# I. Backbone exponents.

⋆ backbones defined as current-carrying bonds (sites) in bus-bar geometry. $N_b \sim L^{d-X_b}$.

$N_b$ − number of backbones, $L$ − finite linear system size; $d$ − spatial dimensionality; $X_b$ − backbone scaling dimension.

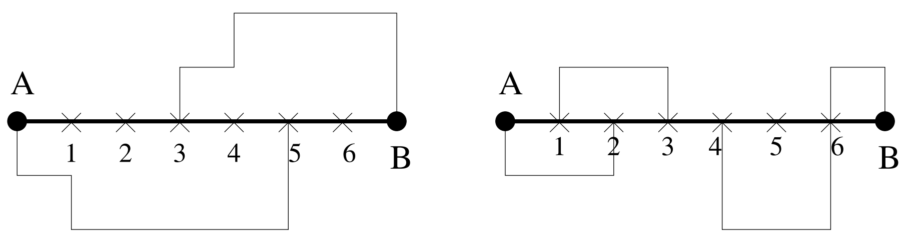
$\Downarrow$

Burning Monte Carlo algorithm.

⋆ path crossing exponents $X_k$. probability $P_k$ of two points connected by *at least* $k$ *independent* paths: $P_k \sim L^{-2X_k}$.

$\Downarrow X_b = X_2$

current Monte Carlo algorithm.




(a) (b)

† † *much more efficient than burning procedures.*

⋆ results.

1): 2D $q$-state Potts model (including Baxter-Wu model).

bond probability $p$ set at random cluster fixed point $^{(r)}$ ($p = 1 - e^{-K}$), and at Potts cluster fixed point $^{(p)}$ ($p = 1$).

| $q$ | $g$ | Mod. | $X_h^{(r)}$ | $X_b^{(r)}$ | $X_h^{(p)}$ | $X_b^{(p)}$ |
|---|---|---|---|---|---|---|
| 0 | 2 | $q = 0$ P | 0 | 3/4? | −3/16 | |
| 1 | 8/3 | Perc. | 5/48 | 0.3566(2) | 0 | 0 |
| 2 | 3 | Ising | 1/8 | 0.2696(3) | 5/96 | 0.0520(3) |
| 3 | 10/3 | $q = 3$ P | 2/15 | 0.2105(3) | 7/80 | 0.0871(7) |
| 4 | 4 | $q = 4$ P | 1/8 | 0.126(1) | 1/8 | 0.1246(5) |
| 4 | 4 | B.W. | 1/8 | 0.136(7) | 1/8 | 0.1239(8) |
| 2 | 14/3 | Trc. I. | 3/40 | 0.0760(15) | 0.0752(3) | 0.0753(8) |

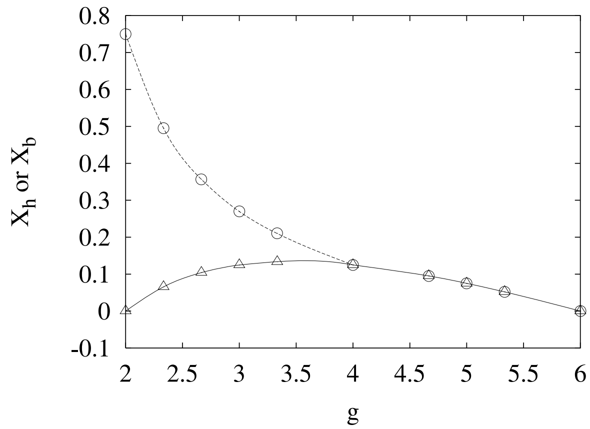
Note: 1), $q = 4$ Potts model is at tricritical point ($K = 1.45790(1), D = 2.478438(2)$); 2), $X_b = 2/3$ for $q = 0$ Potts obtained from Eden tree.

∗ *Q1: can one exactly calculate $X_b$ in 2D?*

Fit formula:

$$P = L^{-2X}(a + bL^{y_1} + cL^{-2} + dL^{-3}) \qquad \textit{or}$$

$$P = L^{-2X}(a + b/\ln L + c + b/\ln^2 L + dL^{-2}) \text{ (P.4 and BW).}$$




† † *$X_b = X_h$ for $g \geq 4$, $X_b > X_h$ for $g < 4$.*

2)*: 3D Ising $X_b = 0.829(4)$.*

*3D percolation and tricritical Ising $X_b =$?*

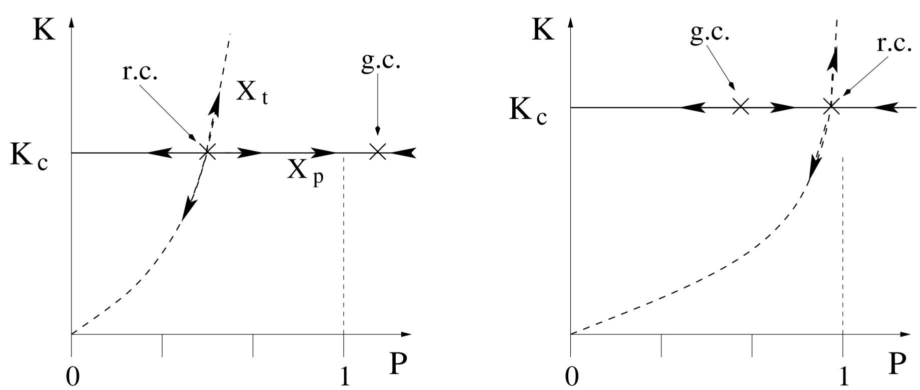
# II. 'Geometric cluster' fixed points.

† † *$X_b = X_h$? depends on 'red-bond' exponent $X_r > 2$?;*

† † *Scaling argument $\Rightarrow X_r = X_p$ (RG exponent along bond-probability direction.*

## $\star$ 2D $q$-state Potts model.

e.g., Ising model ($a$) and tricritical Blume-Capel model ($b$) on square lattice.

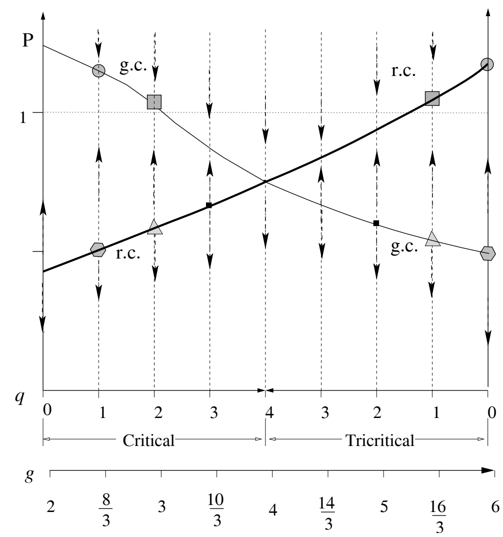



(a) (b)

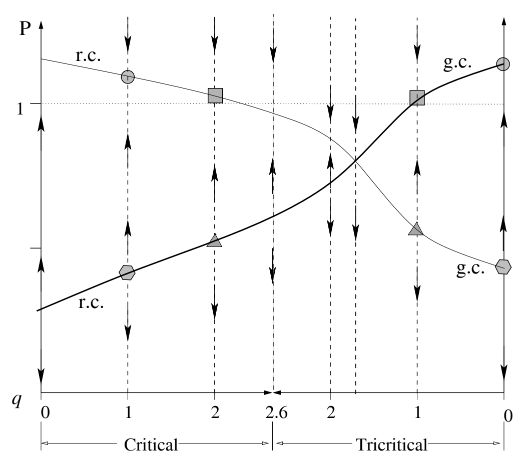
† † *at r.c., $X_p > 0$ for critical branch, $X_p > 0$ for tricritical branch, and $X_p = 0$ for 4-state Potts model.*

•• From Kac formula and other assumptions, we obtain following tables (last two pages).

*RG flows in $p-q$ space:*

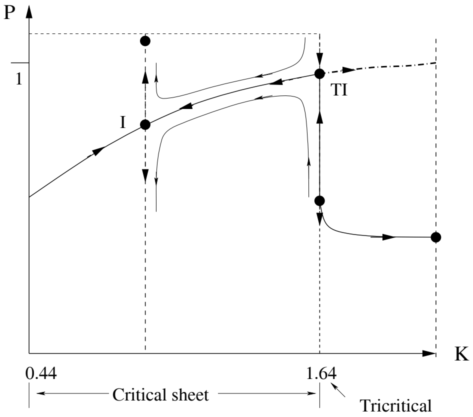



⋆ *for 3D Ising model* $X_p = 0.767(2)$, $X_h^{(g)} = 0.14(1)$?; *for tricritical Blume-Capel model* $X_P > 0$, *so that RG flows:*

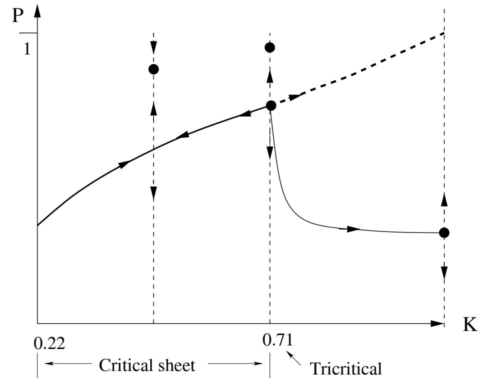



*Q2, for 3D* $q = 0$ *Potts, are followings correct? at criticality,* $X_t = 3$ *and* $X_h = 0$? *at tricriticality,* $X_t = 0.5183$ *and* $X_h = 0.14(1)$?

*Q3, for Blume-Capel model, RG flows and percolation threshold are show in following figures, are they correct?*



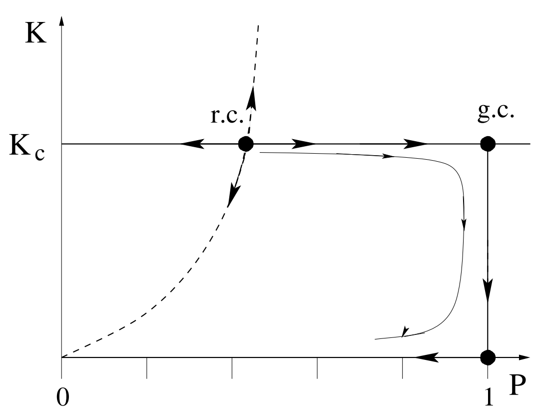
2D



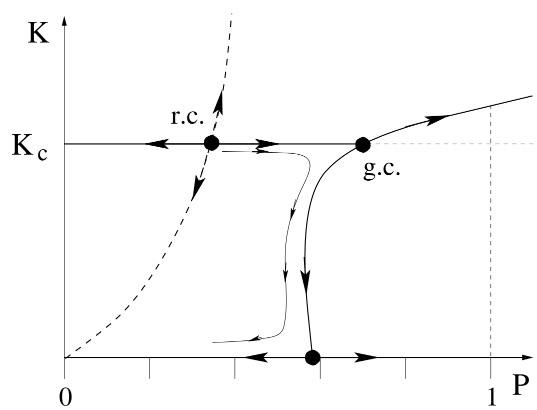
3D

*Q4, for tricritical 3D $q = 2$ Potts, since it behaves mean-field-like, can one calculate $X_b$?*

*Q5, concerning Ising models on triangular and simple-cubic lattices, $(a)$ and $(b)$, respectively, are following RG flows correct?*



(a)



(b)

† † *for Ising clusters ($P = 1$), percolation thresholds coincide with $K_c$ for triangular lattice, but not for simple-cubic lattice.*