# SUSTech Young Mathematicians Academic Forum

Organisers: Hu, Yong - SUSTech Liang, Yongqi - Université Paris 7 Wang, Xiaozong - Université Paris-Sud

July 22 - 28, 2018 at SUSTech Shenzhen, Guangdong, China Wisdom Valley, Building No. 3, Conference Room  $415\ 3415$ 

	2		Schedule	· · · · · · · · · · · · · · · · · · ·		
	23/07 Monday	24/07 Tuesday	25/07 Wednesday	26/07 Thursday	27/07 Friday	28/07 Saturday
09:00 - 10:00	SM-211 Zicheng Qian	SM-215 D.Izquierdo	SM-220 Shun Tang	SM-224 Wen-Wei Li	SM-228 Jie Liu	2 talks for learning seminar
10:00 - 10:30	Tea/coffee break	Tea/coffee break	Tea/coffee break	Tea/coffee break	Tea/coffee break	
10:30 - 11:30	SM-212 Yiwen Ding	SM-216 G.Lucchini Arteche	SM-221 Jinbo Ren	SM-225 Lizao Ye	SM-229 Haowu Wang	
11:30 - 13:00		Lunch				
13:50 - 14:50		SM-217 Yang Cao			-	
14:30 - 15:30	SM-213 Hui Gao		SM-222 Shenghao Sun	SM-226 Lei Zhang		
14:50 - 15:20		Tea/coffee break				
15:30 - 16:00	Tea/coffee break		Tea/coffee break			
15:20 - 16:20		SM-218 Yisheng Tian			Free Afternoon	End
16:00 - 17:00	SM-214 R.Abdellatif		SM-223 Xu Shen	SM-227 Daxin Xu		
16:20 - 16:40		Tea/coffee break				
17:00 - 17: 20	Group Photo					
16:40 - 17:40		SM-219 Yigeng Zhao			1	
17:20 - 19:00	Dinner			Dir	nner	
18:00		Conference dinner				

### **Titles and Abstracts**

# 1 (SM-211) Mod p local global compatibility for $GL_n(\mathbb{Q}_p)$ in Fontaine-Laffaille case

Qian, Zicheng Université Paris-Sud

Abstract : Let p be a prime number, n > 2 an integer, and  $\mathbb{F}$  a CM field in which p splits completely. Assume that a continuous automorphic Galois representation  $r : Gal(\mathbb{Q}/\mathbb{F}) \to GL_n(\mathbb{F}_p)$  satisfies certain genericity conditions at a place w above p, and that every subquotient of  $r|_{Gal(\mathbb{Q}_p/\mathbb{F}_w)}$  of dimension > 2 is FontaineCLaffaille generic. We show that the isomorphism class of  $r|_{Gal(\mathbb{Q}_p/\mathbb{F}_w)}$  is determined by  $GL_n(\mathbb{F}_w)$ -action on a space of mod p algebraic automorphic forms cut out by the maximal ideal of a Hecke algebra associated to r. In particular, we show that the wildly ramified part of  $r|_{Gal(\mathbb{Q}_p/\mathbb{F}_w)}$ is determined by the action of Jacobi sum operators (seen as elements of  $\mathbb{F}_p[GL_n(\mathbb{F}_p)]$ ) on this space. This is a joint work with Chol park if r is upper triangular at w and joint work in progress with Bao V. Le Hung, Chol park, Daniel Le and Stefano Morra in the general case.

### 2 (SM-212) L-invariants and p-adic Langlands program

#### Ding, Yiwen Peking University

**Abstract**: Let V be a special p-adic Galois representation. By p-adic Hodge theory, one can associate to V the so-called Fontaine-Mazur L-invariants. A task in p-adic Langlands program is to understand their counterpart on the automorphic side. In this talk, we will first review Breuil's initial work on L-invariants in  $GL_2(\mathbb{Q}_p)$ -case, and then report some recent progress (joint with Breuil) on higher L-invariants.

# 3 (SM-213) Locally analytic vectors and overconvergent $(\varphi, \tau)$ modules

#### Gao, Hui University of Helsinki

**Abstract**: The  $(\varphi, \tau)$ -modules, as constructed by Caruso, is an analogue of the more classical  $(\varphi, \Gamma)$ -modules, and they also classify *p*-adic Galois representations of  $G_K$  (where *K* is for example a finite extension of  $\mathbb{Q}_p$ , and  $G_K$  the Galois group). In this talk, we study locally analytic vectors in some period rings and in the  $(\varphi, \tau)$ -modules; this enables us to establish the overconvergence property of the  $(\varphi, \tau)$ -modules. This is joint work with Léo Poyeton.

# 4 (SM-214) Coefficients systems and local representation theory

#### Abdellatif, Ramla Université de Picardie Jules Verne

**Abstract**: Let  $\mathcal{G}$  be a connected reductive group defined over a non-archimedean local field F and  $G = \mathcal{G}(F)$  be its group of F-points. In their seminal work, Schneider and Stuhler introduced a category of G-equivariant objects on the Bruhat-Tits building of G, called G-equivariant coefficient systems, to better understand irreducible smooth complex representations of G. Since then, this theory of coefficient systems keeps inspiring further developments, not only for complex representations nor for reductive groups.

This talk will start by a short review of the main points of Schneider-Stuhler's work. I will then explain some results coming from two different settings that both used this theory as a beacon : the first one relates to smooth representations of G in characteristic p, where p denotes the residual characteristic of F; it was initiated by Paskunas, further developed by several authors, and is now a well-established tool in this context. The other one relates to complex representations of Kac-Moody groups and is at the core of a current joint project with Hébert, aiming to define such coefficients systems on the good analogue of Bruhat-Tits buildings in this setting, namely the *masures*.

## 5 (SM-215) Vanishing theorems and Brauer-Hasse-Noether exact sequences for higher-dimensional fields

Izquierdo, Diego Ecole Normale Supérieure de Paris

**Abstract**: When one wants to study the arithmetic of a given field K, it is often useful to understand the cohomology of the Galois module of roots of unity  $\mathbb{Q}/\mathbb{Z}(1)$  or, more generally, the cohomology of its twists  $\mathbb{Q}/\mathbb{Z}(r)$ . In this talk, we will be interested in the situation when K is a finite extension of the Laurent series field in m variables  $k((x_1, ..., x_m))$  with coefficients in a finite field, a p-adic field or a number field. We will discuss some vanishing theorems as well as some exact sequences that play the role of the Brauer-Hasse-Noether exact sequence for the field K.

# 6 (SM-216) Bad places for the Brauer-Manin obstruction on homogeneous spaces

Lucchini Arteche, Giancarlo Universidad de Chile

Abstract : The Brauer-Manin obstruction explains (partially) the lack of density of rational points of a variety X in the product of points over the different completions of the base field. A conjecture by Colliot-Thélne states that this obstruction should be the only one for rationally connected varieties. As a consequence, there should exist a finite set of "bad places" out of which one actually has this density property. In this talk, I will show how to explicitly describe such a set for homogeneous spaces of semisimple simply connected groups with finite stabilizers. The idea is to translate the Brauer group and its associated Brauer-Manin pairing in terms of group cohomology and then generalize previous work by Bogomolov and Colliot-Thélne on the unramified classes of this group.

### 7 (SM-217) Obstruction to strong approximation

#### Cao, Yang MPIM Bonn

**Abstract :** The arithmetic duality (resp. class field theory) provide exactly the Brauer-Manin obstruction to strong approximation for torus (resp. trivial torus). From this example, I will talk about the Brauer-Manin obstruction and descent obstruction to strong approximation.

### 8 (SM-218) Rational points on tori over p-adic function fields

#### Tian, Yisheng Université Paris-Sud

**Abstract :** In this talk, I will explain some results on the existence of rational points on tori defined over a p-adic function field. More precisely, I will begin with global duality theorems and then introduce a cohomological obstruction to the local-global principle for tori. I will also talk about a reciprocity obstruction to weak approximation for tori and the ingredients of proof. Finally, I will give a comparison of the above two cohomological obstructions for tori.

# 9 (SM-219) A new approach to higher dimensional class field theory

#### Zhao, Yigeng Universität Regensburg

**Abstract**: For a scheme X of arithmetic interest, the class field theory of X aims to describe its abelian fundamental group in terms of arithmetic data on X. In this talk, we give a direct method by using higher ideles and duality theorems. This is a joint work with Moritz Kerz.

### 10 (SM-220) Singular Lefschetz-Riemann-Roch theorem

Tang, Shun Capital Normal University

**Abstract :** In this talk, I will introduce a Lefschetz-Riemann-Roch theorem for singular projective schemes which admit diagonalizable group scheme actions. I will also compare this result with a fixed point formula of Lefschetz type due to R. W. Thomason.

# 11 (SM-221) Mathematical logic and its applications in number theory

### Ren, Jinbo IHES

Abstract : Many interesting classical mathematical problems such as

a) Finding rational solutions of the so-called trigonometric diophantine equation  $F(\cos x, \sin x) = 0$ , where F(X, Y) is an irreducible two-variable polynomial with rational coefficients,

b) Studying algebraicity of values of hypergeometric functions at algebraic numbers,

can be regarded as special cases of the Zilber-Pink conjecture in number theory. In this talk, I will explain how we use tools from mathematical logic to attack this conjecture. In particular, I will present some partial results toward the Zilber-Pink conjecture, including those proved by Christopher Daw and myself.

# 12 (SM-222) *l*-independence, perverse sheaves, and some related problems

#### Sun, Shenghao Tsinghua University

Abstract : In the theory of etale cohomology, there is a type of problems called "independence of l". Basically, it asks whether any family of l-adic sheaves, though for varying primes l but which are compatible in some sense, remains compatible under any cohomological operation. One particular l-independence that we expect after the operation is applied is the domain of lissity, and this question has a variant with respect to the perverse t-structure. We will explain the relevant notions and state the theorem. Time permitting, we will also discuss some related questions in the end.

### 13 (SM-223) *p*-adic period domains and the Fargues-Rapoport conjecture

#### Shen, Xu Chinese Academy of Sciences AMSS

**Abstract :** In his 1970 ICM report, Grothendieck asked the question to describe the p-adic analogues of Griffiths period domains. In this talk, we will review some constructions for these p-adic period domains, following recent developments in p-adic Hodge theory. We will then explain some ideas in a proof of the Fargues-Rapoport conjecture about the structure of certain p-adic period domains. This is joint work with Miaofen Chen and Laurent Fargues.

# 14 (SM-224) Representation theory on homogeneous spaces a survey

#### Li, Wen-Wei Chinese Academy of Sciences AMSS

**Abstract :** In this talk I will introduce the basic notions of harmonic analysis on homogeneous spaces, which is nowadays understood as some relative version of Langlands program. The theory of spherical embeddings plays a pivotal role here. If time permits, I will also discuss some possible generalizations of the basic local results.

### 15 (SM-225) Wilson loop via Koszul duality

#### Ye, Lizao Université de Lorraine

**Abstract :** Perturbative quantum field theory studies formal functions on the moduli space of fields. These functions are called observables, they form a Beilinson-Drinfeld factorisation algebra, which quantifies the shifted Poisson structure on classical observables.

We are interested in a special type of observable called Wilson loops (which plays important role in Langlands program), their support is a loop instead of a point. We will explain how Koszul duality permits their construction via representations of Yangian. This talk is based on works of Costello et al.

### 16 (SM-226) A crystalline incarnation of Berthelots conjecture

Zhang, Lei Freie Universität Berlin

Abstract : Berthelots conjecture predicts that under a proper and smooth morphism of varieties in characteristic p, the higher direct images of an F-overconvergent isocrystal are F-overconvergent isocrystals. In a joint work with Valentina Di Proietto and Fabio Tonini, we prove that this is true for crystalline isocrystals. As an application we prove a Künneth formula for the crystalline fundamental group.

### 17 (SM-227) On higher direct images of convergent isocrystals

### Xu, Daxin California Institute of Technology

**Abstract**: Let k be a perfect field of characteristic p > 0 and W the ring of Witt vectors of k. In this article, we give a new proof of the Frobenius descent for convergent isocrystals on a variety over k relative to W. This proof allows us to deduce an analogue of the de Rham complexes comparison theorem of Berthelot without assuming a lifting of the Frobenius morphism. As an application, we prove a version of Berthelots conjecture on the preservation of convergent isocrystals under the higher direct image by a smooth proper morphism of k-varieties.

# 18 (SM-228) Positivity of the second Chern classes of Fano manifolds

#### Liu, Jie Université de Nice Sophia-Antipolis

Abstract : A projective manifold is called Fano if its anticanonical divisor is ample. Fano manifolds are of great interest in numerous parts of mathematics such as birational/algebraic geometry, differential geometry, arithmetic geometry, etc, due to the fascinating symmetries they possess. Moreover, if X is Fano manifold of Picard number one, it is conjectured that the tangent bundle of X should be stable, and then the famous Bogomolov inequality holds for X. In this talk, I will explain how to get a weaker version of Bogomolov inequality for Fano manifolds with Picard number one using rational curves without assuming the stability of tangent bundles. If time allows, I will show how this inequality can be applied to the study of the explicit geometry of Fano manifolds of coindex four.

### 19 (SM-229) Non-existence of 2-reflective modular forms

Wang, Haowu Université des Sciences et Technologie de Lille - Lille 1

**Abstract**: An even lattice L of signature (2, n) is called 2-reflective if it admits a holomorphic modular form whose support of zero divisor is contained in the Heegner divisor determined by the (-2)-vectors in L. Reflective modular forms have applications in the classification of Lorentzian Kac-Moody algebras and in the theory of moduli spaces (Kodaira dimension and uniruledness of orthogonal modular varieties). In this talk we will give a formula expressing the weight of 2-reflective modular forms and

will prove that there is no 2-reflective lattice when  $n \ge 15$  and  $n \ne 19$  except the even unimodular lattices of signature (2, 18) and (2, 26).

We will start with the notions of orthogonal modular forms, vector valued modular forms and Jacobi forms. Then we will introduce Borcherds products and will explain the relation between vector valued modular forms and Jacobi forms. Finally, we will prove our main results and present an application.

For more details, see https://arxiv.org/abs/1801.09590.