A NOVEL NETWORK METHOD DESIGNING MULTIRATE FILTER BANKS AND WAVELETS

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ABSTRACT

A new unified method for designing both paraunitary cosine-modulated FIR filter banks and cosine-modulated wavelets is proposed in this paper. This problem has been formulated as a quadratic-constrained least-squares (QCLS) minimization problem in which all constraint matrices are symmetric and positive definite. Furthermore, a specific analog neural network whose energy function is chosen as the combined cost of the QCLS minimization problem is built for our design problem in real time. It is quite easy and efficiency to obtain the analysis and synthesis filters with high stopband attenuation and cosine modulated wavelets with compact support by this method. A number of simulations show the effectiveness of this method and the correctness of the theoretical analysis in this paper.

Keyword: neural network, cosine-modulated FIR filter bank, quadratic-constrained least-square, wavelets

1. INTRODUCTION

Recently, the cosine-modulated filter bank (CMFB) with perfect reconstruction (PR) character in the field of multirate signal processing has emerged as an attractive choice of filter bank (FB) with respect to implementation cost and design saving. It is shown[2,5] that the 2M polyphase components of the prototype filter can be grouped into M power-complementary pairs, where each pair is implemented as a two-channel lossless lattice filter bank. The lattice coefficients are optimized to minimize the stopband attenuation of the prototype filter, but this is a highly nonlinear optimization problem with respect to lattice coefficients. Consequently, it is difficult to obtain the PR CMFB with high stopband attenuation. Recently, several authors[4,5,9] formulate the design problem as a quadratic-constrained least squares (QCLS) problem and obtain the CMFB with high stopband attenuation. In this paper, we convert the QCLS problem in [4,5] into a very simple format whose constraint matrices are of symmetric and positive definite properties, and then recast them into an analog neural network framework. After that, we can not only obtain cosine-modulated filter banks with high stopband attenuation but also realize real-time filter bank design in terms of modern analog VLSI technology.

It is shown[10] that filter banks satisfying regularity conditions can be used to generate orthonormal bases of compactly supported wavelets. These regularity conditions can also be translated into some additional constraints imposed on the frequency response of the lowpass filter of the CMFB so as to design cosine-modulated compactly supported wavelets.

This paper is organized as follows. In section II, we present the quadratic-constraint (QC) formulation of the design problem of multirate cosine-modulated filter banks and wavelets. In section III, we build a specific analog neural network to solve the optimization design in real time. Two examples are given in section VI. Finally, we conclude this paper with some remarks.

2. QUADRATIC-CONSTRAINT FORMULATIONS

2.1. QC Formulation for Cosine-modulated Filter Bank

The impulse responses of the analysis and synthesis filters of the cosine-modulated filter bank, \( h_k(n) \) and \( f_k(n) \), are cosine-modulated versions of a prototype filter denoted by \( p_0(n) \), which can be given by

\[
h_k(n) = 2p(n) \cos\left(2k + l\right) \frac{\pi}{2M} \left(n - \frac{N - k - l}{2}\right) + (-l)^k \frac{\pi}{2}
\]
where $N$ denotes the filter length, $M$ denotes the number of channels of the filter bank, $0 \leq n \leq N-I$ and $0 \leq k \leq M - 1$.

Here, the lengths of filters, whose z-transform of impulse response can be expressed as $H_k(z)$ and $F_k(z)$, respectively, are the same and are assumed to be multiples of $2M$, that is, $N = 2mM$. As shown in [2,4], when the prototype filter has linear-phase, the PR conditions for the CMFB with even $M$ can be written as

$$G_k(z)G_k(z) + G_{M-k}(z)G_{M-k}(z) = \frac{1}{M} \quad \text{for even } M \quad (2a)$$

where $0 \leq k \leq \frac{M}{2} - 1$, $G_k(z)$ are the polyphase components of the prototype filter, $p_0(n)$ denotes the impulse response of prototype filter. Hereafter we only consider the even $M$ case since the odd $M$ case is very similar.

It is well known that a simple filter design method is directly optimizing the impulse response $p_0(n)$ of filter banks under the condition of minimizing objective function subject to required constraints (e.g., PR conditions). For this purpose, the PR conditions in (2) should be transformed and written as a set of QC in terms of $p_0(n)$

$$\mathbf{p}_+^T \mathbf{C}_{k,r} \mathbf{p}_+ = 1 \quad (3)$$

where $\mathbf{p}_+ = [p_0(0), p_0(1), \cdots, p_0(mM-1)]^T$, and $\mathbf{C}_{k,r}$ is a family of symmetric and positive definite matrices. Note that $k$ goes from $0$ to $M/2-1$ whereas $r$ is in the range of $0$ to $m-1$, there therefore are $mM/2$ constraints in Eq.(3). In addition, it is evident that there is a normalized condition included in (3).

Because of limited space, the detailed derivation process of Eq.(3) is omitted here. Interesting readers can refer to the relevant materials and references [5-9].

Since the system is most approximate perfect reconstruction in this case and according to the PR theory of multirate filter bank, it is sufficient to only minimize the stopband energy of the prototype filter under the quadratic-constraints of Eq.(3) in even $M$ case. The cost of stopband energy of the prototype filter, expressed by $\phi = \int_{\pi/2M}^{\pi} |p_0(e^{i\omega})|^2 d\omega$, can be further simplified, in vector and matrix notation, as

$$\phi = \mathbf{p}_+^T \mathbf{Q} \mathbf{p}_+ \quad (4)$$

where $\mathbf{Q}$ is a real, symmetric, and positive-definite matrix whose entry ought to be specified by $M$, $N$ and $\varepsilon$ which denotes the bandwidth of transition band of the prototype filter.

### 2.2. QC Formulation for Orthonormal Cosine-modulated Wavelet

It is evident that filter banks that satisfy regularity conditions can be used to generate orthonormal bases of compactly supported wavelets according to literature. Here, we want to obtain a cosine modulated PR filter bank where lowpass filter $H_0(z)$ is maximally regular. It has been shown [10] that an $M$th-band orthonormal wavelet can be obtained from an $M$-channel PR filter bank by using an infinite tree structure. Moreover, the $M$-th channel wavelet would have $L$ vanishing moments if and only if the function $H_0(e^{i\omega})$ has zeros of order $L$ at frequencies $\omega = 2l\pi/M$, $1 \leq l \leq M - 1$ and $H_0(l) = \sqrt{M}$.

Through a lot of cumbersome formula derivations, the former condition of above can be further expressed in terms of the filter’s coefficient vector $\mathbf{P}_+$ as follows

$$\left| \frac{d}{d\omega} H_0(e^{i\omega}) \right|^2_{\omega=0} = \mathbf{p}_+^T \mathbf{W}_0 \mathbf{p}_+ = 0 \quad (5)$$

Note that $\mathbf{W}_0$ whose diagonal elements are real is Hermitian matrix because $\mathbf{W}_0^H = \mathbf{W}_0$. Thus, Eq.(5) can be further simplified as

$$\mathbf{p}_+^T \Re(\mathbf{W}_0) \mathbf{p}_+ = 0 \quad (6)$$

where function $\Re(\cdot)$ takes the real part of its augment.

On the other hand, the latter of the above conditions, $H_0(l) = \sqrt{M}$, could be also converted into the form of

$$\mathbf{p}_+^T \mathbf{W}_0 \mathbf{p}_+ = 1 \quad (7)$$

where $\mathbf{W}_0$ is a symmetric and positive definite matrix.

By combining (6) with (7) together, we can obtain a group of equivalent regularity conditions in $\mathbf{P}_+$ which are as follows

$$\begin{cases} \mathbf{p}_+^T \mathbf{R}_0 \mathbf{p}_+ = 1 \\ \mathbf{p}_+^T \mathbf{R}_{kl} \mathbf{p}_+ = 1 \end{cases} \quad (8)$$

where $\mathbf{R}_0 = \mathbf{W}_0$ and $\mathbf{R}_{kl} = \gamma \Re(\mathbf{W}_{kl}) + \mathbf{W}_0$.

The choice of the scalar $\gamma$ must be done to guarantee that $\mathbf{R}_{kl}$ is a set of absolute diagonally dominant matrices. Therefore, $\mathbf{R}_{kl}$ are positive definite matrices according to classical matrix theory.

It is evident that Eq.(8) includes $L(M-I) + I$ additional constrained conditions which ensure that
designed cosine modulated wavelet has L vanishing moments and then the property of compact support. In summary, by gathering the regularity conditions with the PR conditions in Eq.(3), we are able to design the required cosine modulated PR wavelet with specified vanishing moments.

2.3. United QC Formulation for PR CMFB and Wavelets

From the discussion of two subsections above, we have known that both the PR conditions of cosine-modulated filter banks and regularity conditions imposed on its lowpass analysis filter of prototype filter can be converted into a set of same formation of quadratic-constraints whose constraint matrices are symmetric and positive definite as shown in (3) and (8). Thus, the design problem of both CMFB and its corresponding wavelets can be unified as a unification method that can be expressed as a kind of QCLS minimization problem subject to a set of quadratic-constraints, that is

\[ h_{opt} = \min \mathbf{p}^T \mathbf{Qp} \quad \text{s.t.} \quad \begin{cases} \text{Eq.(3)} \\ \text{Eq.(8)} \end{cases} \]  

(9)

By solving (9) for impulse response of prototype filter and considering Eq.(1), we can obtain PR cosine-modulated wavelets with given vanishing moments. Obviously, this method is able to impose arbitrary order of vanishing moments on the designed cosine-modulated wavelets according to the actual requirements.

3. NEURAL NETWORK FOR DESIGNING FB AND WAVELET

In recent years, due to the pioneering work of Hopfield and Kennedy et. al[1], neural networks implemented by analog VLSI technology have been received extensive studies and applied to a variety of problems where processing time is critical[1,3,7,8,9]. Right here, we would like to use this method to deal with the design problem in hand.

3.1. Energy Function

According to Eq.(9), we can build the cost of the constrained optimization problem and map it as the energy function of an analog neural network

\[ J(X, \beta) = X^TQX + \sum_{i=1}^{L} \beta_i p_i(X) \]  

(10)

where \( \beta_i \) is a positive constant, \( p_i(X) \) is a penalty function.

Note that qualified penalty function \( p_i(X) \) should be satisfied such conditions: (a) it is a continuous and differentiable function of unknown column vector \( X \) which is corresponded to the vector \( \mathbf{P}_{i} \) in (9), (b) \( p_i(X) \geq 0, \forall X \), (c) \( p_i(X) = 0 \iff X' \mathbf{A}_i X = 1 \), where \( \mathbf{A}_i \) is referred to as a constraint matrix. An ordinary choice of \( p_i(X) \) is given by \( p_i(X) = (X' \mathbf{A}_i X - 1)^2 \).

3.2. Neural Network’s Dynamics

According to the energy function in (10), the neural network dynamics should be such that the time derivative of \( J \) is negative. Thus, we can define the motion equation for the \( k \)th neuron as

\[ \frac{d u_k(t)}{dt} = - \frac{\partial J}{\partial v_k(t)} \quad k = 1, \ldots, I. \]  

(11a)

\[ v_k(t) = \text{f}(u_k(t)) \quad k = 1, \ldots, I. \]  

(11b)

where \( u_k(t) \) and \( v_k(t) \) are the input and output of the \( k \)th neuron at instant \( t \), individually, \( f(u) \) is a monotonically increasing activation function, and \( I \) is the number of neurons in the network.

The time derivative of energy function of the neural network can be given by

\[ \frac{dJ}{dt} = \sum_{i=1}^{I} \frac{\partial J}{\partial v_k(t)} \cdot f'(u_k(t)) \cdot \frac{du_k(t)}{dt} \]  

(12)

where \( f'(u) \) is the derivative of \( f(u) \) with respect to its augment \( u \).

It can be easily deduced from Eqs.(11) and (12) that the neural network with dynamics given by (11) has stable stationary points at the local minima of \( J \).

3.3. Performance Analysis

In this subsection, we will analyze the Lyapunov stability, global convergence property and QCLS optimization computation ability of the proposed neural network above. For space saving, Only necessary and important theorems without proofs are given to make our problem be complete here. Interesting readers, please refer to the references[5,8,9].

For convenient expressions of followings, we need to define the feasible solution space of (10), which consists of all the equilibrium points of \( J \), as

\[ \Theta = \left\{ X \mid \mathbf{QX} = \sum_{i=1}^{I} \mathbf{A}_i X, \text{where} V_{i}(X) = 2\beta_i \left(1-X' \mathbf{A}_i X \right) \right\} \]

Theorem 1: Suppose \( V = V(t, v_o) \) is the solution of
Theorem 2: \( \overline{X} \) is a stable global minimizer of \( J \) in (10) if and only if \( \overline{X} \) satisfies \( \|X^i\|_M = \max_{X \in \Theta} \|X^i\|_M \), \( i = 1, \ldots, l \), where \( \|X\|_M \) stands for weighted norm of \( X \) with respect to matrix \( M \).

In summary, according to theorems of above, we can conclude that the neural network described by (11) has only one unique global stable minimizer while the other equilibrium points are unstable minimizers. Thus, the design of paraunitary CMFB and wavelets can be in deed carried out by our neural network method with random initialization. The output of the neural network is just the impulse response coefficients of the prototype filter of the designed filter bank and wavelet when it is settled down. Furthermore, the whole design process may be accomplished in magnitude order of circuit time constant when it is implemented by advanced analog VLSI technology.

4. SIMULATIONS

4.1. Design Example I

In order to verify our analyses and derivations in preceding sections, we design an 8-channel PR CMFB as the first example by using the proposed neural network when \( M = 8 \) and \( m = 3 \). Thus, filter length is \( N = 48 \). When we choose the penalty factor to be 1000 and initialize network randomly, the designing network can start to evolve and settle down in magnitude order of circuit time constant (about hundreds of nanoseconds). The magnitude responses of the designed prototype filter \( P(z) \) and corresponding analysis filter bank \( H_k(z) \) are shown in Figure 1.

For convenient comparison, we also plot an 8-channel PR CMFB designed by conventional two-channel lossless lattice (TC_LAT) parameterized method in Fig.1(a) in dashed line. It can be seen that the stopband attenuation by our NN design method is about 3dB lower than those by the conventional TC_LAT method. The constraint errors in this case are very small and can be considered to be negligible for many practical purposes even though penalty factors are large finite constants.

4.2. Design Example II

For \( M = 8 \) and \( m = 3 \), we design the compactly supported orthonormal cosine-modulated wavelets with two vanishing moments. The optimized scaling function and seven compact support wavelet functions by the proposed neural network method are shown in Figure 2.
5. CONCLUSION

This paper proposed a new unified method for designing both paraunitary cosine-modulated FIR filter banks and cosine-modulated compactly supported wavelets. This problem is equivalent to an QCLS minimization problem in which all constraint matrices are symmetric and positive definite. For the propose of efficiently solving the QCLS optimization problem, we built an analog neural network and analyzed its performance in detail. By using the neural network to solve the corresponding minimization problem, we easily and flexibly obtained cosine modulated filter banks with larger stopband attenuation, and cosine modulated compactly supported wavelets. At last part of this paper, two computer simulations were given to support our theory and method.

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Biography

Ying Tan was born in Yingshan county, Sichuan, China, in 1964. He received his B.S., M.S., and Ph.D. degrees in 1985, 1988, and 1997, respectively. During 1990-1994 he was a research scientist and lecturer. He has published about 60 academic papers and four books. Now he is an associate professor and postdoctoral research fellow in University of Science and Technology of China (USTC), Hefei, P. R. China. His current research interests include neural network theory and its applications, intelligent computational science, multirate signal processing, wavelet transform, image processing, pattern recognition, intelligent systems etc.