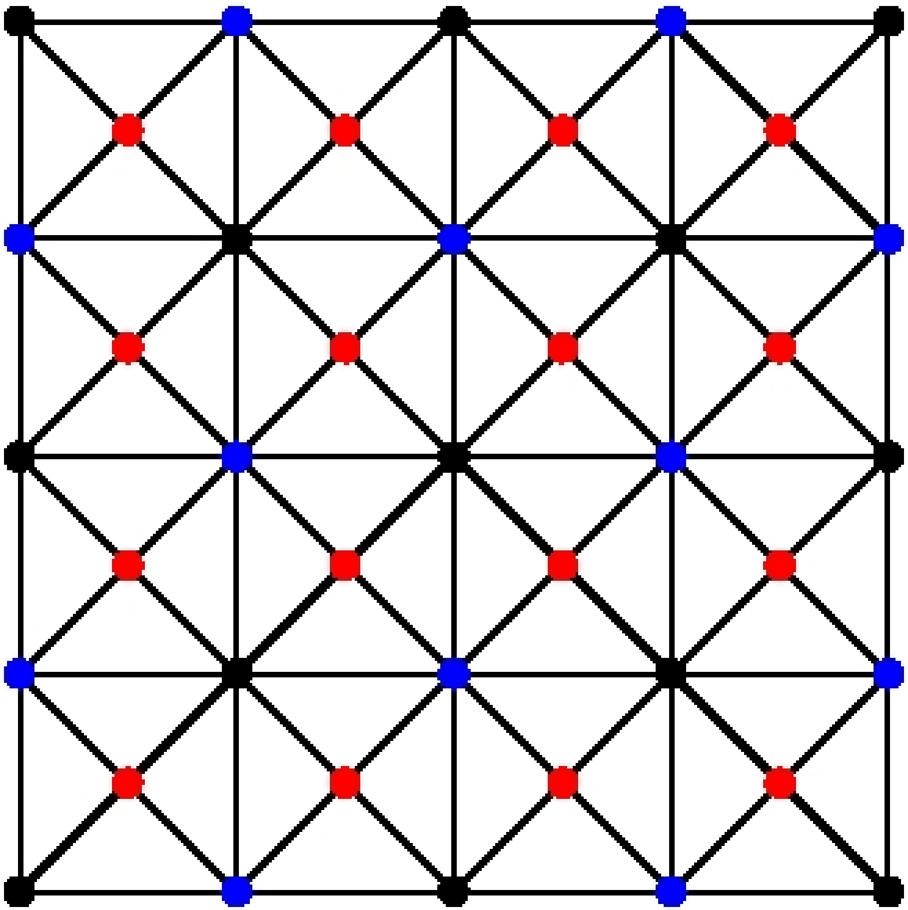


union jack lattice



bisected hexagonal lattice

union jack lattice



$$G = (V, E)$$

$$G^* = (V^*, E^*)$$

$$\hat{G} = (V \cup V^*, \hat{E})$$

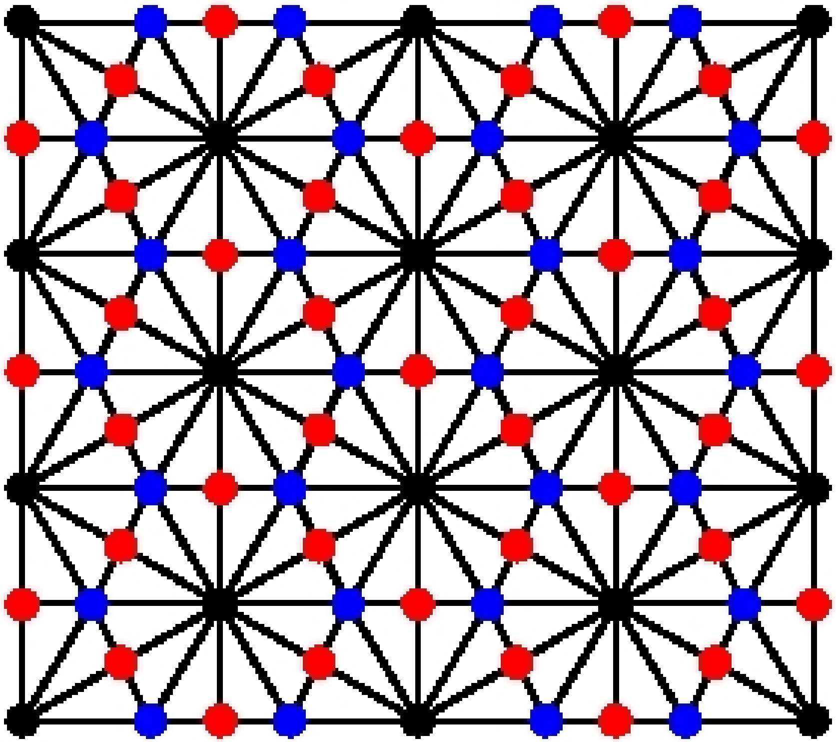
$$\tilde{G} = (V \cup V^*, \tilde{E})$$

$$G = G^* = \hat{G}$$

$$= square\ lattice$$

$$\tilde{G} = union\ jack\ lattice$$

bisected hexagonal lattice



$$G = (V, E)$$
$$= triangular$$

$$G^* = (V^*, E^*)$$
$$= hexagonal$$

$$\hat{G} = (V \cup V^*, \hat{E})$$
$$= diced$$

$$\tilde{G} = (V \cup V^*, \tilde{E})$$
$$= bisected \;\; hexagonal$$

Exact identities

- Argument 1.

Ising AF has a nonzero-temperature phase transition on union jack lattice.

F. Y. Wu and K. Y. Lin
Ising Model on the union jack lattice as a free fermion model
J. Phys. A: Math. Gen. 20(1987)

Exact identities

- Argument 2.

q=4 AF Potts T=0 $\Leftrightarrow$ q=9 F Potts at $v = e^J - 1 = 3$

$(\tilde{G})$ $(G \text{ or } G^*)$

non-critical $\Leftarrow$ non-critical

Exact identities

- union-jack lattice

  G and G^* : square ------- self-dual

  q=9 F Potts at v=3 is at 1st-order transition point.

- bisected hexagonal lattice

  G and G^* : triangular ------- hexagonal

  q=9 F Potts at v=3 is :

  disordered ---------- hexagonal

  ordered ---------- triangular

Exact identities

- Argument 3.

some 2D AF models T=0 $\Leftrightarrow$ "height" model

height model: "smooth"(ordered) / "rough"(critical)

So these AF model must either be ordered / critical at T=0.

Phase Transition

- Based on the above arguments:

4-state AF Potts model on $\tilde{G}$ has an order-disorder transition at finite temperature.

- Universality:

✓ G is self-dual:
The symmetry is S4 $\times$ Z2.
The universality class is a 4-state Potts model plus an Ising model (decoupled).

✓ G is not self-dual:
The symmetry is S4, and the universality class is that of a 4-state Potts model.

Transfer-matrix Method for union jack lattice

- q=4

$$c(v)(central\ charge):$$

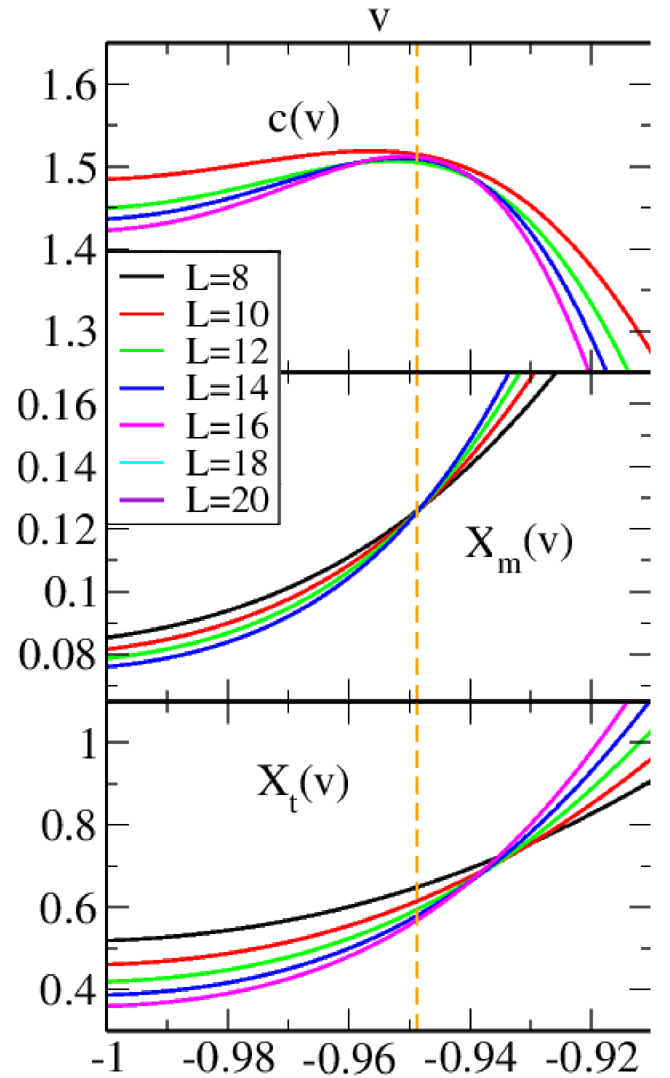
$$v_c = -0.944(5),\quad c = 1.510(5)$$

$$X_m(v)(magnetic\ exponent)$$

and $X_t(v)(thermal\ exponent)$:

$$v_c = -0.9488(3),\ X_m = 0.1255(6),$$

$$X_t = 0.51(2)$$



Transfer-matrix Method for union jack lattice

- T=0

$c(q):$

$$q_0 = 3.63(2), \quad c = 1.43(1)$$
$$q_c = 4.330(5), \quad c = 1.63(1)$$

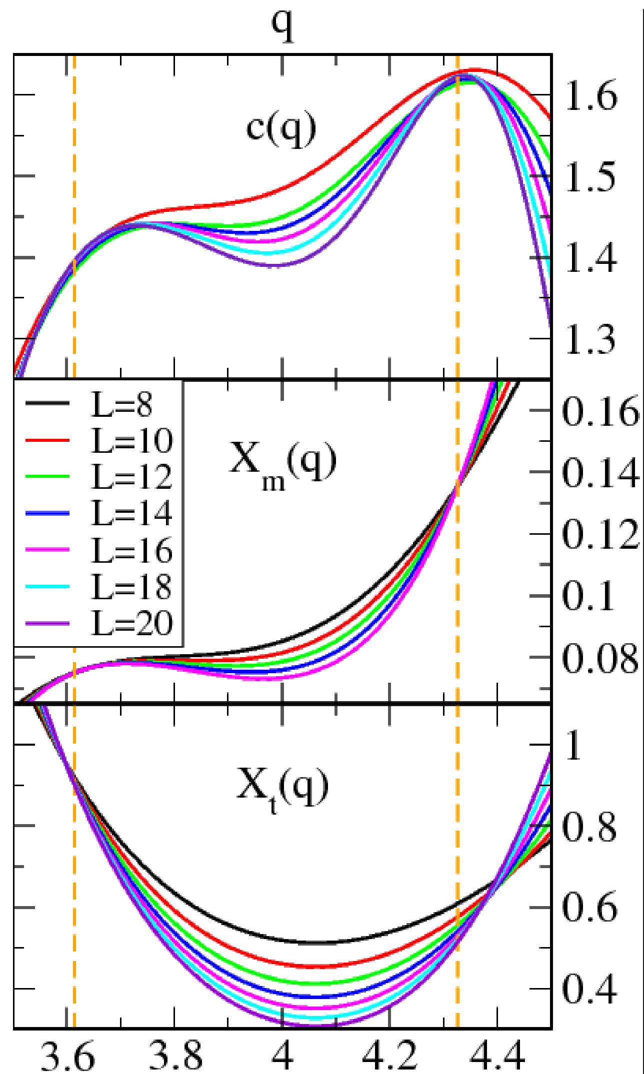
$X_m(q)$ and $X_t(q):$

$$q_0 = 3.616(6), \; X_m = 0.0751(3),$$

$$X_t = 0.88(2);$$

$$q_c = 4.326(5), \; X_m = 0.134(2),$$

$$X_t = 0.52(3).$$



Transfer-matrix Method for bisected hexagonal lattice

- q=4

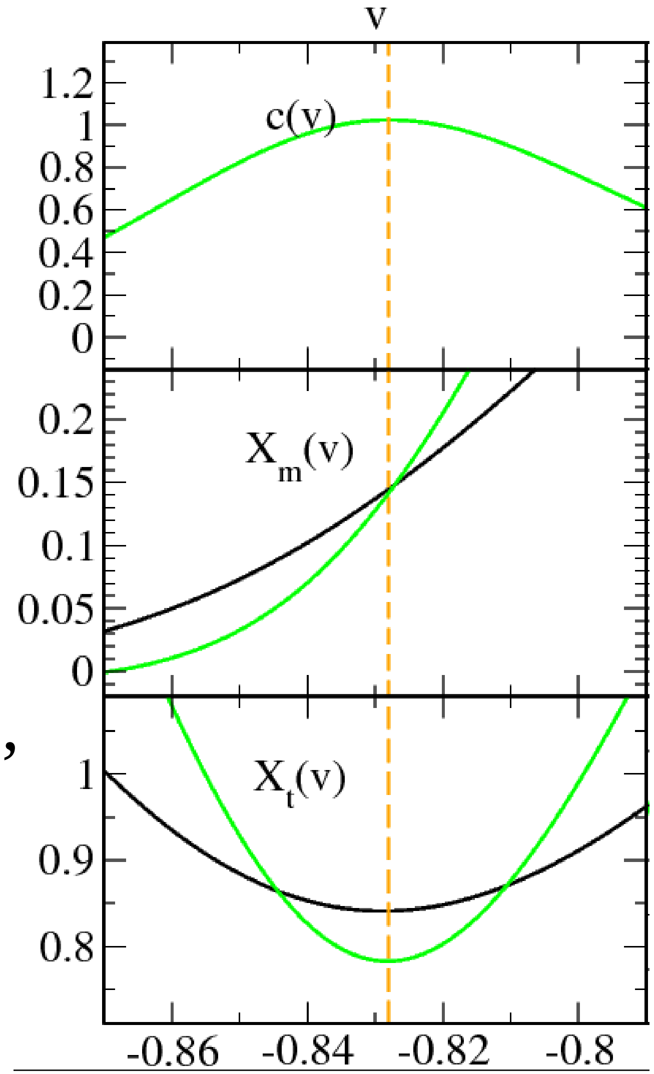
$c(v)$:

$$v_c = -0.8281(1), \;\; c = 1.000(5)$$

$X_m(v)$ and $X_t(v)$:

$$v_c = -0.8280(1), \;\; X_m = 0.15(1),$$

$$X_t = 0.65(10)$$



Transfer-matrix Method for bisected hexagonal lattice

- T=0

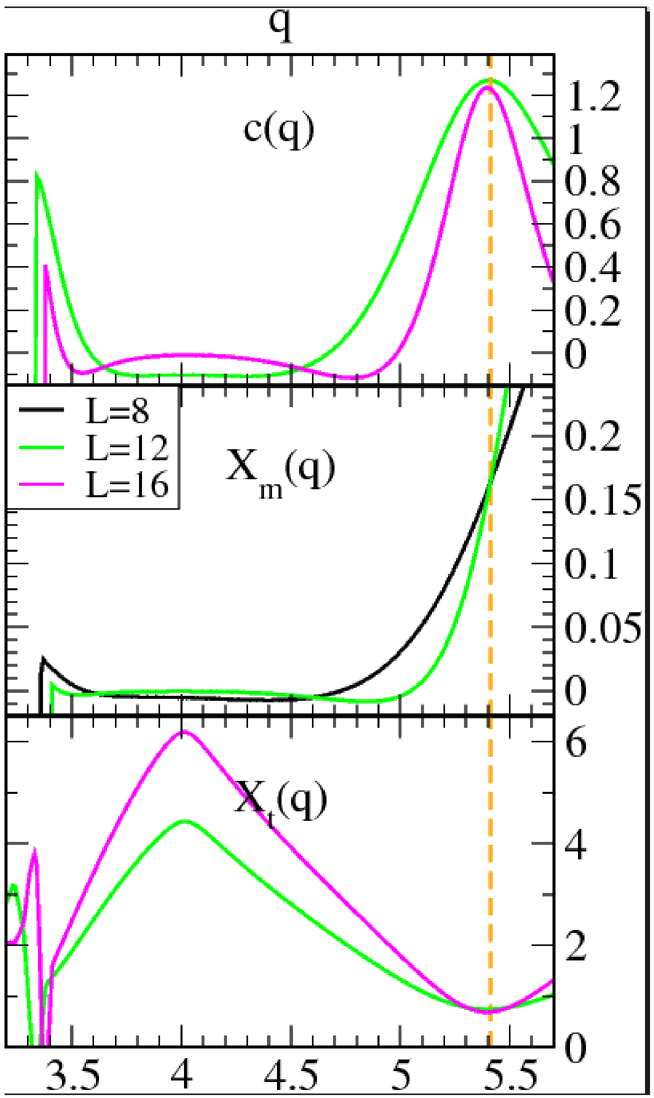
$c(q)$:

$$q_c = 5.395(10), \quad c = 1.20(5)$$

$X_m(q)$ and $X_t(q)$:

$$q_c = 5.397(5), \quad X_m = 0.15(1),$$

$$X_t = 0.6(1).$$



MC method for q=4 on union jack lattice

- susceptibility observables

$$magnetization\ on\ sublattices:$$

$$M_{i,\alpha} = \sum_{x \in V_i} \delta_{\sigma(x),\alpha} \quad i = A, B, C, \quad \alpha = 1, 2, 3, 4$$

$$matrix\ of\ susceptibility:$$

$$\chi_{ij} = \frac{1}{|V|}\left[\frac{3}{4}\sum_{\alpha=1}^{4}\left\langle M_{i,\alpha} M_{j,\alpha}\right\rangle - \frac{1}{3}|V_i||V_j|\right] \qquad i, j = A, B, C$$

$$and\ also\ the\ eigenvalues\ of\ the\ susceptibility\ matrix:$$

$$\lambda_1(\chi),\ \lambda_2(\chi),\ \lambda_3(\chi)$$

MC method for q=4 on union jack lattice

- specific-heat observables

$$\textit{the energy on each subset of edges:}$$

$$\mathrm{E}_i = \sum_{<xy>\in E_i} \delta_{\sigma(x),\sigma(y)} \quad i = A, B, C$$

$$\textit{matrix of specific-heat-like quantities :}$$

$$C_{ij} = \frac{1}{|E|} \left[\left\langle E_i E_j \right\rangle - \left\langle E_i \right\rangle \left\langle E_j \right\rangle \right] \qquad i, j = A, B, C$$

$$\textit{and also the eigenvalues of the matrix:}$$

$$\lambda_1(C),\ \lambda_2(C),\ \lambda_3(C)$$

MC method for q=4 on union jack lattice

- renormalization exponents

$$y_{h1} = 1.87 \qquad y_{h2} = 1.39 \qquad y_{t1} = 1.50 \qquad y_{t2} = 0.81$$

- my conjectures

$$y_{h1} = 15/8$$

$$y_{h2} = 2y_{h1} - d = 3/2 \quad \text{with log correction}$$

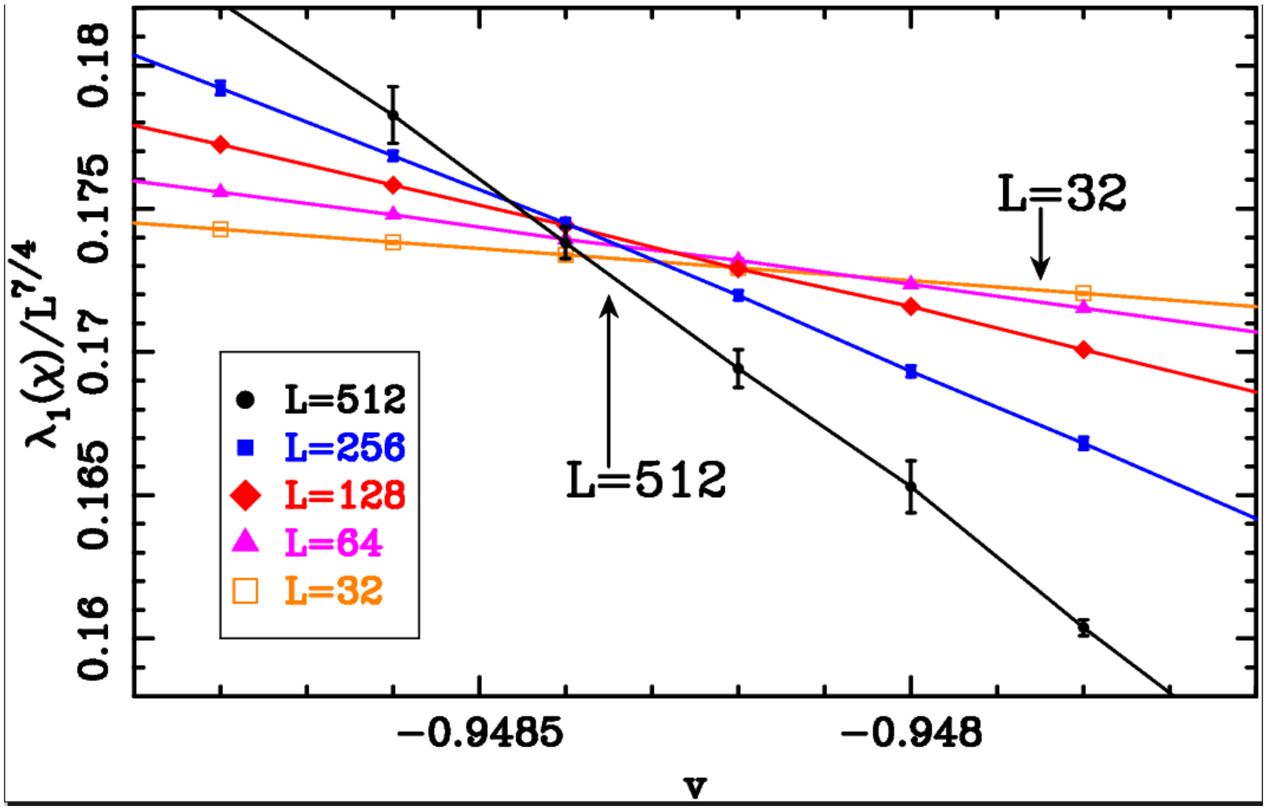
$$(\ b^{y_{h2}} \to b^{3/2}(\ln b)^{-1/4} \ )$$

$$y_{t1} = 3/2$$

$$y_{t2} = 2y_{t1} - d = 1 \quad \text{with log correction}$$

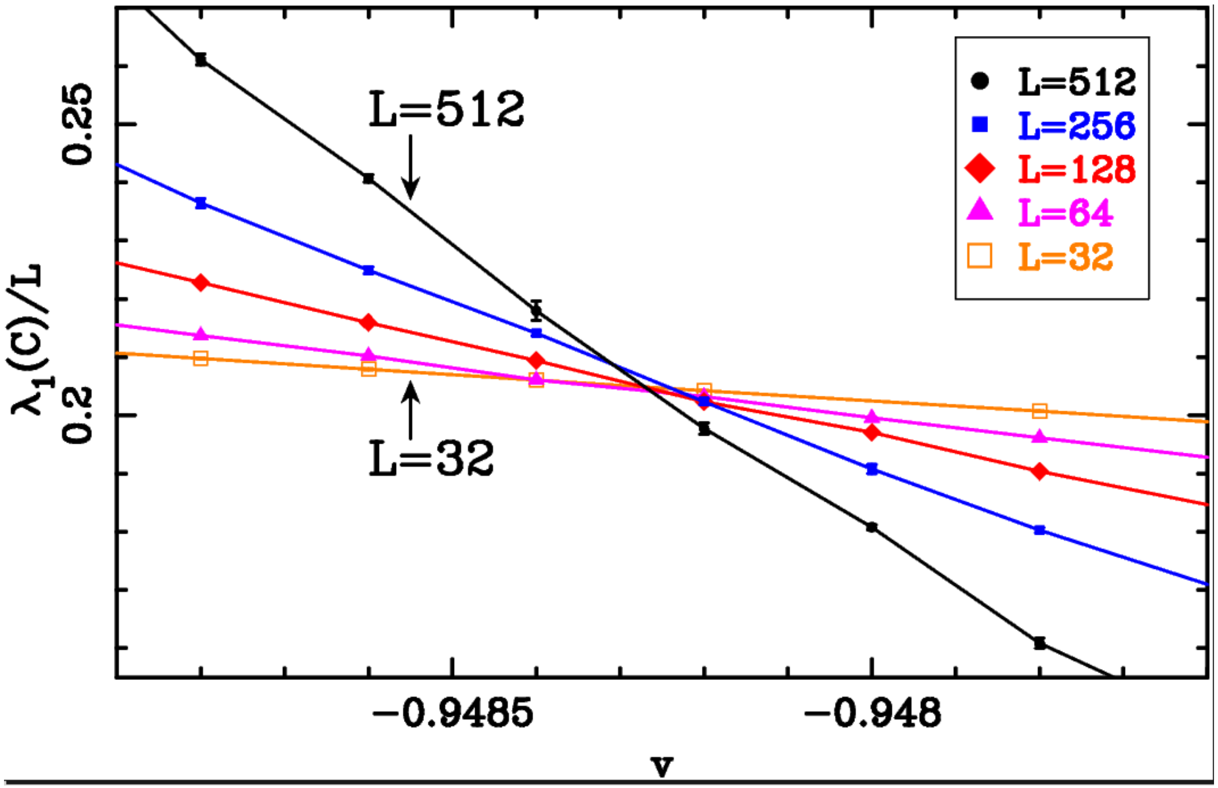
$$(\ b^{y_{t2}} \to b(\ln b)^{-3/2} \ )$$

MC method for q=4 on union jack lattice



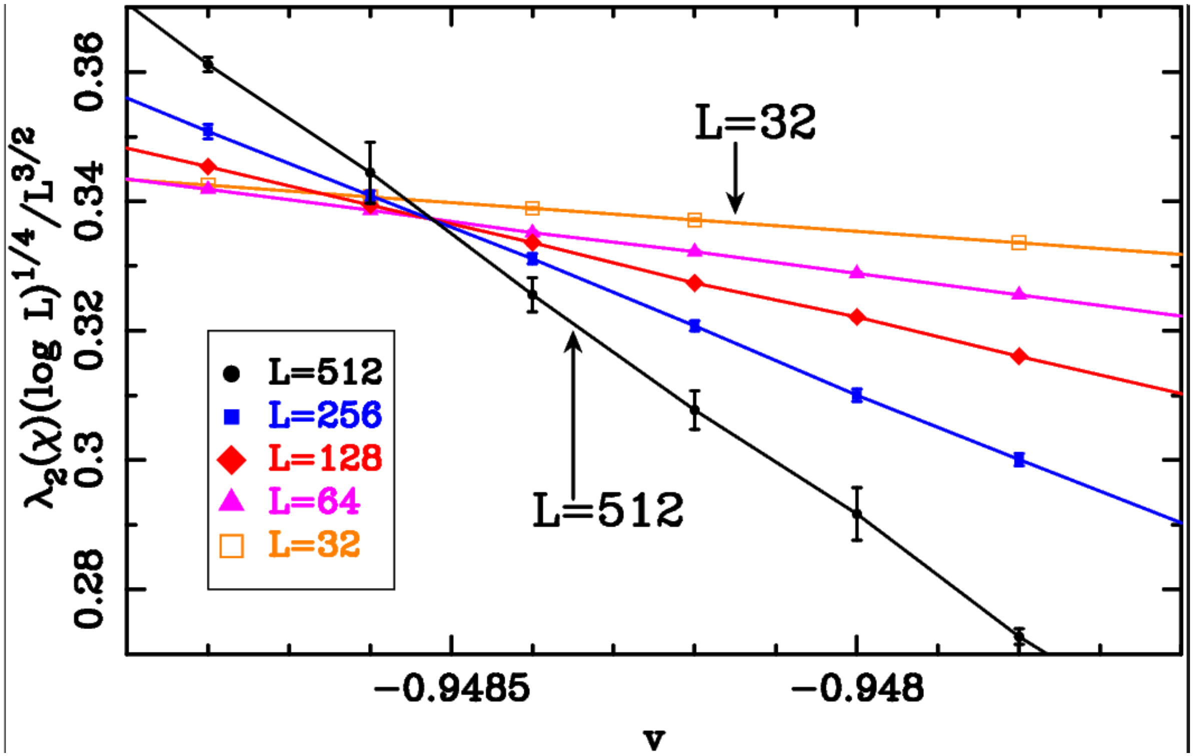
$\lambda_1(\chi)$: $\gamma/\upsilon = 2 - 2X_m = 7/4$ $\quad v_c = -0.9485(1)$

MC method for q=4 on union jack lattice



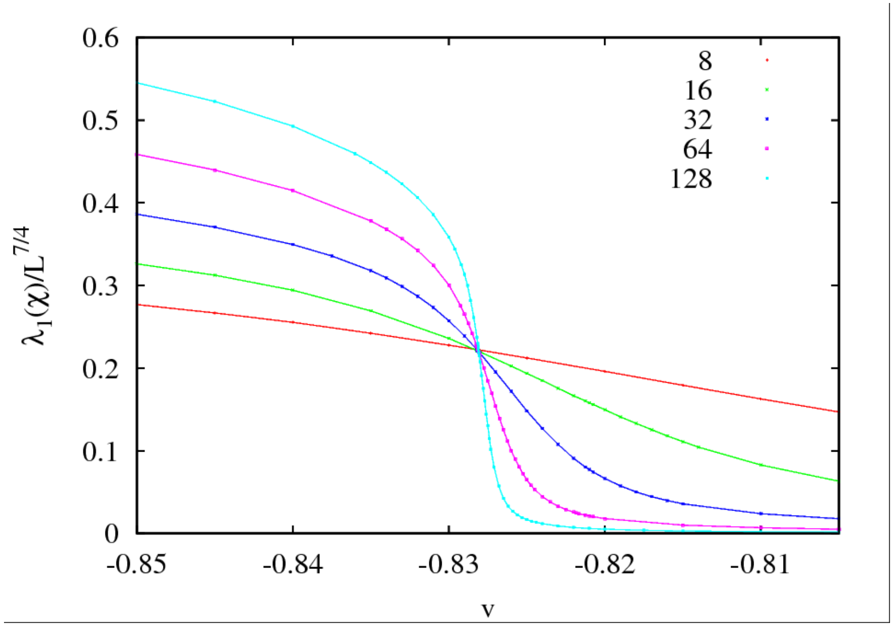
$$\lambda_1(C): \quad \alpha/\upsilon = 2 - 2X_t = 1 \qquad v_c = -0.9483(2)$$

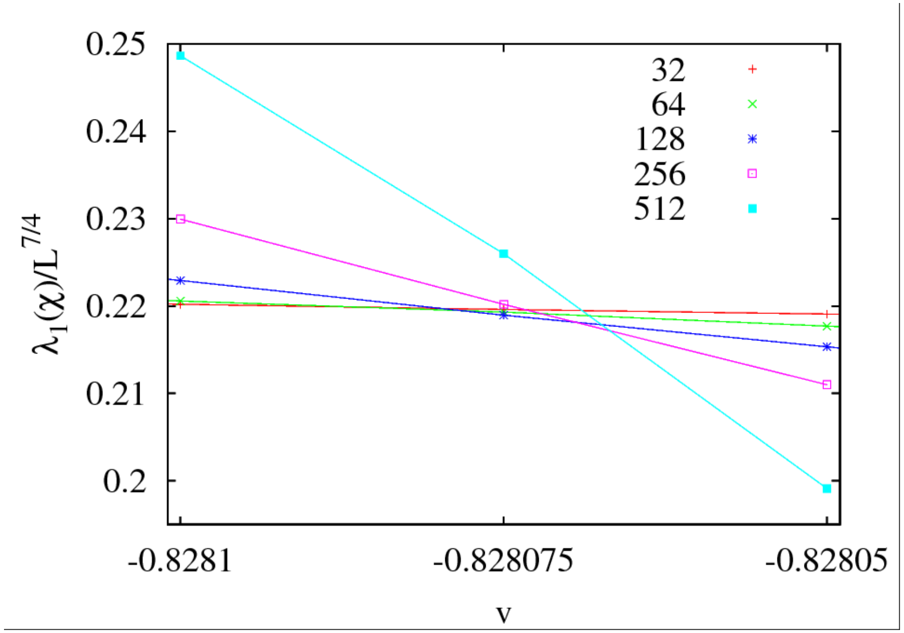
MC method for q=4 on union jack lattice



$$\lambda_2(\chi) \propto L^{3/2}(\ln L)^{-1/4} \qquad v_c = -0.9485(2)$$

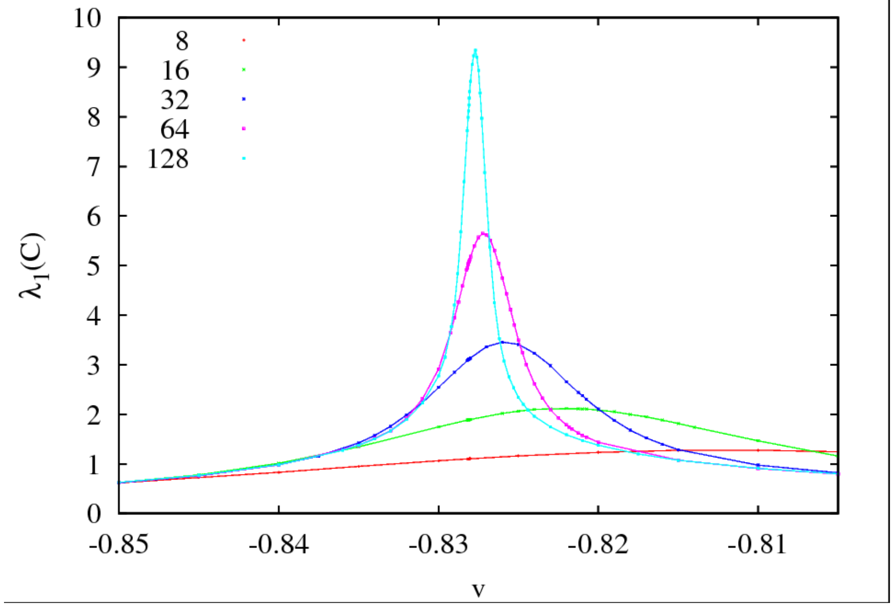
MC method for q=4 on bisected hexagonal lattice

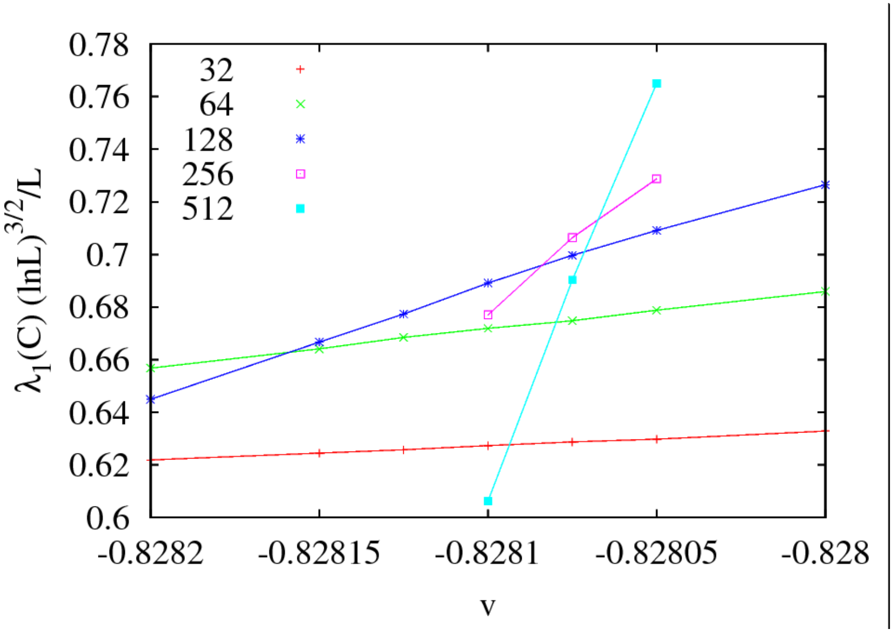




$$\lambda_1(\chi) \propto L^{7/4}(\ln L)^{-1/8} \qquad v_c = -0.828066(4)$$

MC method for q=4 on bisected hexagonal lattice

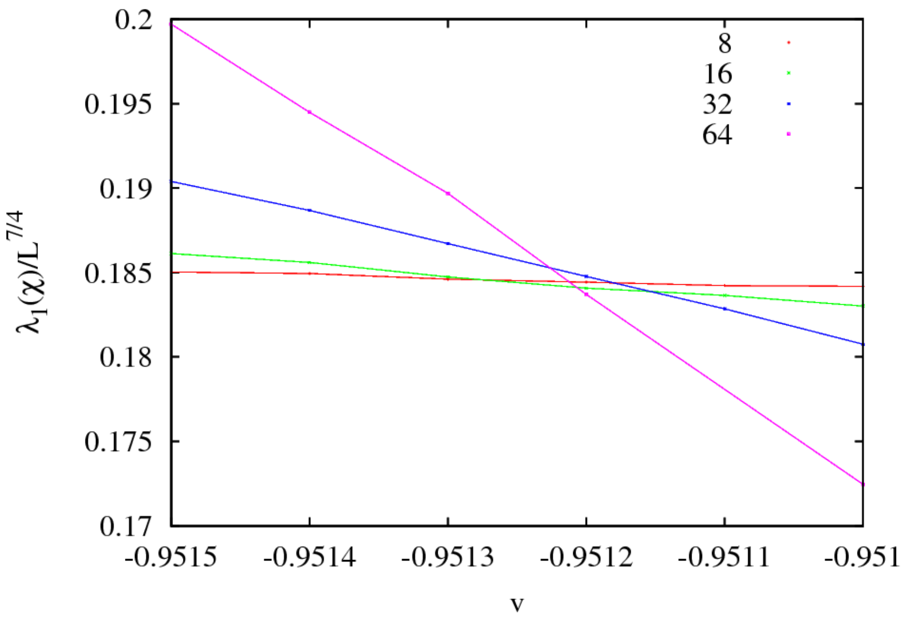


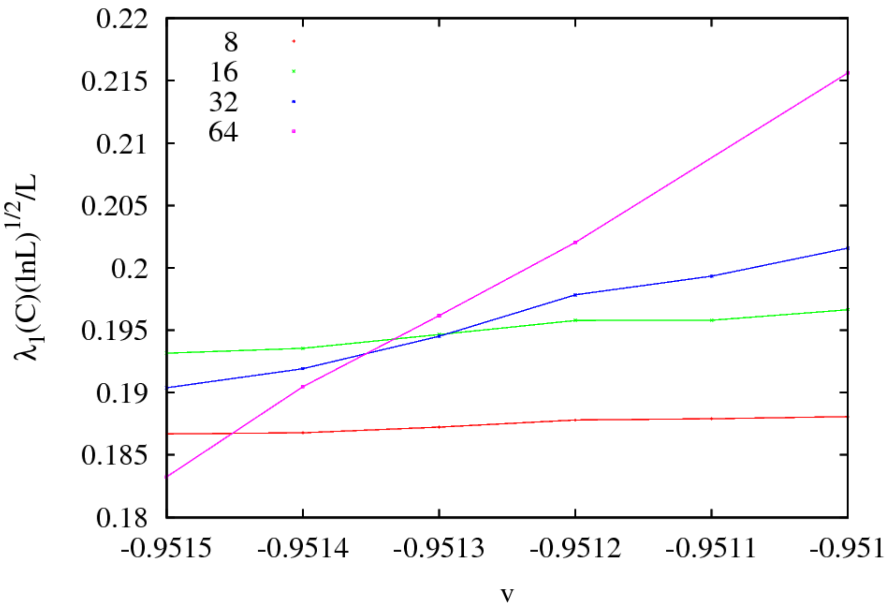


$$\lambda_1(C) \propto L(\ln L)^{-3/2}$$

MC method for q=5 on bisected hexagonal lattice

preliminary MC results :





$$v_c = -0.95132(2), \qquad X_m = 0.113(4), \qquad X_t = 0.495(5)$$

Conclusion

- an analytical existence argument for a finite-temperature phase transition in a class of 4-state Potts antiferromagnets;
- a prediction of the universality class;
- large-scale numerics, using two complementary techniques, to determine critical exponents;
- determination of q_0 and q_c as well as v_c ;
- the surprising prediction of a finite-temperature phase transition also for q = 5 on the BH lattice .

Future work

- General q
- Different lattices
- Different models

$\Rightarrow$

- Is there any phase transition at finite temperature, of what order?
- If so, how about the critical exponents and the universality?

Thank you!