Flow Sensitive Analysis

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Most content comes from http://cs.au.dk/~amoeller/spa/

Agenda

- **Constant propagation analysis**
- Live variables analysis
- Available expressions analysis
- Very busy expressions analysis
- Reaching definitions analysis
- Initialized variables analysis

Constant Propagation Optimization

```
var x,y,z;
x = 27;
y = input,
z = 2*x+y;
if (x<0) { y=z-3; } else { y=12 }
output y;
```

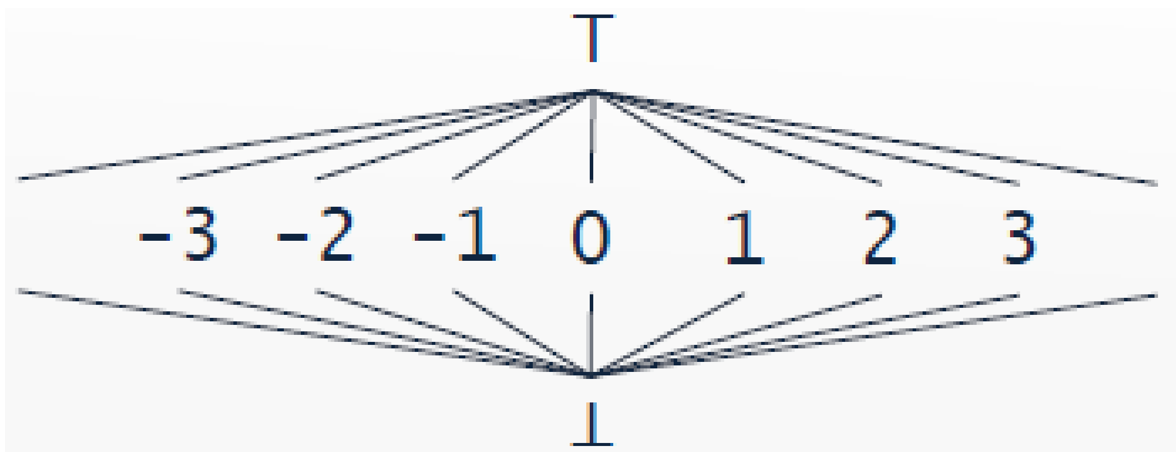
```
var x,y,z;
x = 27;
y = input;
z = 54+y;
if (0) { y=z-3; } else { y=12 }
output y;
```

```
var y;
y = input;
output 12;
```

https://github.com/cs-au-dk/TIP/blob/master/src/tip/analysis/ConstantPropagationAnalysis.scala

Constant Propagation Analysis

- Determine variables with a constant value
- Flat lattice:



Constraints for Constant Propagation

- Essentially as for the Sign analysis…

- Abstract operator for addition:

$$\overline{+}(n,m) = \begin{cases} \bot & \text{if } n=\bot \lor m=\bot \\ \top & \text{else if } n=\top \lor m=\top \\ n+m & \text{otherwise} \end{cases}$$

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- **Live variables analysis**
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Live Variables Analysis

- A variable is *live* at a program point its value may be read later in the remaining execution

- Undecidable, but the property can be conservatively approximated

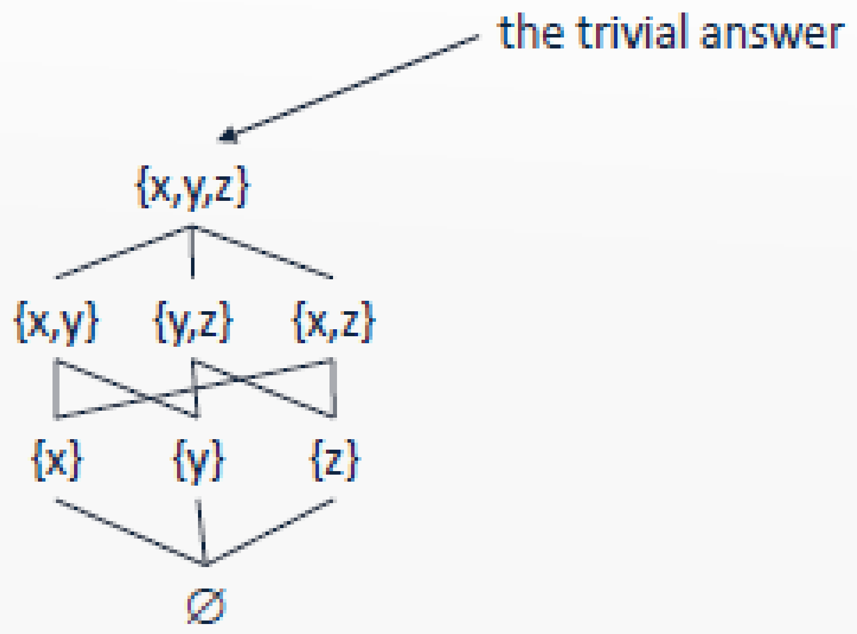
- The analysis must only reply *dead* if the variable is really dead
  - No need to store the values of dead variables

A Lattice for Liveness

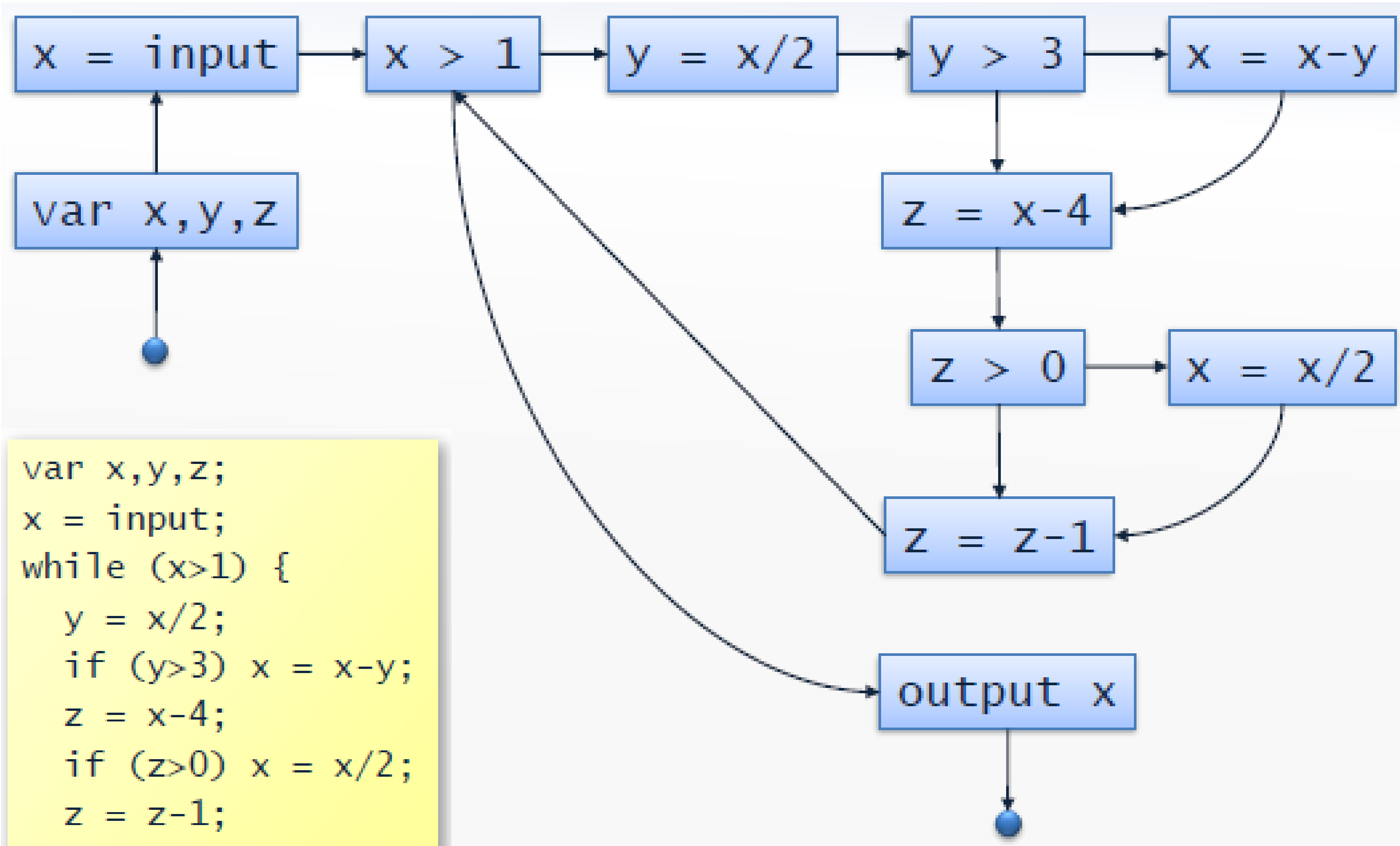
- A powerset lattice of program variables

```
var x,y,z;
x = input;
while (x>1) {
  y = x/2;
  if (y>3) x = x-y;
  z = x-4;
  if (z>0) x = x/2;
  z = z-1;
}
output x;
```

$$L = (2^{\{x,y,z\}}, \subseteq)$$



The Control Flow Graph




```
var x,y,z;
x = input;
while (x>1) {
  y = x/2;
  if (y>3) x = x-y;
  z = x-4;
  if (z>0) x = x/2;
  z = z-1;
}
output x;
```

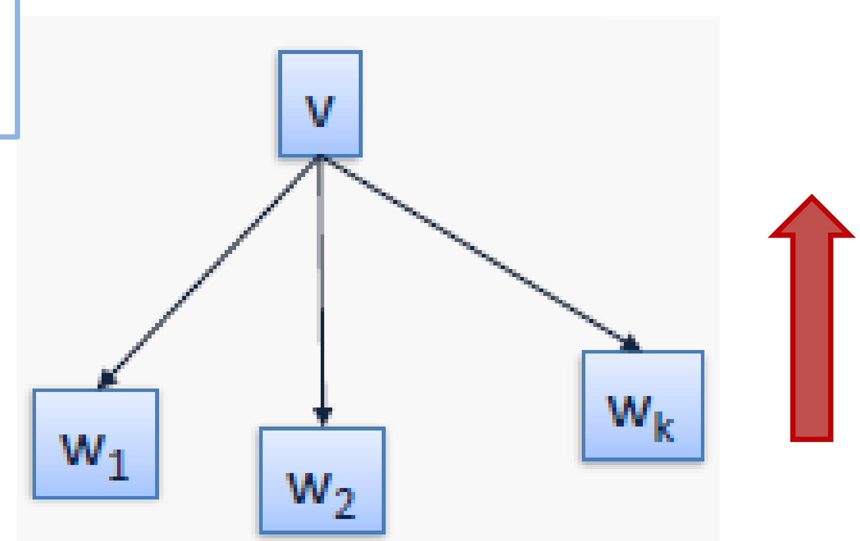
Setting Up

- For every CFG node v we have a variable ⟦v⟧
  - the subset of program variables that are live at the program point *before* v

- Since the analysis is conservative, the computed set may be *too large*

分析出的是**可能的**活跃变量集合

- Auxiliary definition
  - $\mathrm{JOIN}(v) = \bigcup_{w \in succ(v)} [\![w]\!]$



Liveness Constraints

- For the exit node

  $[\![exit]\!] = \emptyset$

- For conditions and output

  $[\![\text{if } (E)]\!] = [\![\text{while } E]\!] = [\![\text{output } E]\!] = JOIN(v) \cup vars(E)$

- For assignments

  $[\![x = E]\!] = JOIN(v) \backslash \{x\} \cup vars(E)$

- For variable declarations

  $[\![\text{var } x_1, \cdots, x_n]\!] = JOIN(v) \backslash \{x_1, \cdots, x_n\}$

- For all other nodes

  $[\![v]\!] = JOIN(v)$

vars(*E*) = variables occurring in *E*

right-hand sides are monotone since *JOIN* is monotone, and ...

Generated Constraints

⟦var x,y,z⟧ = ⟦z=input⟧ \ {x,y,z}
⟦x=input⟧ = ⟦x>1⟧ \ {x}
⟦x>1⟧ = (⟦y=x/2⟧ ∪ ⟦output x⟧) ∪ {x}
⟦y=x/2⟧ = (⟦y>3⟧ \ {y}) ∪ {x}
⟦y>3⟧ = ⟦x=x-y⟧ ∪ ⟦z=x-4⟧ ∪ {y}
⟦x=x-y⟧ = (⟦z=x-4⟧ \ {x}) ∪ {x,y}
⟦z=x-4⟧ = (⟦z>0⟧ \ {z}) ∪ {x}
⟦z>0⟧ = ⟦x=x/2⟧ ∪ ⟦z=z-1⟧ ∪ {z}
⟦x=x/2⟧ = (⟦z=z-1⟧ \ {x}) ∪ {x}
⟦z=z-1⟧ = (⟦x>1⟧ \ {z}) ∪ {z}
⟦output x⟧ = ⟦*exit*⟧ ∪ {x}
⟦*exit*⟧ = ∅

$\llbracket exit \rrbracket = \emptyset$

$\llbracket \text{if } (E) \rrbracket = \llbracket \text{while } E \rrbracket = \llbracket \text{output } E \rrbracket$
$= JOIN(v) \cup vars(E)$

$\llbracket x = E \rrbracket = JOIN(v) \backslash \{x\} \cup vars(E)$

$\llbracket \text{var } x_1, \cdots, x_n \rrbracket = JOIN(v) \backslash \{x_1, \cdots, x_n\}$

$\llbracket v \rrbracket = JOIN(v)$

```
var x,y,z;
x = input;
while (x>1) {
  y = x/2;
  if (y>3) x = x-y;
  z = x-4;
  if (z>0) x = x/2;
  z = z-1;
}
output x;
```

Least Solution

$[\![\texttt{var x,y,z}]\!] = [\![\texttt{z=input}]\!] \setminus \{x,y,z\}$

$[\![\texttt{x=input}]\!] = [\![\texttt{x>1}]\!] \setminus \{x\}$

$[\![\texttt{x>1}]\!] = ([\![\texttt{y=x/2}]\!] \cup [\![\texttt{output x}]\!]) \cup \{x\}$

$[\![\texttt{y=x/2}]\!] = ([\![\texttt{y>3}]\!] \setminus \{y\}) \cup \{x\}$

$[\![\texttt{y>3}]\!] = [\![\texttt{x=x-y}]\!] \cup [\![\texttt{z=x-4}]\!] \cup \{y\}$

$[\![\texttt{x=x-y}]\!] = ([\![\texttt{z=x-4}]\!] \setminus \{x\}) \cup \{x,y\}$

$[\![\texttt{z=x-4}]\!] = ([\![\texttt{z>0}]\!] \setminus \{z\}) \cup \{x\}$

$[\![\texttt{z>0}]\!] = [\![\texttt{x=x/2}]\!] \cup [\![\texttt{z=z-1}]\!] \cup \{z\}$

$[\![\texttt{x=x/2}]\!] = ([\![\texttt{z=z-1}]\!] \setminus \{x\}) \cup \{x\}$

$[\![\texttt{z=z-1}]\!] = ([\![\texttt{x>1}]\!] \setminus \{z\}) \cup \{z\}$

$[\![\texttt{output x}]\!] = [\![\mathit{exit}]\!] \cup \{x\}$

$[\![\mathit{exit}]\!] = \varnothing$

$[\![\mathit{entry}]\!] = \varnothing$

$[\![\texttt{var x,y,z}]\!] = \varnothing$

$[\![\texttt{x=input}]\!] = \varnothing$

$[\![\texttt{x>1}]\!] = \{x\}$

$[\![\texttt{y=x/2}]\!] = \{x\}$

$[\![\texttt{y>3}]\!] = \{x,y\}$

$[\![\texttt{x=x-y}]\!] = \{x,y\}$

$[\![\texttt{z=x-4}]\!] = \{x\}$

$[\![\texttt{z>0}]\!] = \{x,z\}$

$[\![\texttt{x=x/2}]\!] = \{x,z\}$

$[\![\texttt{z=z-1}]\!] = \{x,z\}$

$[\![\texttt{output x}]\!] = \{x\}$

$[\![\mathit{exit}]\!] = \varnothing$

Many non-trivial answers!

Optimizations

- Variables y and z are never live at the same time
  ➔they can share the same variable location
- The value assigned in z=z-1 is never read
  ➔the assignment can be skipped

```
var x,y,z;
x = input;
while (x>1) {
   y = x/2;
   if (y>3) x = x-y;
   z = x-4;
   if (z>0) x = x/2;
   z = z-1;
}
output x;
```

```
var x,yz;
x = input;
while (x>1) {
   yz = x/2;
   if (yz>3) x = x-yz;
   yz = x-4;
   if (yz>0) x = x/2;
}
output x;
```

- better register allocation
- a few clock cycles saved

Time Complexity (for the naive algorithm)

- With n CFG nodes and k variables:
  - the lattice L^n has height $k \cdot n$
  - so there are at most $k \cdot n$ iterations
- Subsets of Vars(the variables in the program) can be represented as bitvectors:
  - each element has size k
  - each ∪, \, = operation takes time $O(k)$
- Each iteration uses $O(n)$ bitvector operations:
  - so each iteration takes time $O(k \cdot n)$
- Total time complexity: $O(k^2n^2)$

- Exercise: what is the complexity for the worklist algorithm?

L^n是CFG中n个node要计算的程序点状态的取值的范围

一次迭代的状态转移函数f: $L^n \rightarrow L^n$

Agenda

- Constant propagation analysis
- Live variables analysis
- **Available expressions analysis**
- Very busy expressions analysis
- Reaching definitions analysis
- Initialized variables analysis

Available Expressions Analysis

- A (non-trivial) expression is *available* at a program point if its current value has already been computed earlier in the execution

- The approximation generally includes *too few* expressions
  - The analysis can only report *"available"* if the expression is definitely available
  - No need to re-compute available expressions (e.g. common subexpression elimination)

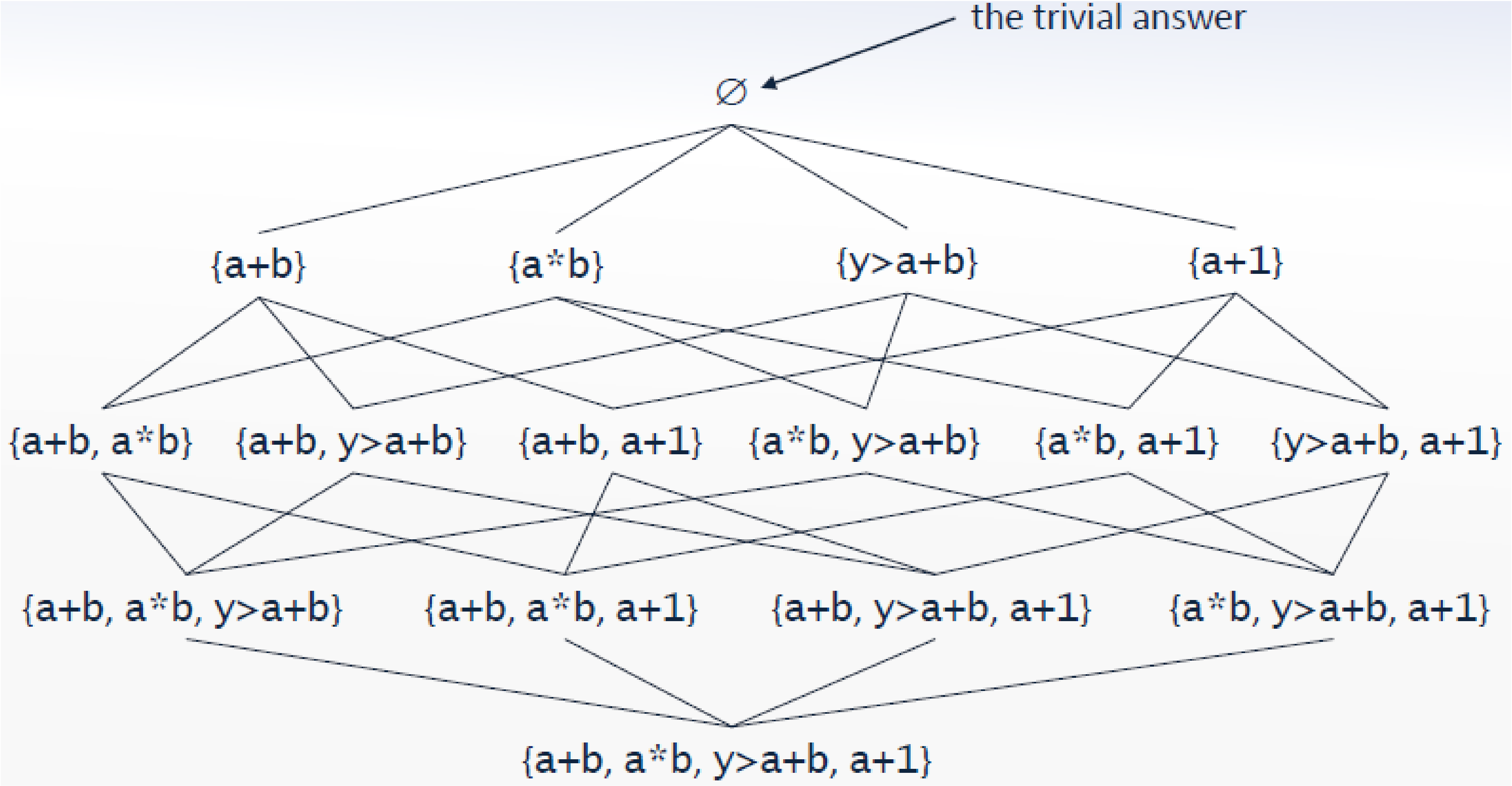
A Lattice for Available Expressions

A reverse powerset lattice of nontrivial expressions

```
var x,y,z,a,b;
z = a+b;
y = a*b;
while (y > a+b) {
  a = a+1;
  x = a+b;
}
```

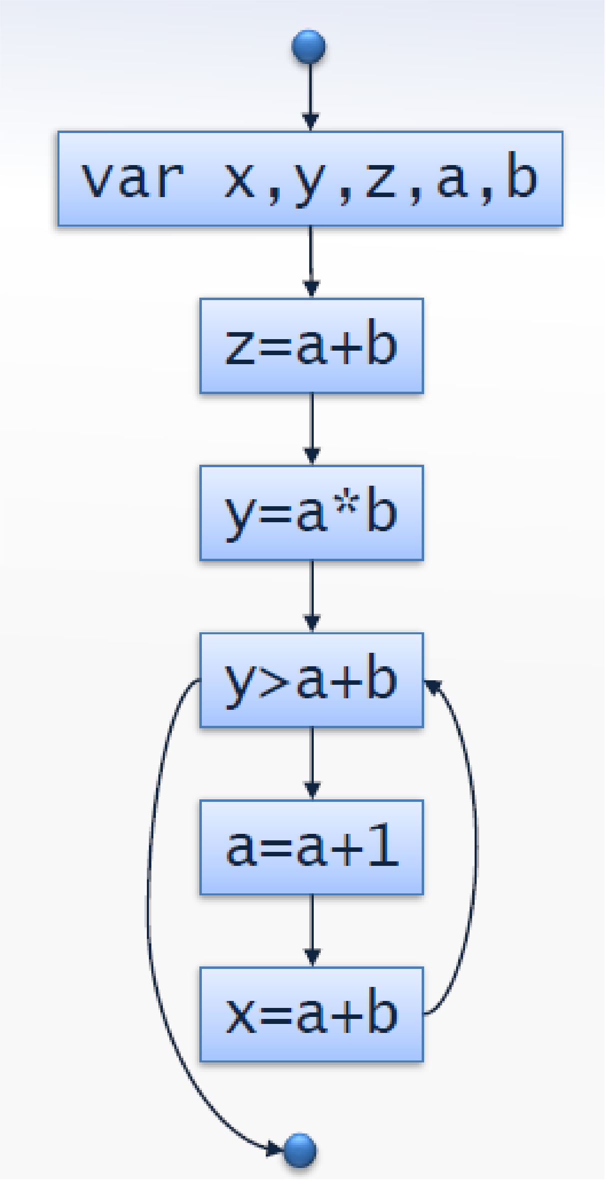
$$L = (2^{\{a+b,\ a*b,\ y>a+b,\ a+1\}}, \supseteq)$$

Reverse Powerset Lattice



Flow Graph

```
var x,y,z,a,b;
z = a+b;
y = a*b;
while (y > a+b) {
  a = a+1;
  x = a+b;
}
```



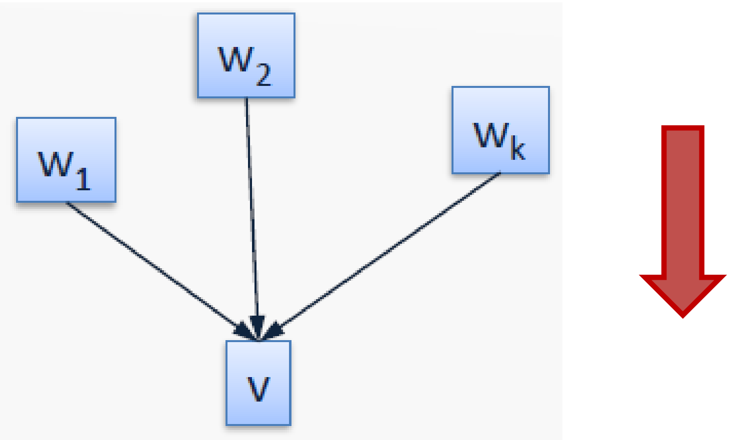
Setting Up

- For every CFG node v we have a variable $[\![v]\!]$
  - the subset of expressions that are available at the program point *after* v

- Since the analysis is conservative, the computed set may be *too small*

- Auxiliary definition

$$\mathrm{JOIN}(v) = \bigcap_{w \in pred(v)} [\![w]\!]$$



Auxiliary Functions

- The function $X{\downarrow}x$ removes all expressions from X that contain a reference to the variable *x*

- The function *exps*(*E*) is defined as:
  - $exps(intconst) = \emptyset$
  - $exps(x) = \emptyset$
  - $exps(\text{input}) = \emptyset$
  - $exps(E_1 op\ E_2) = \{E_1\ op\ E_2\} \cup exps(E_1) \cup exps(E_2)$
    but don't include expressions containing input

Availablity Constraints

- For the entry node

  $[\![entry]\!] = \emptyset$

- For conditions and output

  $[\![\text{if } (E)]\!] = [\![\text{while } E]\!] = [\![\text{output } E]\!] = JOIN(v) \cup exps(E)$

- For assignments

  $[\![x = E]\!] = (JOIN(v) \cup exps(E)) \downarrow x$

- For all other nodes

  $[\![v]\!] = JOIN(v)$

Generated Constraints

$[\![entry]\!] = \emptyset$

$[\![if\ (E)]\!] = [\![while\ E]\!] = [\![output\ E]\!]$

$= JOIN(v) \cup exps(E)$

$[\![x = E]\!] = (JOIN(v) \cup exps(E)) \downarrow x$

$[\![v]\!] = JOIN(v)$

$[\![entry]\!] = \emptyset$

$[\![\texttt{var x,y,z,a,b}]\!] = [\![entry]\!]$

$[\![\texttt{z=a+b}]\!] = exps(\texttt{a+b})\downarrow\texttt{z}$

$[\![\texttt{y=a*b}]\!] = ([\![\texttt{z=a+b}]\!] \cup exps(\texttt{a*b}))\downarrow\texttt{y}$

$[\![\texttt{y>a+b}]\!] = ([\![\texttt{y=a*b}]\!] \cap [\![\texttt{x=a+b}]\!]) \cup exps(\texttt{y>a+b})$

$[\![\texttt{a=a+1}]\!] = ([\![\texttt{y>a+b}]\!] \cup exps(\texttt{a+1}))\downarrow\texttt{a}$

$[\![\texttt{x=a+b}]\!] = ([\![\texttt{a=a+1}]\!] \cup exps(\texttt{a+b}))\downarrow\texttt{x}$

$[\![exit]\!] = [\![\texttt{y>a+b}]\!]$

```
var x,y,z,a,b;
z = a+b;
y = a*b;
while (y > a+b) {
   a = a+1;
   x = a+b;
}
```

Least Solution

⟦*entry*⟧ = ∅

⟦var x,y,z,a,b⟧ = ⟦*e*

⟦z=a+b⟧ = *exps*(a+b)↓z

⟦y=a*b⟧ = (⟦z=a+b⟧ ∪ *e*

⟦y>a+b⟧ = (⟦y=a*b⟧ ∩ ⟦

⟦a=a+1⟧ = (⟦y>a+b⟧ ∪ *e*

⟦x=a+b⟧ = (⟦a=a+1⟧ ∪ *e*

⟦*exit*⟧ = ⟦y>a+b⟧

⟦*entry*⟧ = ∅

⟦var x,y,z,a,b⟧ = ∅

⟦z=a+b⟧ = {a+b}

⟦y=a*b⟧ = {a+b, a*b}

⟦y>a+b⟧ = {a+b, y>a+b}

⟦a=a+1⟧ = ∅

⟦x=a+b⟧ = {a+b}

⟦*exit*⟧ = {a+b}

Many non-trivial answers!

Optimizations

- We notice that a+b is available before the loop
- The program can be optimized (slightly):

```
var x,y,z,a,b;
z = a+b;
y = a*b;
while (y > a+b) {
  a = a+1;
  x = a+b;
}
```

```
var x,y,x,a,b,aplusb;
aplusb = a+b;
z = aplusb;
y = a*b;
while (y > aplusb) {
  a = a+1;
  aplusb = a+b;
  x = aplusb;
}
```

引入临时变量记录表达式的值，便于在表达式所引用的变量修改后重新计算并记录新值，也便于后面实施复写传播，发现更多优化机会

Agenda

- Constant propagation analysis
- Live variables analysis
- Available expressions analysis
- **Very busy expressions analysis**
- Reaching definitions analysis
- Initialized variables analysis

Very Busy Expressions Analysis

- A (nontrivial) expression is *very busy* if it will definitely be evaluated before its value changes

一个表达式在程序点非常忙当它无论沿

哪条路径从那个点到终止点都会被计算

- The approximation generally includes *too few* expressions
  - the answer "*verybusy*" must be the true one
  - Very busy expressions may be pre-computed (e.g. loop hoisting)

- Same lattice as for available expressions

An Example Program

```
var x,a,b;
x = input;
a = x-1;
b = x-2;
while (x > 0) {
  output a*b-x;
  x = x-1;
}
output a*b;
```

The analysis shows that a*b is very busy

Code Hoisting

```
var x,a,b;
x = input;
a = x-1;
b = x-2;
while (x > 0) {
  output a*b-x;
  x = x-1;
}
output a*b;
```

```
var x,a,b,atimesb;
x = input;
a = x-1;
b = x-2;
atimesb = a*b;
while (x > 0) {
  output atimesb-x;
  x = x-1;
}
output atimesb;
```

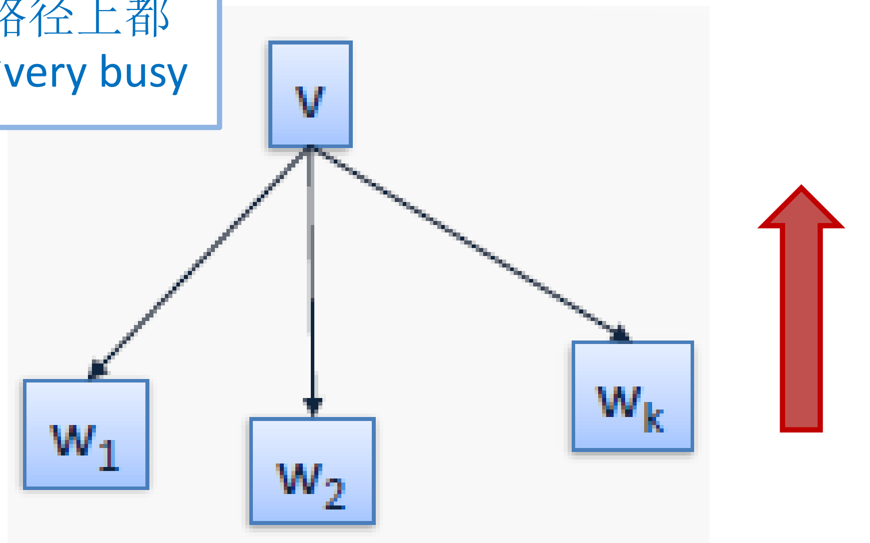
Setting Up

- For every CFG node v we have a variable $[\![v]\!]$
  - the subset of expressions that are very busy at the program point *before* v

- Since the analysis is conservative, the computed set may be *too small*

必须其后的每条路径上都very busy才能称为very busy

- Auxiliary definition
  - $\text{JOIN}(v) = \bigcap_{w \in succ(v)} [\![w]\!]$



Very Busy Constraints

- For the exit node

$[\![exit]\!] = \emptyset$

- For conditions and output

$[\![\text{if } (E)]\!] = [\![\text{while } E]\!] = [\![\text{output } E]\!] = JOIN(v) \cup exps(E)$

- For assignments

$[\![x = E]\!] = JOIN(v) \downarrow x \cup exps(E)$

- For all other nodes

$[\![v]\!] = JOIN(v)$

Agenda

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Reaching Definitions Analysis

- The *reaching definitions* for a program point are those assignments that may define the current values of variables

- The conservative approximation may include *too many* possible assignments

A Lattice for Reaching Definitions

The powerset lattice of assignments

$$L = (2^{\{x=input,\, y=x/2,\, x=x-y,\, z=x-4,\, x=x/2,\, z=z-1\}}, \subseteq)$$

```
var x,y,z;
x = input;
while (x > 1) {
  y = x/2;
  if (y>3) x = x-y;
  z = x-4;
  if (z>0) x = x/2;
  z = z-1;
}
output x;
```

Reaching Definitions Constraints

- The function $X{\downarrow}x$ removes assignments to x from X

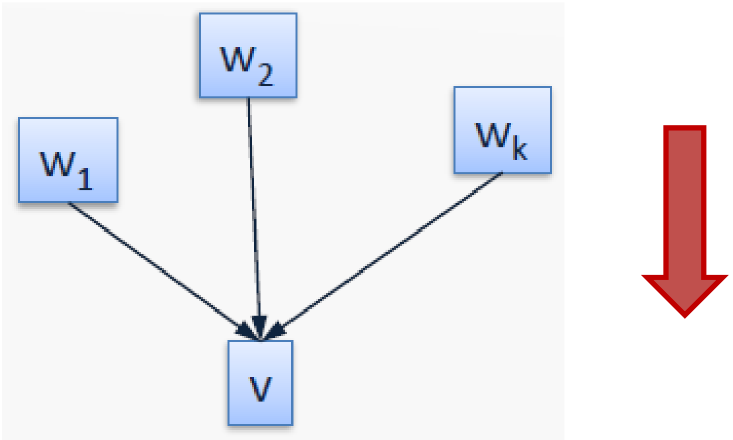
- For assignments

  $[\![x = E]\!] = JOIN(v) \downarrow x \cup \{x = E\}$

- For all other nodes

  $[\![v]\!] = JOIN(v)$

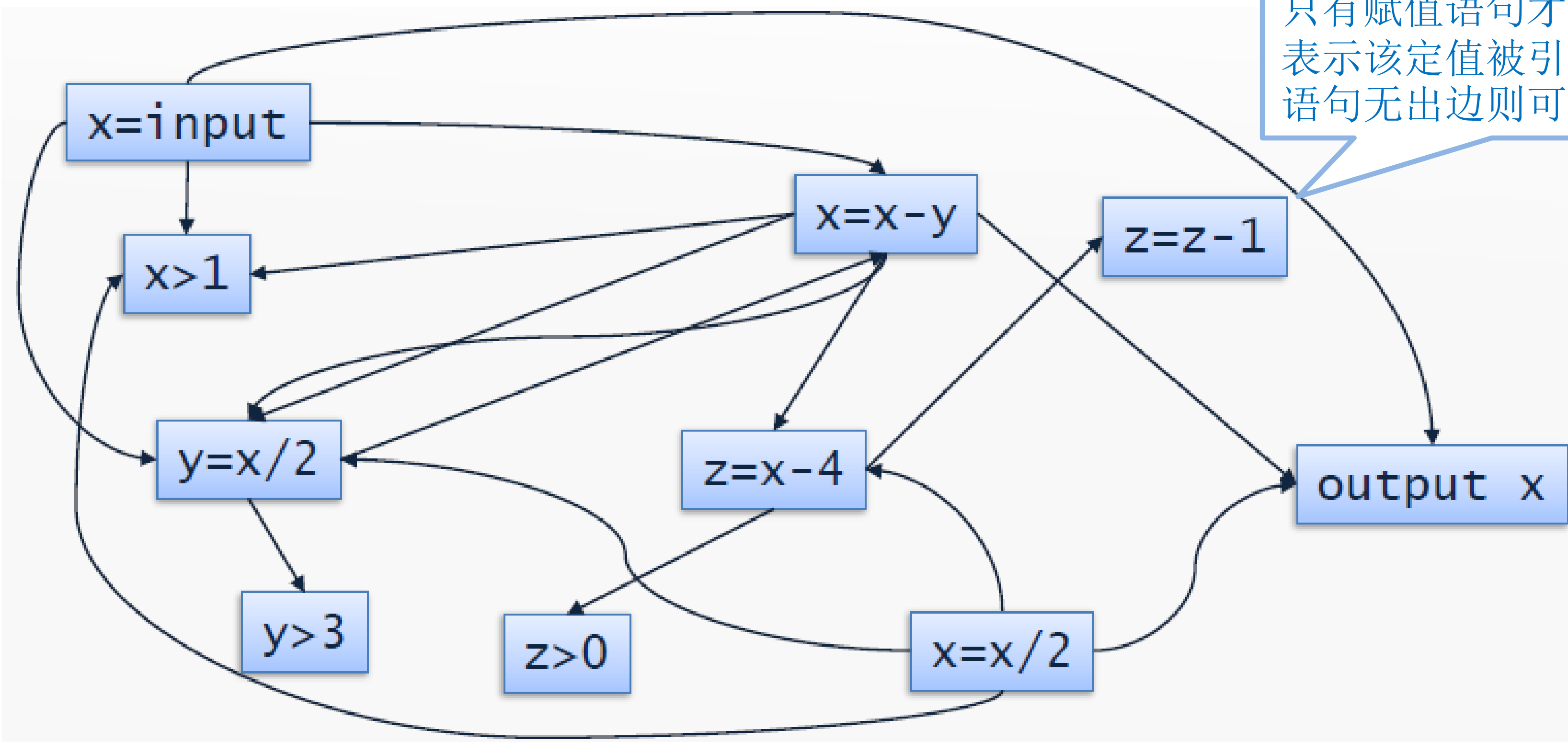
- Auxiliary definition
  - $\text{JOIN}(v) = \cup_{w \in pred(v)} [\![w]\!]$



Def-use Graph

- Reaching definitions define the def-use graph:
  - like a CFG but with edges from *def* to *use* nodes
  - basis for *dead code elimination* and *code motion*

只有赋值语句才可能有出边，表示该定值被引用；若定值语句句无出边则可删除



Forward vs. Backward

- A *forward* analysis:
  - computes information about the *past* behavior
  - examples: available expressions, reaching definitions

- A *backward* analysis:
  - computes information about the *future* behavior
  - examples: liveness, very busy expressions

May vs. Must

- A *may* analysis:
  - describes information that is *possibly* true
  - an *over*-approximation
  - examples: liveness, reaching definitions

- A *must* analysis:
  - describes information that is *definitely* true
  - an *under*-approximation
  - examples: available expressions, very busy expressions

Classifying Analyses

	forward	backward
may	example: reaching definitions $[\![v]\!]$ describes state after v $\mathrm{JOIN}(v) = \bigsqcup_{w \in pred(v)} [\![w]\!] = \bigcup_{w \in pred(v)} [\![w]\!]$	example: liveness $[\![v]\!]$ describes state before v $\mathrm{JOIN}(v) = \bigsqcup_{w \in succ(v)} [\![w]\!] = \bigcup_{w \in succ(v)} [\![w]\!]$
must	example: available expressions $[\![v]\!]$ describes state after v $\mathrm{JOIN}(v) = \bigsqcup_{w \in pred(v)} [\![w]\!] = \bigcap_{w \in pred(v)} [\![w]\!]$	example: very busy expressions $[\![v]\!]$ describes state before v $\mathrm{JOIN}(v) = \bigsqcup_{w \in succ(v)} [\![w]\!] = \bigcap_{w \in succ(v)} [\![w]\!]$

Agenda

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- Reaching definitions analysis
- **Initialized variables analysis**

Initialized Variables Analysis

- Compute for each program point those variables that have *definitely* been initialized in the *past*
- (Called *definite assignment* analysis in Java and C#)
- ➔*forward must analysis*
- Reverse powerset lattice of all variables

$$\mathrm{JOIN}(v) = \bigcap_{w \in pred(v)} [\![w]\!]$$

- For assignments: $[\![x = E]\!] = JOIN(v) \cup \{x\}$
- For all others: $[\![v]\!] = JOIN(v)$