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Effects of detachment and size of particles in totally asymmetric simple exclusion processes

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Abstract

In this article, the effects of irreversible detachments of particles in totally asymmetric simple exclusion processes (TASEPs) with extended particles which occupy more than one lattice site, are investigated. First, an approximate mean-field theory is used to calculate phase diagrams and density profiles. The results show that the detachment and the size of the particles have distinct effects on the stationary phases in the two sublattices divided by the detachment, especially in the mc/hd and the hd/hd phase. Here, symbols "mc", "hd", and "ld" are initials of maximal current, high density, and low density, respectively, and "mc/hd" represents the stationary state that the left sublattice is in the maximum-current (mc) phase while the right sublattice is in the high density (hd) phase. When the detachment rate is very large, there are four stationary phases, including ld/ld, ld/hd, mc/ld, and mc/hd phases. When the value of the detachment rate is in the middle range, the hd/hd phase occurs, and hence there exist five stationary phases. When the rate is very small, the mc/hd phase disappears, and there are only four phases again. These theoretical results qualitatively and quantitatively agree with computer Monte Carlo simulations especially in the case of large value of the detachment rate.

Keywords: TASEPs; Detachment; Extended particle; Density profile

1. Introduction

In recent years, totally asymmetric simple exclusion processes (TASEPs) have become a subject of increasing scientific interest in physics, chemistry and biology [1,2]. They serve as simple models for collective phenomena ranging from traffic flows to molecular motors [3–7]. The TASEPs are one-dimensional lattice gas models where particles move preferentially in one direction and interact only through hard-core exclusion. Despite their simple rules, the TASEPs still exhibit very rich dynamical behavior. Examples include boundary induced phase transitions [8], spontaneous symmetry breaking [9,10], and phase segregations [11,12].

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Recently, systems without particle conservation in the bulk have attracted much attention. In Ref. [13], the effects of a single detachment or attachment site in the bulk of an asymmetric simple exclusion process were investigated in detail for the transportation of single particles, and complex dynamics were analyzed. In Refs. [14–17], the interplay of the totally asymmetric exclusion process with local absorption/desorption kinetics of single particles acting at all sites, termed "Langmuir kinetics" (LK) was considered. The combined processes of TASEPs and LK show the new feature of a local shock in the density profile of the stationary state. Further, the TASEPs with extended particles that can occupy more than one lattice site have been investigated by mean-field and various continuum approaches [18–22]. The extended particle model is believed to be of physical relevance in realistic biological processes such as ribosomes moving along mRNA, large molecules or vesicles moving along filaments or microtubules and RNA translation. For example, ribosomes typically cover 10–12 codon sites but move only to one codon at one time. Effects of local inhomogeneity in the TASEPs with extended objects have also been investigated [20].

To our knowledge, the effect of irreversible detachments on dynamics of the TASEPs with extended objectives has not been systematically studied yet. In this paper, we present a simple detachment model in which the inhomogeneity is at the middle of the lattice. The goal of this work is to investigate the mixed effect of detachments and size of particles on the density profiles in the system of TASEPs. Most of our results will be derived from mean-field calculations and Monte Carlo simulations. The paper is organized as follows. In Section 2, a model is presented to describe the detachment of TASEPs with extended particles. Section 3 summarizes some known results for homogeneous TASEPs with extended particles; some calculations for our model from a simple approximate theory are presented. In Section 4, the associated phase diagrams are obtained by Monte Carlo simulations, and a comparison between the numerical results and the theoretical predictions is made. A brief summary is given in Section 5.

2. Model

Consider a chain of N lattice sites with open boundary conditions at both ends and a bunch of identical particles of size $l \geqslant 1$ moving through the lattice from the left to the right (N is here a large *even* number). The particles, each of which covers $l \geqslant 1$ lattice sites, interact through hard-core repulsion. For convenience, we shall locate the position of a particle by its left edge. A particle enters the system from the left with rate α if the first l sites are empty, and exits the lattice with rate β if it sits at site N. In the bulk, the particle at site i will always move to the next site $i+1$ as long as site $i+l$ is unoccupied. In addition, when a particle is at the special site k, which is far away from the boundaries, it will jump off the lattice with rate q, if site $k+2l-1$ is covered, otherwise it can jump to the next site with unit rate. In this work, we set $k = N/2$ (as shown in Fig. 1), and define the particle density as the coverage density. Namely, the density is given by $\rho = Ml/N$, where M is the number of the extended particles on the lattice. Here we are mainly interested in the asymptotic behavior in the limit $N \gg 1, l$.

The model reduces to the homogeneous TASEP with extended particles [18] if rate $q = 0$, the normal TASEP with detachment [13] if size $l = 1$, and the simple TASEP if $q = 0$ and $l = 1$. For the last case, the model can be investigated analytically by the matrix-product ansatz [1,2].

3. Mean-field approximation

The TASEPs with a local inhomogeneity have been extensively investigated [23–27,29]. In this case, a simple mean-field approximation is found to be rather successful in describing the associated stationary phases.

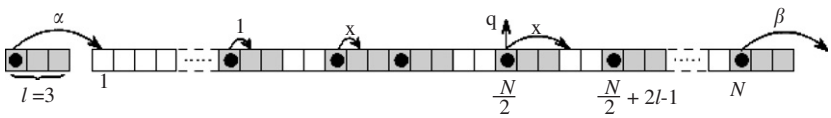

Fig. 1. Model for TASEPs with extended particles and detachment. System with size $l = 3$ particles is shown. Hopping rate in the bulk is 1 and the detachment rate at special site $N/2$ is q. Entrance and exit rates are α and β, respectively.

The mean-field theory involves two plausible assumptions [13,20,28]: the effect of a local inhomogeneity is just to divide the whole system into two coupled TASEPs without defect, and the density correlations between two sublattices can be neglected. Since the above assumptions do not depend on particle sizes, we follow the procedures in Refs. [13,20,28], and also consider the system as two coupled homogeneous TASEPs with extended particles.

According to Refs. [18,21], there are three stationary phases in the TASEPs with extended particles: the maximal current, entry-limited current, and exit-limited current phases. The latter two are also referred to the low-density (ld) and the high-density (hd) phases. The steady-state currents J and densities, together with the values of entrance rate α and exit rate β, in these stationary phases are described by

- Maximal-current (mc) phase is in range $\alpha \geqslant 1/(\sqrt{l}+1)$ and $\beta \geqslant 1/(\sqrt{l}+1)$, and has

$$J = \frac{1}{(\sqrt{l}+1)^2}, \quad \rho = \frac{\sqrt{l}}{\sqrt{l}+1}. \tag{1}$$

- High-density (hd) phase is in range $\alpha > \beta$ and $\beta < 1/(\sqrt{l}+1)$, and has

$$J = \frac{\beta(1-\beta)}{1+(l-1)\beta}, \quad \rho = 1-\beta. \tag{2}$$

- Low-density (ld) phase is in range $\alpha < \beta$ and $\alpha < 1/(\sqrt{l}+1)$, and has

$$J = \frac{\alpha(1-\alpha)}{1+(l-1)\alpha}, \quad \rho = \frac{\alpha l}{1+(l-1)\alpha}. \tag{3}$$

The local inhomogeneity is at site $N/2$ in the present model, and thus the left sublattice will have the entrance rate α and an effective exit rate β_{eff} at site $i = N/2$, given by

$$\beta_{eff} = q + (1 - \rho_{N/2+2l-1}), \tag{4}$$

where $\rho_{N/2+2l-1}$ is the coverage density of site $N/2+2l-1$. The first term q is the rate for a particle at $N/2$ to jump off the lattice. The second term describes that a particle will move into the right lattice site with rate 1 as long as sites from $N/2+l$ to $N/2+2l-1$ are empty. The probability of this event is equal to $1-\rho_{N/2+2l-1}$, because an empty site at $N/2+2l-1$ guarantees that sites from $N/2+l$ to $N/2+2l-1$ are empty.

If the particles are considered to be uniformly distributed among the whole right sublattice in the steady state (we expect that neglect of boundary effects will affect the locations of phase diagrams, but does not have qualitative consequences), Eq. (4) then reduces to

$$\beta_{eff} = q + 1 - \rho_r, \tag{5}$$

where subscript r represents the right sublattice.

Analogously, the right sublattice will have the exit rate β and an effective entrance rate α_{eff}, given by

$$\alpha_{eff} = \frac{\rho_l}{l}, \tag{6}$$

where ρ_l is the coverage density of the left sublattice in a steady state. The factor $1/l$ is due to the fact that a particle covers l lattice sites.

Since each sublattice can have three stationary phases, there are nine potential phases for the whole system with irreversible detachment. Nevertheless, since the current on the right sublattice must be smaller than that on the left sublattice due to detachment, the right sublattice cannot reach the mc state. Thus, the three associated phases—the ld/mc, hd/mc, and mc/mc phases, cannot occur. Here, we have used symbol "ld/mc" to represent that the left (right) sublattice is in the low-density (maximal-current) phase; the same applies to symbols "hd/mc" and "mc/mc". The conditions for the remaining six phases can be

found by using the properties of the TASPs with extended particles and Eqs. (5) and (6). We shall discuss them sequentially:

1. The high-density/low-density hd/ld phase. Following Refs. [18,21], this phase occurs only if

$$\alpha > \beta_{eff}, \quad \beta_{eff} < \frac{1}{\sqrt{l}+1},$$
$$\alpha_{eff} < \beta, \quad \alpha_{eff} < \frac{1}{\sqrt{l}+1}. \tag{7}$$

The particle density ρ_l (ρ_r) on the left (right) sublattice is given by

$$\rho_l = 1 - \beta_{eff}, \quad \rho_r = \frac{\alpha_{eff} l}{1 + (l-1)\alpha_{eff}}. \tag{8}$$

Substituting Eq. (8) into Eqs. (5) and (6), we can derive the effective exit rate from the left lattice and the effective entrance rate to the right lattice as

$$\alpha_{eff} = \frac{1}{l} - \frac{2l + (l-1)q - \sqrt{(l-1)^2(q^2-4q) + 4(2l-1)}}{2l(l-1)},$$
$$\beta_{eff} = \frac{2l + (l-1)q + \sqrt{(l-1)^2(q^2-4q) + 4(2l-1)}}{2(l-1)} \quad (l \neq 1). \tag{9}$$

With some simple calculations, one finds that Eqs. (7) and (8) do not have any common solution. Thus, we conclude that the hd/ld phase does not occur.

2. The low-density/low-density (ld/ld) phase. This occurs in range

$$\alpha < \frac{1}{\sqrt{l}+1}, \quad \alpha < \beta_{eff},$$
$$\alpha_{eff} < \frac{1}{\sqrt{l}+1}, \quad \alpha_{eff} < \beta. \tag{10}$$

The densities are

$$\rho_l = \frac{\alpha l}{1 + (l-1)\alpha}, \quad \rho_r = \frac{\alpha_{eff} l}{1 + (l-1)\alpha_{eff}}. \tag{11}$$

From Eqs. (5), (6) and (11), we have the effective entrance α_{eff} and exit rates β_{eff} as

$$\alpha_{eff} = \frac{\alpha}{1 + (l-1)\alpha}, \quad \beta_{eff} = q + 1 - \frac{\alpha l}{1 + 2(l-1)\alpha}. \tag{12}$$

Using conditions (10), the parameter ranges for ld/ld phase for a given value of q can be given by

$$\alpha < \frac{1}{\sqrt{l}+1}, \quad \beta > \frac{\alpha}{1 + (l-1)\alpha}. \tag{13}$$

3. The low-density/high-density (ld/hd) phase. It has

$$\alpha < \frac{1}{\sqrt{l}+1}, \quad \alpha < \beta_{eff},$$
$$\alpha_{eff} > \beta, \quad \beta < \frac{1}{\sqrt{l}+1}. \tag{14}$$

The densities are

$$\rho_l = \frac{\alpha l}{1 + (l-1)\alpha}, \quad \rho_r = 1 - \beta, \tag{15}$$

and the effective entrance and exit rates are

$$\alpha_{eff} = \frac{\alpha}{1+(l-1)\alpha}, \quad \beta_{eff} = q+\beta. \tag{16}$$

The existence conditions are given by

$$\alpha < q+\beta, \quad \alpha < \frac{1}{\sqrt{l}+1},$$
$$\beta < \frac{\alpha}{1+(l-1)\alpha}, \quad \beta < \frac{1}{\sqrt{l}+1}. \tag{17}$$

Therefore, the ld/hd phase is in the region of $\alpha < 1/(\sqrt{l}+1)$ and $\beta < \alpha/(1+(l-1)\alpha)$.

4. The mc/ld phase occurs if (according to Refs. [18,21])

$$\alpha > \frac{1}{\sqrt{l}+1}, \quad \beta_{eff} > \frac{1}{\sqrt{l}+1},$$
$$\alpha_{eff} < \frac{1}{\sqrt{l}+1}, \quad \alpha_{eff} < \beta. \tag{18}$$

The densities are

$$\rho_l = \frac{\sqrt{l}}{\sqrt{l}+1}, \quad \rho_r = \frac{\alpha_{eff} l}{1+(l-1)\alpha_{eff}}. \tag{19}$$

Substituting Eq. (19) into Eqs. (5) and (6) yields the effective entrance and exit rates

$$\alpha_{eff} = \frac{1}{\sqrt{l}(\sqrt{l}+1)}, \quad \beta_{eff} = q+1-\frac{l}{2l-1+\sqrt{l}}. \tag{20}$$

Finally, the parameter ranges for the mc/ld phase is given by

$$\alpha > \frac{1}{\sqrt{l}+1}, \quad \beta > \frac{1}{\sqrt{l}(\sqrt{l}+1)}, \tag{21}$$

for any given value of q.

5. The mc/hd phase is given by

$$\alpha > \frac{1}{\sqrt{l}+1}, \quad \beta_{eff} > \frac{1}{\sqrt{l}+1},$$
$$\alpha_{eff} > \beta, \quad \beta < \frac{1}{\sqrt{l}+1}, \tag{22}$$

with densities profiles

$$\rho_l = \frac{\sqrt{l}}{\sqrt{l}+1}, \quad \rho_r = 1-\beta. \tag{23}$$

Using Eqs. (5) and (6), we obtain

$$\alpha_{eff} = \frac{1}{\sqrt{l}(\sqrt{l}+1)}, \quad \beta_{eff} = q+\beta. \tag{24}$$

This phase condition can be obtained by performing similar calculations. The mc/hd phase exists in the following region:

$$\frac{1}{\sqrt{l}+1} - q < \beta < \frac{1}{\sqrt{l}(\sqrt{l}+1)}, \quad \alpha > \frac{1}{\sqrt{l}+1}. \tag{25}$$

According to the above inequalities, when $q > 1/(\sqrt{l}+1)$, the mc/hd phase occur for $\alpha > 1/(\sqrt{l}+1)$ and $0 < \beta < 1/\sqrt{l}(\sqrt{l}+1)$, and when $1/l((l+1)/(\sqrt{l}+1)-1) < q < 1/(\sqrt{l}+1)$, the mc/hd phase only exists in

the region of $\alpha > 1/(\sqrt{l}+1)$ and $1/(\sqrt{l}+1) - q < \beta < 1/\sqrt{l}(\sqrt{l}+1)$. But, when the value of q is smaller than $1/l((l+1)/(\sqrt{l}+1)-1)$, there is no mc/hd phase in the phase diagram.

6. The high-density/high-density (hd/hd) phase is described by

$$\alpha > \beta_{eff}, \quad \beta_{eff} < \frac{1}{\sqrt{l}+1},$$

$$\alpha_{eff} > \beta, \quad \beta < \frac{1}{\sqrt{l}+1} \tag{26}$$

and

$$\rho_l = 1 - \beta_{eff}, \quad \rho_r = 1 - \beta. \tag{27}$$

Substituting this last equation into Eqs. (5) and (6) yields the effective entrance and exit rates

$$\alpha_{eff} = \frac{1-q-\beta}{l}, \quad \beta_{eff} = q + \beta. \tag{28}$$

Comparing these expressions with the inequalities (26), we obtain the conditions of the hd/hd phase as following. For $\alpha < 1/(\sqrt{l}+1)$

$$\alpha > q, \quad \beta < \alpha - q, \tag{29}$$

while for $\alpha > 1/(\sqrt{l}+1)$

$$\beta < \frac{1-q}{l+1},$$

$$\beta < \frac{1}{\sqrt{l}+1} - q. \tag{30}$$

The inequalities (29) and (30) determine the conditions for the existence of the hd/hd phase. According to the inequalities (30), the hd/hd phase exists only for $q < 1/(\sqrt{l}+1)$. But for the exit rate β, there are two possibilities. When $(1-q)/(l+1) > 1/(\sqrt{l}+1) - q$ is satisfied, inequalities $\beta < 1/(\sqrt{l}+1) - q$ and $q > 1/l((l+1)/(\sqrt{l}+1)-1)$ are obtained. However, if $(1-q)/(l+1) < 1/(\sqrt{l}+1) - q$ is satisfied, there are inequalities $\beta < (1-q)/(l+1)$ and $q < 1/l((l+1)/(\sqrt{l}+1)-1)$. Therefore the hd/hd phase appear in the following region, depending on the value of q. The hd/hd phase occurs for $\beta < \alpha - q$ in the case of $\alpha < 1/(\sqrt{l}+1)$; and for $\alpha > 1/(\sqrt{l}+1)$ and $\beta < 1/(\sqrt{l}+1) - q$ if the inequality $1/l((l+1)/(\sqrt{l}+1)-1) < q < 1/(\sqrt{l}+1)$ is satisfied; and for $\alpha > 1/(\sqrt{l}+1)$ and $\beta < (1-q)/(l+1)$ if $q < 1/l((l+1/(\sqrt{l}+1)-1)$ is met. However if $q > 1/(\sqrt{l}+1)$, there is no hd/hd phase in the phase diagram.

In summary, the phase diagrams of the model in Section 2 depend both on the detaching rate q and the particle size l. If $q > 1/(\sqrt{l}+1)$, there are four stationary phases: the mc/ld, mc/hd, ld/ld, and ld/hd phases; the hd/hd phase does not occur. When $1/l((l+1)/(\sqrt{l}+1)-1) < q < 1/(\sqrt{l}+1)$, the hd/hd phase emerges, and there are five stationary phases in total.

When $q < 1/l((l+1)/(\sqrt{l}+1)-1)$ is satisfied, the mc/hd phase cannot occur, there remains four phases. Nevertheless, we notice that the situation $1/(\sqrt{l}(\sqrt{l}+1)) < \beta < (1-q)/(l+1)$ is allowed, where both the mc/ld and the hd/hd phases are given by the above mean-field theory. Thus, the approximate method cannot be valid. This is a consequence of neglect of boundary correlations; namely, we have simply substituted the coverage density on boundaries by the bulk density in the above calculations. In reality, when the value of the detaching rate q is very small, the coverage density near the local inhomogeneity is larger than that in the bulk of the sublattice, therefore the deviations may be resulted in. In case of $q < 1/l((l+1)/(\sqrt{l}+1)-1)$, the condition $\beta < 1/(\sqrt{l}+1)$ is satisfied, therefore there is a hd phase in the right sublattice. In the left sublattice, when the particle is on the site $N/2$, it must make l jumps with rate q and one jump with rate 1, because the density in the right sublattice is high. The effective probability for the particle to jump off per time step is $\beta_q = (lq+1)/(l+1)$. The condition $q < 1/l((l+1)/(\sqrt{l}+1)-1)$ yields $\beta_q < 1/(\sqrt{l}+1)$, therefore the hd phase

appears in the left sublattice. Thus, in the region of $1/(\sqrt{l}(\sqrt{l}+1))<\beta<(1-q)/(l+1)$ under the condition $q<1/l((l+1)/(\sqrt{l}+1)-1)$, there is the hd/hd phase in the phase diagram.

4. Monte Carlo simulations

In order to test the predictions of the mean-field theory, we performed Monte Carlo simulations for a chain of $N = 3000$ lattice sites with particle size $l = 3$. The random sequential updating is used. In each Monte Carlo step (MCs), there is a pool containing M particles plus a free particle, where M is the number of the extended particles on the lattice before this MCs is updated. The $M + 1$ particles are chosen at random. If a particle on the normal lattice is selected, it attempt to move forward. But if a particle on the lattice site $N/2$ is chosen, it can jump completely into the right subsystem with unit rate if the site $N/2 + 2l - 1$ is not covered, otherwise it detaches from the lattice with rate q and if a particle is free, it can enter the lattice at left end with rate α if the first l sites are empty. Simulations are begun with an empty lattice and run until steady states are reached. In the process of simulations the steady states are reached after 4×10^7 MCs. And density profiles are obtained by averaging over every 100 MCs in additional 2×10^6 MCs after steady states are attained.

According to the mean-field theory discussed earlier, the phase diagram of the present model depends on detaching rate q. For instance, for $q>1/(\sqrt{l}+1)$, there exist four stationary phases. For $l = 3$, the inequality $q>1/(\sqrt{l}+1)$ implies $q>0.366$. Simulations were performed with $l = 3$, $N = 3000$, and $q = 0.7$. The density profiles are shown in Fig. 2, in which the solid lines correspond to the mean-field predictions. This confirms the mean-field calculations.

Fig. 3 shows the density profiles calculated by the approximate approach and those obtained by Monte Carlo simulations with the same parameter values of Fig. 2. The ld/ld, mc/hd, mc/ld, and ld/hd phases are presented in Fig. 3(a)–(d), respectively. As shown in Fig. 3, it is found that an excellent qualitative and quantitative agreement between the theoretical results and Monte Carlo simulations, has been obtained. In Fig. 3(a) and (c), we also observe that the density profiles split into approximately two branches near the right edge of the system. This phenomenon has also been obtained in the TASEPs with particles of arbitrary size in Refs. [18,21]. Especially, the bifurcation can also be observed near the right of the left sublattice in the mc/hd density profile, as shown in Fig. 3(b).

The decrease of the value of q causes another phase, i.e., the hd/hd phase in the phase diagram, as shown in Fig. 4. When the value of the detaching rate q is smaller than $1/(\sqrt{l}+1)$, and larger than $1/l((l+1)/(\sqrt{l}+1)-1)$, i.e., $1/l((l+1)/(\sqrt{l}+1)-1) = 0.155<q<1/(\sqrt{l}+1) = 0.366$, according to the above theoretical results, the hd/hd phase exists in the region where when $\alpha<1/(\sqrt{l}+1) = 0.366$, $\beta<\alpha-q$, however when $\alpha>1/(\sqrt{l}+1) = 0.366$, $\beta<1/(\sqrt{l}+1)-q$. The Monte Carlo simulations show an excellent

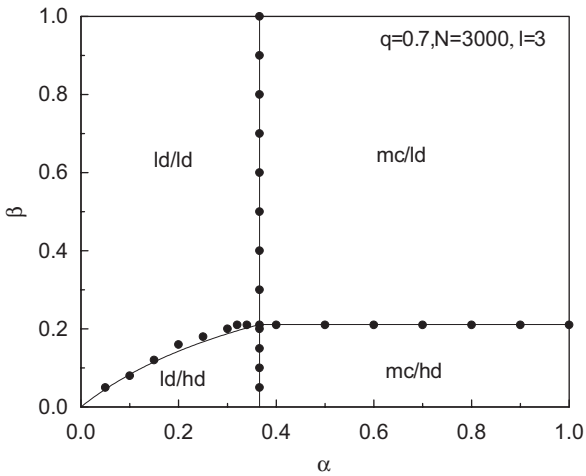

Fig. 2. Phase diagram for the TASEPs with extended particles and irreversible detachments for $q>1/(\sqrt{l}+1)$. The points correspond to the phase transition boundaries obtained by Monte Carlo simulations, and the lines are derived from an approximate method.

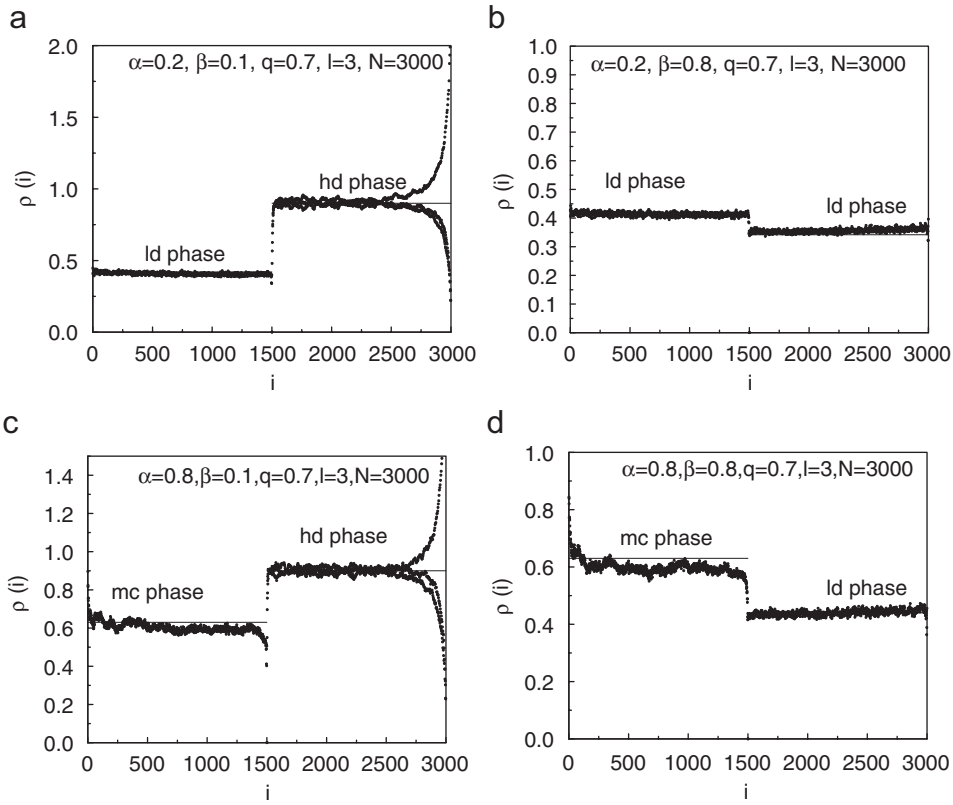

Fig. 3. Sample density profiles from simulations for the system of size $q > 1/(\sqrt{l}+1)$. (a) ld/hd phase with $\alpha = 0.2$ and $\beta = 0.1$; (b) ld/ld phase with $\alpha = 0.2$, $\beta = 0.8$; (c) mc/hd phase with $\alpha = 0.8$, $\beta = 0.1$; (d) mc/ld phase with $\alpha = 0.8$, $\beta = 0.8$.

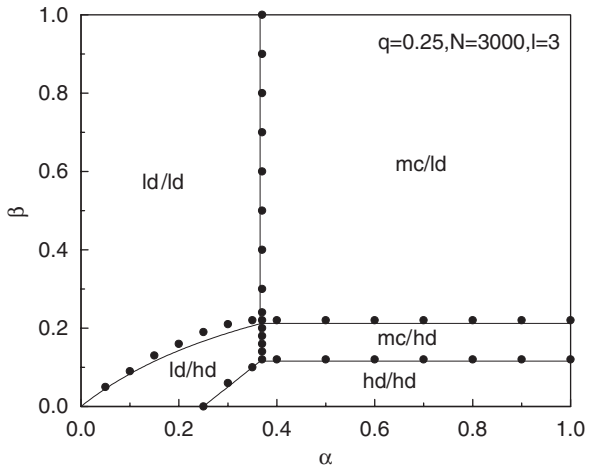

Fig. 4. Phase diagram for the TASEPs with extended particles and irreversible detachments for $1/l((l+1)/(\sqrt{l}+1)-1) < q < 1/(\sqrt{l}+1)$. The points correspond to the phase transition boundaries obtained by Monte Carlo simulations, and the lines are derived from an approximate method.

agreement with the theoretical results in the case of $1/l((l+1)/(\sqrt{l}+1)-1) < q < 1/(\sqrt{l}+1)$. In Fig. 4, other four phases are not affected by the decrease of the value of q, except the boundary between mc/hd and hd/hd phases.

Fig. 5 shows the density profiles derived from the mean-field theory and computer simulations for $q = 0.25$. As shown in Fig. 5, an excellent agreement has been obtained between the theoretical results and the computer simulations. Only differences can be observed in the region near the boundaries and the special site $N/2$ from which irreversible detachments may occur. There are two reasons to induce these consequences. One is that the density profiles on the boundaries cannot be replaced by the bulk density of the sublattice in the approximate method. For example, as shown in Fig. 5(b), there is a larger difference in density profile in the ld/ld phase, especially on the right sublattice; the other is that the mean-field theory neglects the correlations especially

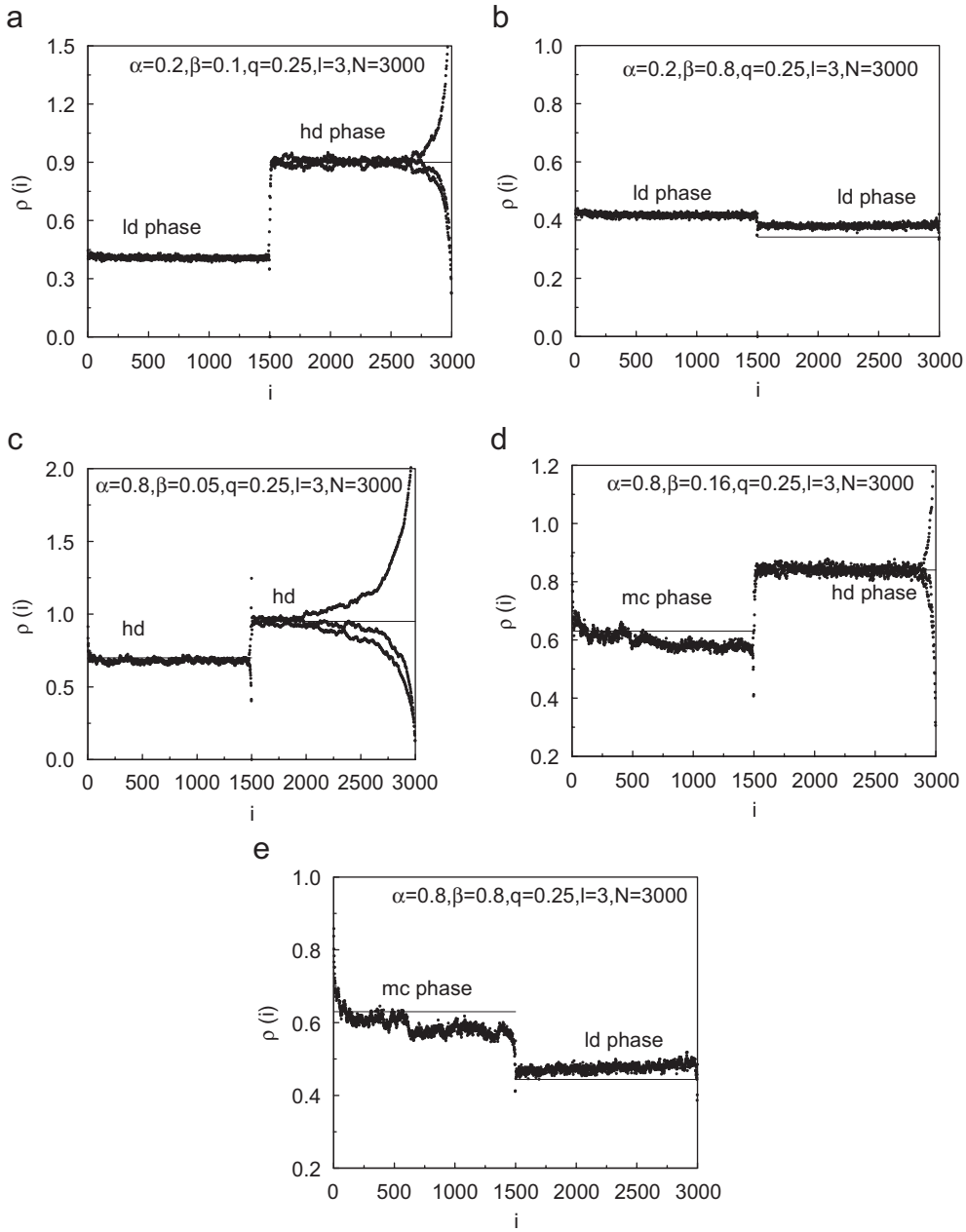

Fig. 5. Sample density profiles from simulations for $1/l((l+1)/(\sqrt{l}+1)-1) < q < 1/(\sqrt{l}+1)$. (a) ld/hd phase with $\alpha = 0.2$, $\beta = 0.1$; (b) ld/ld phase with $\alpha = 0.2$, $\beta = 0.8$; (c) hd/hd phase with $\alpha = 0.8$, $\beta = 0.1$; (d) mc/hd phase with $\alpha = 0.8$, $\beta = 0.16$; (e) mc/ld phase with $\alpha = 0.8$, $\beta = 0.8$.

near the special site $N/2$. The correlations usually have significant effects on the density profiles, and in real system determine the behavior of the system. Moreover, about two branches have been observed near the right boundary of both the right and the left sublattice when there is the hd phase in the corresponding sublattice, as shown in Fig. 5(a), (c) and (d). Particularly, bifurcation can even be observed near the right boundary of the left sublattice when the mc/hd phase occurs in the system, as shown in Fig. 5(d).

Further decrease of the value of q induces decrease of the number of phases. Fig. 6 shows the phase diagram in the case of $q<1/l((l+1)/(\sqrt{l}+1)-1)=0.155$. As shown in Fig. 6, we can observe that the mc/hd phase in the phase diagram disappears and only four stationary phases can be obtained. The approximate theory gives an incompatible results which both the mc/hd and the hd/hd phases are in the region of $\alpha>1/(\sqrt{l}+1)$ and $1/\sqrt{l}(\sqrt{l}+1)<\beta<(1-q)/(l+1)$, and the boundary of the phase ld/hd calculated by the theoretical method is also different from the simulation results, as shown in Fig. 6. In fact, in the region of $1/\sqrt{l}(\sqrt{l}+1)<\beta<(1-$

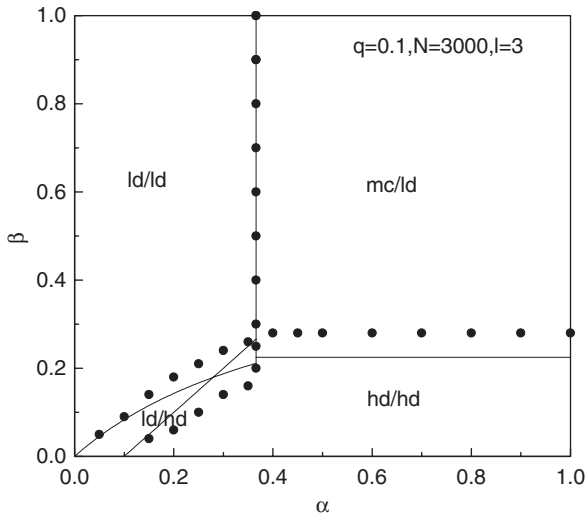

Fig. 6. Phase diagram for the TASEPs with extended particles and irreversible detachments for $q<1/l((l+1)/(\sqrt{l}+1)-1)$. The points correspond to the phase transition boundaries obtained by Monte Carlo simulations, and the lines are derived from an approximate method.

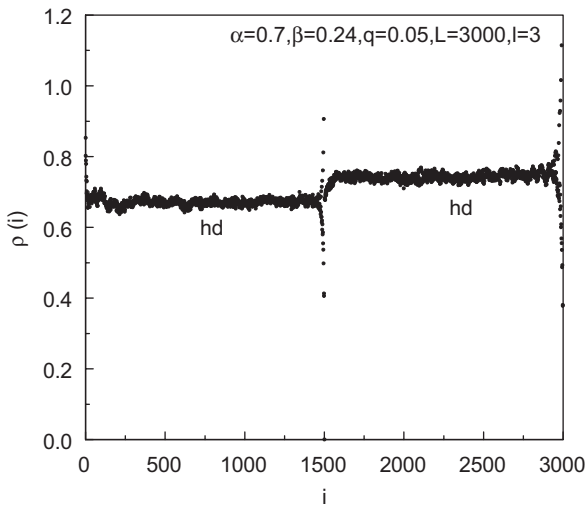

Fig. 7. The hd/hd phase from simulations for $q<1/l((l+1)/(\sqrt{l}+1)-1)$ and $1/\sqrt{l}(\sqrt{l}+1)<\beta<(1-q)/(l+1)$.

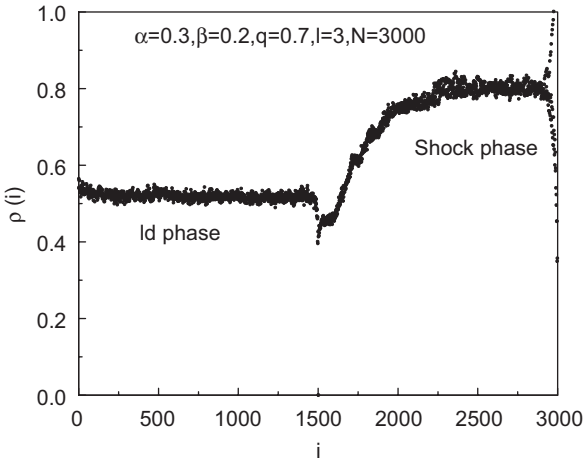

Fig. 8. Sample density profiles near the transition from the ld/hd to the ld/ld phase. The transition is expected to occur at $\beta = 0.1875$. The density profile is a shock wave in the right of the system which is the first-order transition.

$q)/(l+1)$ under the condition $q<1/l((l+1)/(\sqrt{l}+1)-1)$, there is the hd/hd phase in the phase diagram, as shown in Fig. 7. The hd/hd phase is also predicted by a simple approximate method, as discussed above.

In the process of simulations, there are two types of phase transitions in the system. One is the first-order phase transitions involving a jump in the particle density in one of the sublattice, the first-order phase boundaries are those between ld/ld and ld/hd, between ld/hd and hd/hd, and between mc/ld and mc/hd; Other is the second-order phase transitions whose density profiles change continuously, such as boundaries between ld/ld and mc/ld phases, between mc/hd and hd/hd phases, and between mc/hd and ld/hd phases. For the first-order phase transition, the transition lines are easily found. But, for the second-order phase transition, the phase transitions are difficultly confirmed and identified as follows. For example, the transition between the ld/hd phase and the ld/ld phase is expected to produce a shock wave in the right half of the system, as shown in Fig. 8. Similarly, the other phase boundaries can be determined in this way. Visual inspection of the density profiles allows the transition lines to be located within about 0.02 units. As shown in Figs. 2, 4 and 6, these phase boundaries obtained from Monte Carlo simulations accord with theoretically calculated boundaries, although there are small deviations at the first-order phase boundaries and in the case of small value of q.

5. Conclusions

We have investigated the effect of irreversible detachments in totally asymmetric simple exclusion processes with extended particles. The local inhomogeneity is set at the site $N/2$, which is far away from the boundaries. Based on the assumptions that the local inhomogeneity divides the lattice into two coupled totally asymmetric simple exclusion processes without the local defect and that density correlations near the boundary between two sublattices are neglected, we define the effective entrance rate α_{eff} and the exit rate β_{eff}, and obtain two homogeneous totally simple exclusion process with extended particles.

The low-density, the high-density, and the maximal-current phases have been obtained in the TASEPs with extended particles, therefore the detachment may induce nine possible stationary phases in the TASEPs with extended particles. However, three phases, such as the ld/mc, the hd/mc and the mc/mc phases cannot occur due to the fact that the particle current in the right sublattice cannot reach its maximal value because of the effects of the detachment. Furthermore, the approximate theory also indicates that the hd/ld phase cannot exist. The steady phases in the phase diagram depend on the detaching rate q and the size of the particles l. When the rate q is larger than $1/(\sqrt{l}+1)$, i.e., $q>1/(\sqrt{l}+1)$, there are just four stationary phases, such as ld/ld, mc/ld, ld/hd and mc/hd phases. Decreasing the value of q results in the occurrence of the hd/hd phase, therefore there are total five stationary phases in the phase diagram. However, when $q<1/l((l+1)/(\sqrt{l}+1)-1)$, the mc/hd phase disappears, only four phases exist in the phase diagram. The approximate

theory can excellently explain the Monte Carlo simulations, except the case of $q<1/l((l+1)/(\sqrt{l}+1)-1)$. In the case of $q<1/l((l+1)/(\sqrt{l}+1)-1)$, these differences between the theoretical results and computer simulations may be caused by substituting the coverage density of the boundary in the sublattice for the bulk density of the sublattice in the steady state in the process of deriving the boundary conditions. When the value of the detaching rate q is very small, the coverage density near the local inhomogeneity is larger than that in the bulk sublattice, hence results in the deviations. The systematical errors from the mean-field theory may be eliminated by finite-segment mean-field theory (FSMFT) [29]. Moreover, the hydrodynamic method for TASEPs with $l>1$ investigated in Ref. [22] can also be extended to our model. Those can be explored in the future.

Compared with the known results, we find that the detachment and size of the particles leads to distinct effects in the TASEPs. In the TASEPs with $l=1$, the occurrence of the hd/hd phase depends on the value of q [13]. For $q>\frac{1}{2}$, the hd/hd phase does not appear and there are only four phases, while for $q<\frac{1}{2}$, the hd/hd phase is added. But for $l\neq 1$, not only the mc/hd phase but also the hd/hd phase can be determined by the detachment. Obviously, for $l=1$, the results obtained in the paper can accord with the previous results, because of $1/l((l+1)/(\sqrt{l}+1)-1)=0$ and the detachment rate must be positive. In addition, for $l\to\infty$, the expression $1/l((l+1)/(\sqrt{l}+1)-1)\to 0$, the stationary phases in the phase diagram are expected to the same as those for $l=1$. This situation should be further investigated.

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