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Path Sensitivity

Most content comes from http://cs.au.dk/~amoeller/spa/

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Information in Conditions

```
x = input;
y = 0;
z = 0;
while (x>0) {
  z = z+x;
  if (17>y) { y = y+1; }
  x = x-1;
}
```

The interval analysis (with widening) concludes:

$x = [-\infty, \infty]$, $y = [0, \infty]$, $z = [-\infty, \infty]$

Modeling Conditions

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Add artificial "assert" statements:

The statement assert(*E*) models that *E* is *true* in the current program state

- **it causes a runtime error otherwise**
- **but we only insert it where the condition will always be true**

Encoding Conditions

```
x = input;
y = 0;
z = 0;
while (x>0) {
  assert(x>0);
  z = z+x;
  if (17>y) { assert(17>y); y = y+1; }
  else { assert(!(17>y)); }
  x = x-1;
}
assert(!(x>0));
```

preserves semantics since `asserts` are guarded by conditions

(alternatively, we could add dataflow constraints on the CFG *edges*)

Constraints for assert

- A trivial but sound constraint:

$$[\![v]\!] = \mathit{JOIN}(v)$$

- A non-trivial constraint for assert(x>E):

$$[\![v]\!] = \mathit{JOIN}(v)[x \rightarrow gt(\mathit{JOIN}(v)(x), eval(\mathit{JOIN}(v), E))]$$

where

$$gt([l_1,h_1],[l_2,h_2]) = [l_1,h_1] \sqcap [l_2,\infty]$$

- Similar constraints are defined for the dual cases
- More tricky to define for other conditions...

Exploiting Conditions

```
x = input;
y = 0;
z = 0;
while (x>0) {
  assert(x>0);
  z = z+x;
  if (17>y) { assert(17>y); y = y+1; }
  else { assert(!(17>y)); }
  x = x-1;
}
assert(!(x>0));
```

The interval analysis now concludes:

$x = [-\infty, 0]$, $y = [0, 17]$, $z = [0, \infty]$

Branch Correlations

- With assert we have a simple form of *path sensitivity* (sometimes called *control sensitivity*)

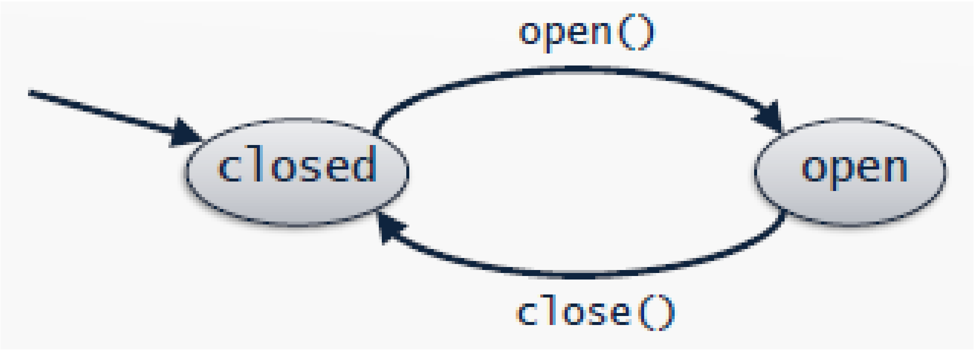
- But it is insufficient to handle *correlation* of branches:

```
if (17 > x) { ... }
... // statements that do not change x
if (17 > x) { ... }
...
```

Open and Closed Files

- Built-in functions open() and close() on a file

- Requirements:
  - never close a closed file
  - never open an open file



- We want a static analysis to check this...(for simplicity, let us assume there is only one file)

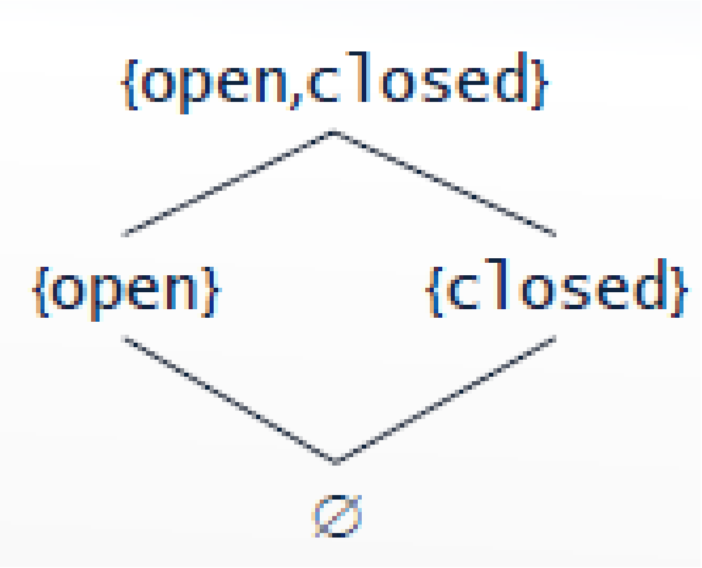
A Tricky Example


```
if (condition) {
  open();
  flag = 1;
} else {
  flag = 0;
}
...
if (flag) {
  close();
}
```

The Naive Analysis (1/2)

- The lattice models the status of the file:

  $L = (\mathcal{P}(\{open, closed\}), \subseteq)$



- For every CFG node, v, we have a constraint variable ⟦v⟧ denoting the status *after* v

$$JOIN(v) = \bigcup_{w \in pred(v)} [\![w]\!]$$

The Naive Analysis(2/2)

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- **Constraints for interesting statements:**

  ⟦*entry*⟧= {closed}

  ⟦open()⟧= {open}

  ⟦close()⟧= {closed}

- **For all other CFG nodes:**

  ⟦v⟧= *JOIN*(v)

- **Before the close() statement the analysis concludes that the file is {open,closed}** ☹

```
if (condition) {
  open();
  flag = 1;
} else {
  flag = 0;
}
...
if (flag) {
  close();
}
```

The Slightly Less Naive Analysis

- We obviously need to keep track of the flag variable
- Our second attempt is the lattice:

$$L = (\mathcal{P}(\{open, closed\}) \times \mathcal{P}(\{flag=0, flag \neq 0\}), \subseteq \times \subseteq)$$

- Additionally, we add assert(...) to model conditionals
- Even so, we still only know that the file is {open,closed} and that flag is {flag=0,flag≠0} ☹

```
if (condition) {
  open();
  flag = 1;
} else {
  flag = 0;
}
...
if (flag) {
  close();
}
```

Enhanced Program


```
if (condition) {
  assert(condition);
  open();
  flag = 1;
} else {
  assert(!condition);
  flag = 0;
}
...
if (flag) {
  assert(flag);
  close();
} else {
  assert(!flag);
}
```

Relational Analysis

- **We need an analysis that keeps track of *relations* between variables**

- **One approach is to maintain *multiple* abstract states per program point, one for each *path context***

  - For the file example we need the lattice:

    L = Paths→ $\mathcal{P}$({open,closed})

    (isomorphic to L=$\mathcal{P}$(Paths× {open,closed}))

    Where Paths = {flag=0,flag≠0} is the set of path contexts

Relational Constraints(1/2)

- For the file statements:

$$[\![entry]\!] = \lambda p.\{\texttt{closed}\}$$

$$[\![\texttt{open()}]\!] = \lambda p.\{\texttt{open}\}$$

$$[\![\texttt{closed()}]\!] = \lambda p.\{\texttt{closed}\}$$

- For `flag` assignments:

"infeasible"

$$[\![\texttt{flag = 0}]\!] = [\texttt{flag=0} \rightarrow \bigcup_{p \in P} JOIN(v)(p),\ \texttt{flag} \neq 0 \rightarrow \emptyset]$$

$$[\![\texttt{flag = } n]\!] = [\texttt{flag} \neq 0 \rightarrow \bigcup_{p \in P} JOIN(v)(p),\ \texttt{flag=0} \rightarrow \emptyset]$$

where n is a non-0 constant number

$$[\![\texttt{flag = } E]\!] = \lambda q. \bigcup_{p \in P} JOIN(v)(p)$$ for any other E

Relational Constraints(2/2)

- For `assert` statements:

$$[\![\texttt{assert(flag)}]\!] = [\texttt{flag}\neq 0 \rightarrow JOIN(v)(\texttt{flag}\neq 0), \texttt{flag}=0 \rightarrow \varnothing]$$

$$[\![\texttt{assert(!flag)}]\!] = [\texttt{flag}=0 \rightarrow JOIN(v)(\texttt{flag}=0), \texttt{flag}\neq 0 \rightarrow \varnothing]$$

- For all other CFG nodes:

$$[\![v]\!] = \mathrm{JOIN}(v) = \lambda p.\ \bigcup_{w\in pred(v)} [\![w]\!](p)$$

Generated Constraints

$$[\![\mathit{entry}]\!] = \lambda p.\{\texttt{closed}\}$$
$$[\![\texttt{condition}]\!] = [\![\mathit{entry}]\!]$$
$$[\![\texttt{assert(condition)}]\!] = [\![\texttt{condition}]\!]$$
$$[\![\texttt{open()}]\!] = \lambda p.\{\texttt{open}\}$$
$$[\![\texttt{flag = 1}]\!] = [\texttt{flag}\neq 0 \to \cup\ [\![\texttt{open()}]\!](p),\ \texttt{flag=0}\to\varnothing]$$
$$[\![\texttt{assert(!condition)}]\!] = [\![\texttt{condition}]\!]$$
$$[\![\texttt{flag = 0}]\!] = [\texttt{flag=0}\to\cup\ [\![\texttt{assert(!condition)}]\!](p),\ \texttt{flag}\neq 0\to\varnothing]$$
$$[\![\texttt{...}]\!] = \lambda p.([\![\texttt{flag = 1}]\!](p) \cup [\![\texttt{flag = 0}]\!](p))$$
$$[\![\texttt{flag}]\!] = [\![\ldots]\!]$$
$$[\![\texttt{assert(flag)}]\!] = [\texttt{flag}\neq 0\to[\![\texttt{flag}]\!](\texttt{flag}\neq 0),\ \texttt{flag=0}\to\varnothing]$$
$$[\![\texttt{close()}]\!] = \lambda p.\{\texttt{closed}\}$$
$$[\![\texttt{assert(!flag)}]\!] = [\texttt{flag=0}\to[\![\texttt{flag}]\!](\texttt{flag=0}),\ \texttt{flag}\neq 0\to\varnothing]$$
$$[\![\mathit{exit}]\!] = \lambda p.([\![\texttt{close()}]\!](p) \cup [\![\texttt{assert(!flag)}]\!](p))$$

Minimal Solution

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	flag = 0	flag ≠ 0
[entry]	{closed}	{closed}
[condition]	{closed}	{closed}
[assert(condition)]	{closed}	{closed}
[open()]	{open}	{open}
[flag = 1]	∅	{open}
[assert(!condition)]	{closed}	{closed}
[flag = 0]	{closed}	∅
[...]	{closed}	{open}
[flag]	{closed}	{open}
[assert(flag)]	∅	{open}
[close()]	{closed}	{closed}
[assert(!flag)]	{closed}	∅
[exit]	{closed}	{closed}

We now know the file is `open` before `close()` ☺

Challenges

- **The static analysis designer must choose Paths**
  - Often as Boolean combinations of predicates from conditionals
  - iterative refinement (e.g. *counter-example guided abstraction refinement*) can be used for gradually finding relevant predicates

- **Exponential blow-up:**
  - for k predicates, we have 2^k different contexts
  - Redundancy often cuts this down

- **Reasoning about assert:**
  - how to update the lattice elements with sufficient precision?
  - Possibly involves heavy-weight theorem proving

Improvements

- **Run auxiliary analyses first, for example:**
  - constant propagation
  - sign analysis

  will help in handling flag assignments

- **Dead code propagation, change**

  $[\![\text{open()}]\!] = \lambda\, p.\{\text{open}\}$

  into the still sound but more precise

  $[\![\text{open()}]\!] = \lambda p.\text{if } JOIN(v)(p)=\varnothing \text{ then } \varnothing \text{ else } \{\text{open}\}$

Thanks