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Control Flow Analysis

Most content comes from http://cs.au.dk/~amoeller/spa/

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Agenda

- **Control flow analysis for TIP with first-class functions**
- Control flow analysis for the λ-calculus
- The cubic framework
- Control flow analysis for object-oriented languages

TIP with first-class functions

```
inc(i) { return i+1; }
dec(j) { return j-1; }
ide(k) { return k; }

foo(n,f) {
  var r;
  if (n==0) { f=ide; }
  r = f(n);
  return r;
}

main() {
  var x,y;
  x = input;
  if (x>0) { y = foo(x,inc); } else { y = foo(x,dec); }
  return y;
}
```

Control Flow Complications

- **First-class functions in TIP complicate CFG construction**
  - Several functions may be invoked at a call site
  - This depends on the dataflow
  - But dataflow analysis first requires a CFG

- **Same situation for other features, e.g.**
  - Function values with free variables (closures)
  - A class hierarchy with objects and methods
  - Prototype objects with dynamic properties

Control Flow Analysis

- **A control flow analysis approximates the call graph**
  - Conservatively computes possible functions at call sites
  - The trivial answer: *all* functions

- **Control flow analysis is usually flow-*insensitive*:**
  - It is based on the AST
  - The call graph can be used for an interprocedural CFG
  - A subsequent dataflow analysis may use the CFG

- **Alternative: use flow-sensitive analysis**
  - Potentially on-the-fly, during dataflow analysis

CFA for TIP with first-class functions

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- **For a computed function call**

  $E(E_1, \ldots, E_n)$

  we cannot immediately see which function is called

- **A coarse but sound approximation**
  - Assume any function with right number of arguments

- **Use CFA to get a much better result**

CFA Constraints

- Tokens are all functions $\{f_1, f_2, \ldots, f_k\}$
- For every AST node, **v**, we introduce the variable $[\![v]\!]$ denoting the set of functions to which **v** may evaluate
- For function definitions $f\,(\ldots)\{\ldots\}$:

$$f \in [\![f]\!]$$

- For assignments $x = E$:

$$[\![E]\!] \subseteq [\![x]\!]$$

CFA Constraints

- **For direct function calls $f\,(E_1, \ldots, E_n)$:**

  $[\![E_i]\!] \subseteq [\![a_i]\!]$ for $i = 1, \ldots, n \wedge [\![E']\!] \subseteq [\![f\,(E_1, \ldots, E_n)]\!]$

  where f is a function with arguments $a_1, \ldots, a_n$ and return expression E'

- For **computed** function calls $E(E_1, \ldots, E_n)$:

  $f \in [\![E]\!] \Rightarrow ([\![E_i]\!] \subseteq [\![a_i]\!]$ for $i = 1, \ldots, n \wedge [\![E']\!] \subseteq [\![E\,(E_1, \ldots, E_n)]\!])$

  for every function f with arguments $a_1, \ldots, a_n$ and return expression E'

  - If consider **typable** programs only：

    Only generate constraints for those functions f for which the call would be type correct

Generated Constraints

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$inc \in [inc]$
$dec \in [dec]$
$ide \in [ide]$
$[ide] \subseteq [f]$
$[f(n)] \subseteq [r]$
$inc \in [f] \Rightarrow [n] \subseteq [i] \wedge [i+1] \subseteq [f(n)]$
$dec \in [f] \Rightarrow [n] \subseteq [j] \wedge [j-1] \subseteq [f(n)]$
$ide \in [f] \Rightarrow [n] \subseteq [k] \wedge [k] \subseteq [f(n)]$
$[input] \subseteq [x]$
$[foo(x,inc)] \subseteq [y]$
$[foo(x,dec)] \subseteq [y]$
$foo \in [foo]$
$foo \in [foo] \Rightarrow [x] \subseteq [n] \wedge [inc] \subseteq [f] \wedge [r] \subseteq [foo(x,inc)]$
$foo \in [foo] \Rightarrow [x] \subseteq [n] \wedge [dec] \subseteq [f] \wedge [r] \subseteq [foo(x,dec)]$
$main \in [main]$

assuming we do not use the special rule for direct calls

```
inc(i) { return i+1; }
dec(j) { return j-1; }
ide(k) { return k; }

foo(n,f) {
  var r;
  if (n==0) { f=ide; }
  r = f(n);
  return r;
}

main() {
  var x,y;
  x = input;
  if (x>0) { y = foo(x,inc); } else { y = foo(x,dec); }
  return y;
}
```

(At each call we only consider functions with matching number of parameters)

Least Solution

$$[\![\mathtt{inc}]\!] = \{\mathtt{inc}\}$$

$$[\![\mathtt{dec}]\!] = \{\mathtt{dec}\}$$

$$[\![\mathtt{ide}]\!] = \{\mathtt{ide}\}$$

$$[\![\mathtt{f}]\!] = \{\mathtt{inc}, \mathtt{dec}, \mathtt{ide}\}$$

$$[\![\mathtt{foo}]\!] = \{\mathtt{foo}\}$$

$$[\![\mathtt{main}]\!] = \{\mathtt{main}\}$$

(the solution is the empty set for the remaining constraint variables)

With this information, we can construct the call edges and return edges in the interprocedural CFG

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- **Control flow analysis for the λ-calculus**
- The cubic framework
- Control flow analysis for object-oriented languages

CFA for the Lambda Calculus

- The pure lambda calculus

$$
\begin{aligned}
E \;\rightarrow\; & \lambda x.E && \text{(function definition)} \\
\mid\; & E_1\, E_2 && \text{(function application)} \\
\mid\; & x && \text{(variable reference)}
\end{aligned}
$$

- Assume all λ-bound variables are distinct
- An ***abstract closure*** λx abstracts the function $\lambda x.E$ in all contexts (values of free variables)
- Goal: for each call site E_1E_2 determine the possible functions for E_1 from the set $\{\lambda x_1, \lambda x_2, \ldots, \lambda x_n\}$

Closure Analysis

A flow-insensitive analysis that tracks function values:

- For every AST node, v, we introduce a variable ⟦v⟧ ranging over **subsets of abstract closures**
- For $\lambda x.E$ we have the constraint

$$\lambda x \in [\![\lambda x.E]\!]$$

- For E_1E_2 we have the *conditional* constraint

$$\lambda x \in [\![E_1]\!] \Rightarrow ([\![E_2]\!] \subseteq [\![x]\!] \wedge [\![E]\!] \subseteq [\![E_1E_2]\!])$$

for every function $\lambda x.E$

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The Cubic Framework

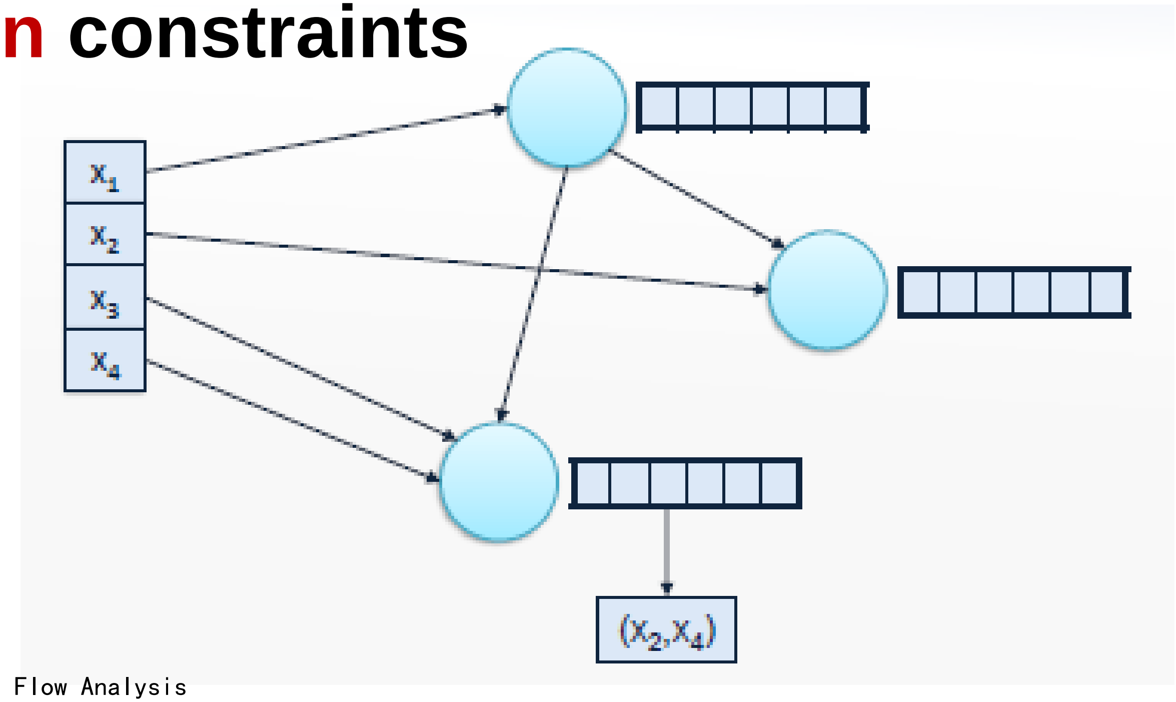
- We have a set of **tokens** $\{t_1, t_2, ..., t_k\}$
- We have a collection of **variables** $\{x_1, ..., x_n\}$ whose values range over subsets of tokens
- A collection of constraints of these forms:
  - $t \in x$
  - $t \in x \Rightarrow y \subseteq z$
- Compute the unique minimal solution
  - This exists since solutions are closed under intersection
- A **cubic time** algorithm exists!

The Solver Data Structure

- Each variable is mapped to a node in a DAG
- Each node has a bitvector in $\{0,1\}^k$
  - initially set to all 0’s
- Each **bit** has a list of pairs of variables
  - used to model conditional constraints

- $t \in x$
- $t \in x \Rightarrow y \subseteq z$

- The DAG **edges** model **inclusion** constraints

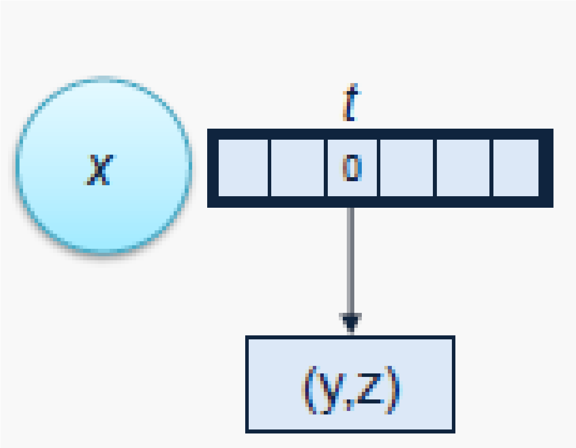


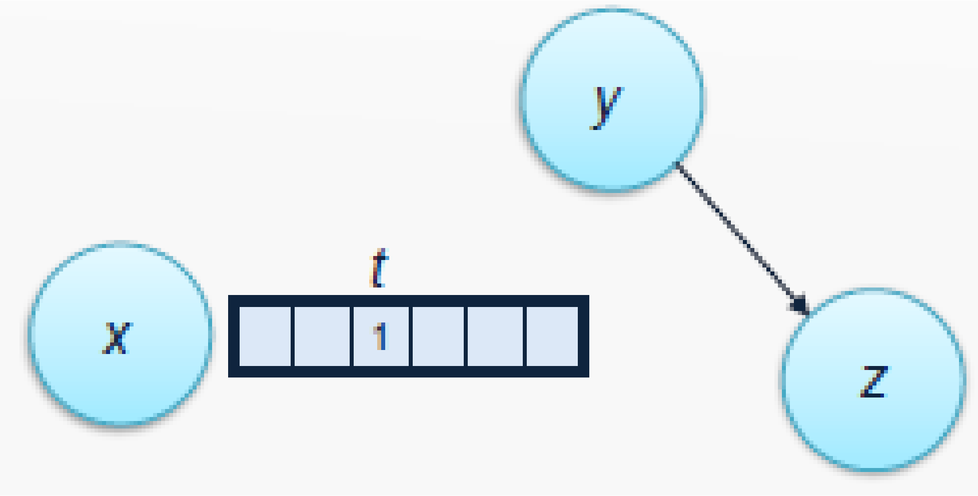
- The bitvectors will at all times directly represent the minimal solution to the constraints seen so far

Adding Constraints (1/2)

- **Constraints of the form $t \in x$:**
  - look up the node associated with *x*
  - set the bit corresponding to *t* to 1
  - if the list of pairs for t is not empty, then add the edges corresponding to the pairs to the DAG

- $t \in x$
- $t \in x \Rightarrow y \subseteq z$



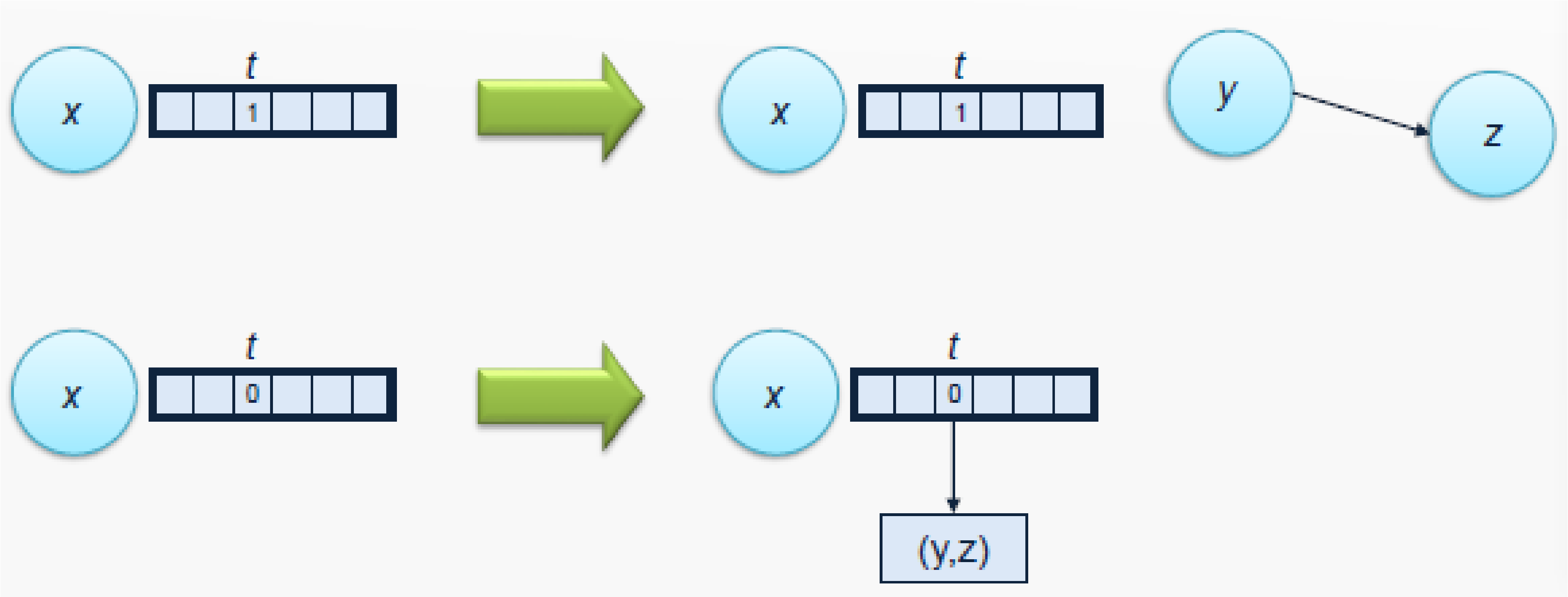


Adding Constraints (2/2)

Constraints of the form $t \in x \Rightarrow y \subseteq z$:

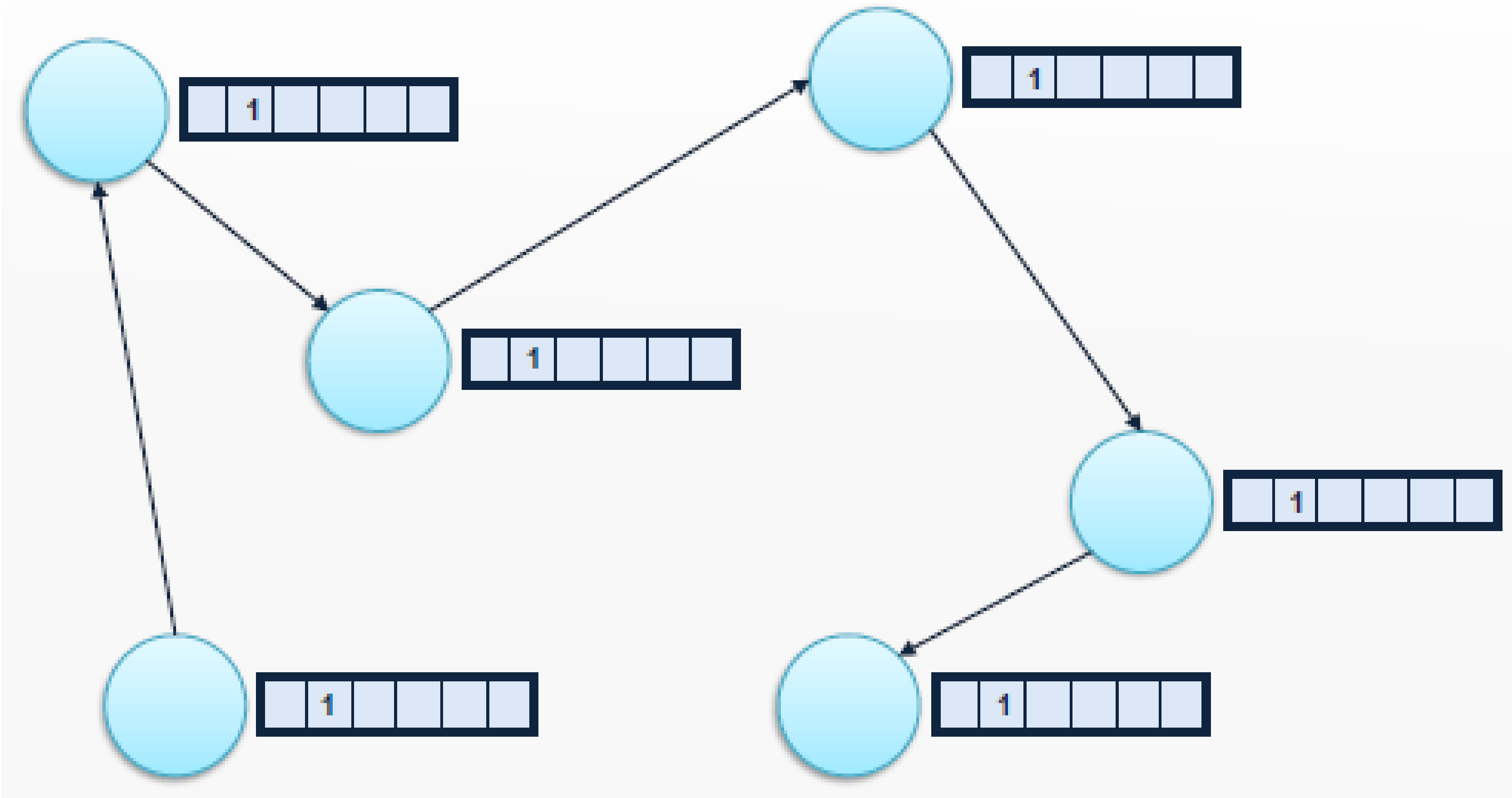
- test if the bit corresponding to t is 1
- if so, add the DAG edge from y to z
- otherwise, add(y,z) to the list of pairs for t

- $t \in x$
- $t \in x \Rightarrow y \subseteq z$



Propagate Bitvectors

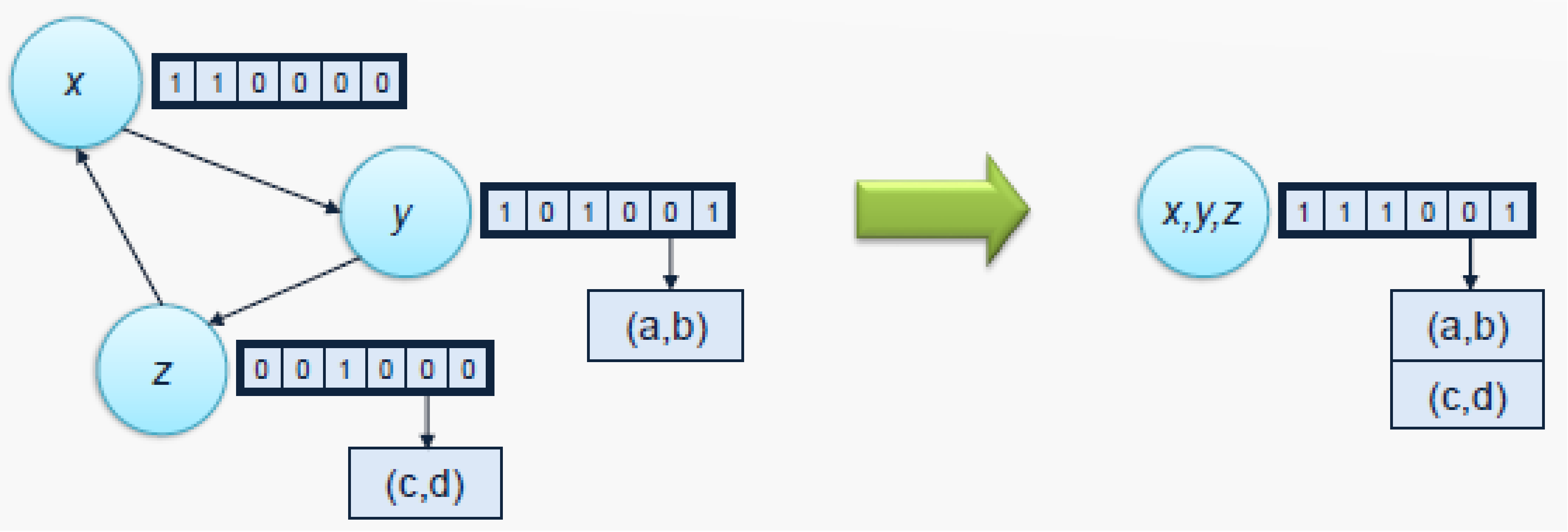
- Propagate the values of all newly set bits along all edges in the DAG



Collapse Cycles

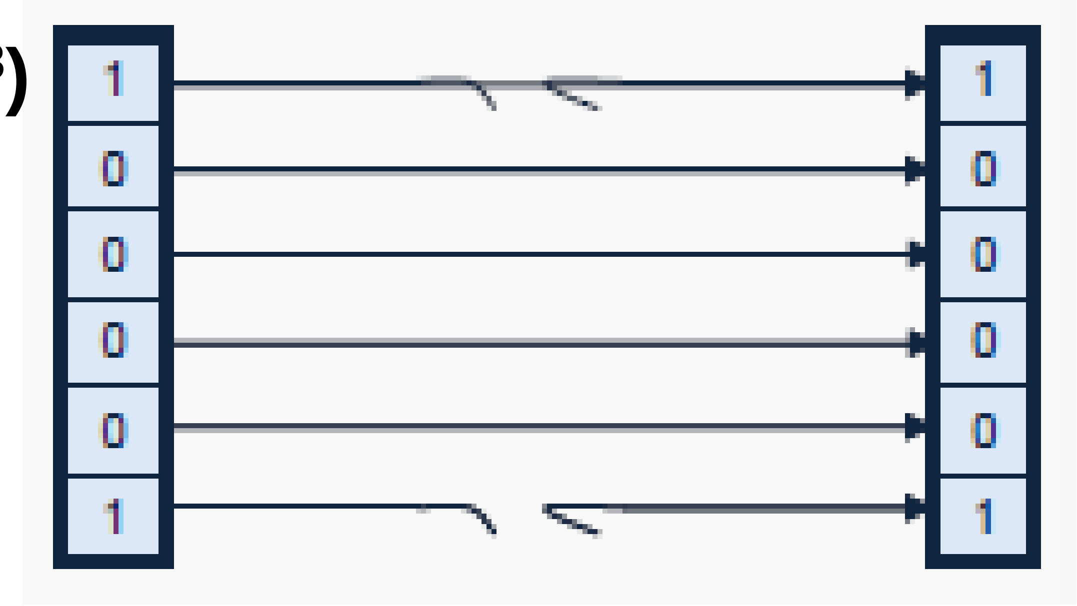
If a newly added edge forms a cycle:

- merge the nodes on the cycle into a single node
- form the union of the bitvectors
- concatenate the lists of pairs
- update the map from variables accordingly



Time Complexity(1/2)

- O(n) functions and O(n) applications, with program size n
- O(n) singleton constraints, O(n^2) conditional constraints
- O(n) nodes, O(n^2) edges, O(n) bits per node
- Total time for bitvector propagation: O(n^3)
- Total time for collapsing cycles: O(n^3)
- Total time for handling lists of pairs: O(n^3)



Time Complexity(1/2)

- **Adding it all up, the upper bound is O(n^3)**

- **This is known as the *cubic time bottleneck*:**
  - Occurs in many different scenarios
  - but O(n^3/log n) is possible…

- **A special case of general set constraints:**
  - Defined on sets of *terms* instead of sets of tokens
  - solvable in time $O(2^{2^n})$

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- **Control flow analysis for object-oriented languages**

Simple CFA for OO (1/3)

- CFA in an object-oriented language:

```
x.m(a,b,c)
```

- Which method implementations may be invoked?

- Full CFA is a possibility...
- But the extra structure allows simpler solutions

Simple CFA for OO (2/3)

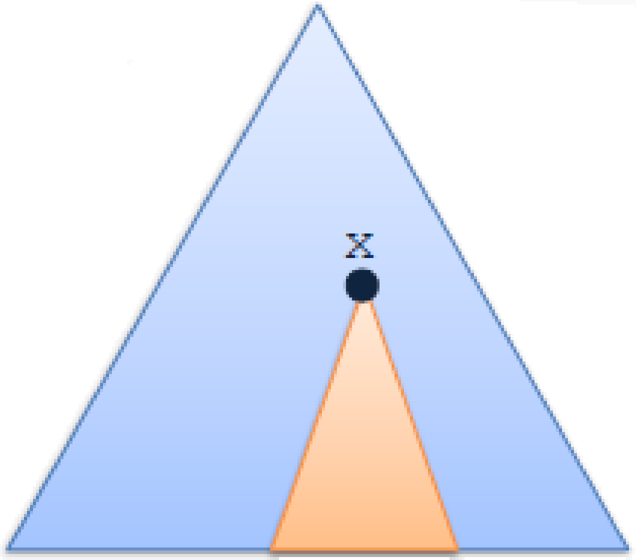
Simplest solution:

- Select all methods named m with three arguments

Class Hierarchy Analysis(CHA):

- Consider only the part of the class hierarchy rooted by the declared type of x

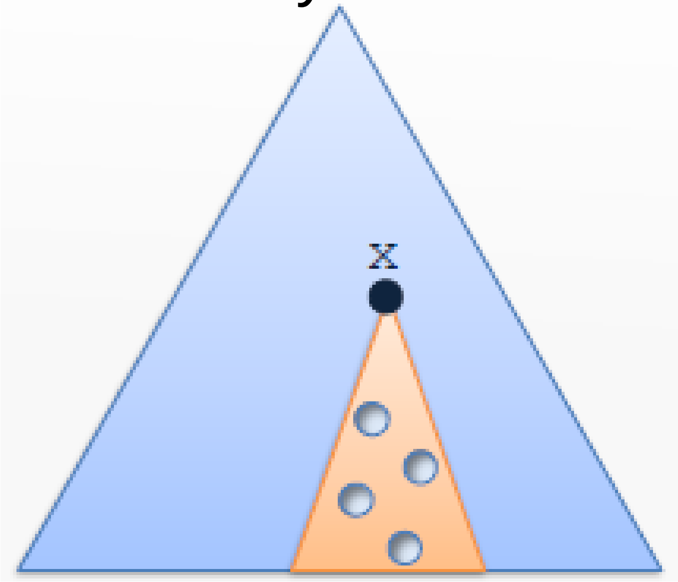
```
Collection<T> c = ...
c.add(e)
```



Simple CFA for OO (3/3)

Rapid Type Analysis (RTA):

- Restrict to those classes that are actually used in the program in **new** expressions
- Start from **main**, iteratively find reachable methods



Variable Type Analysis (VTA):

- perform *intraprocedural* control flow analysis

Thanks