Static Program Analysis

Part 4 – flow sensitive analyses

http://cs.au.dk/~amoeller/spa/

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Agenda

- **Live variables analysis**
- Available expressions analysis
- Very busy expressions analysis
- Reaching definitions analysis
- Constant propagation analysis

Liveness analysis

- A variable is *live* at a program point if its current value may be read in the remaining execution

- This is clearly undecidable, but the property can be conservatively approximated

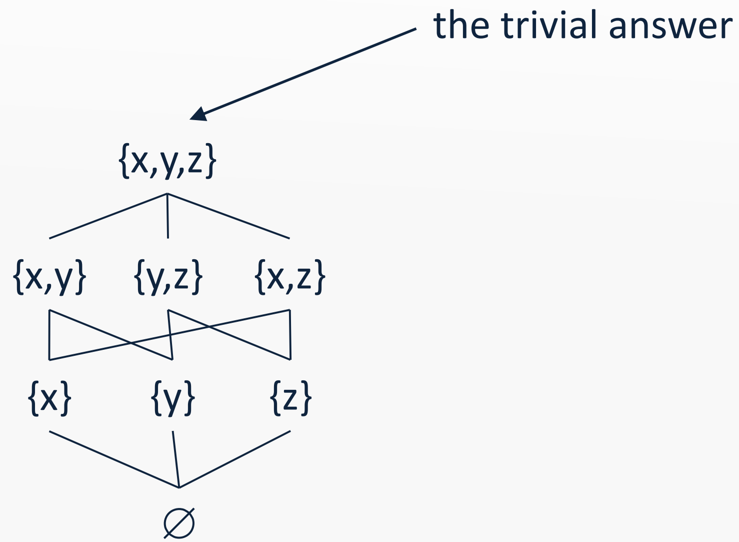
- The analysis must only answer *“dead”* if the variable is really dead
  - no need to store the values of dead variables

A lattice for liveness

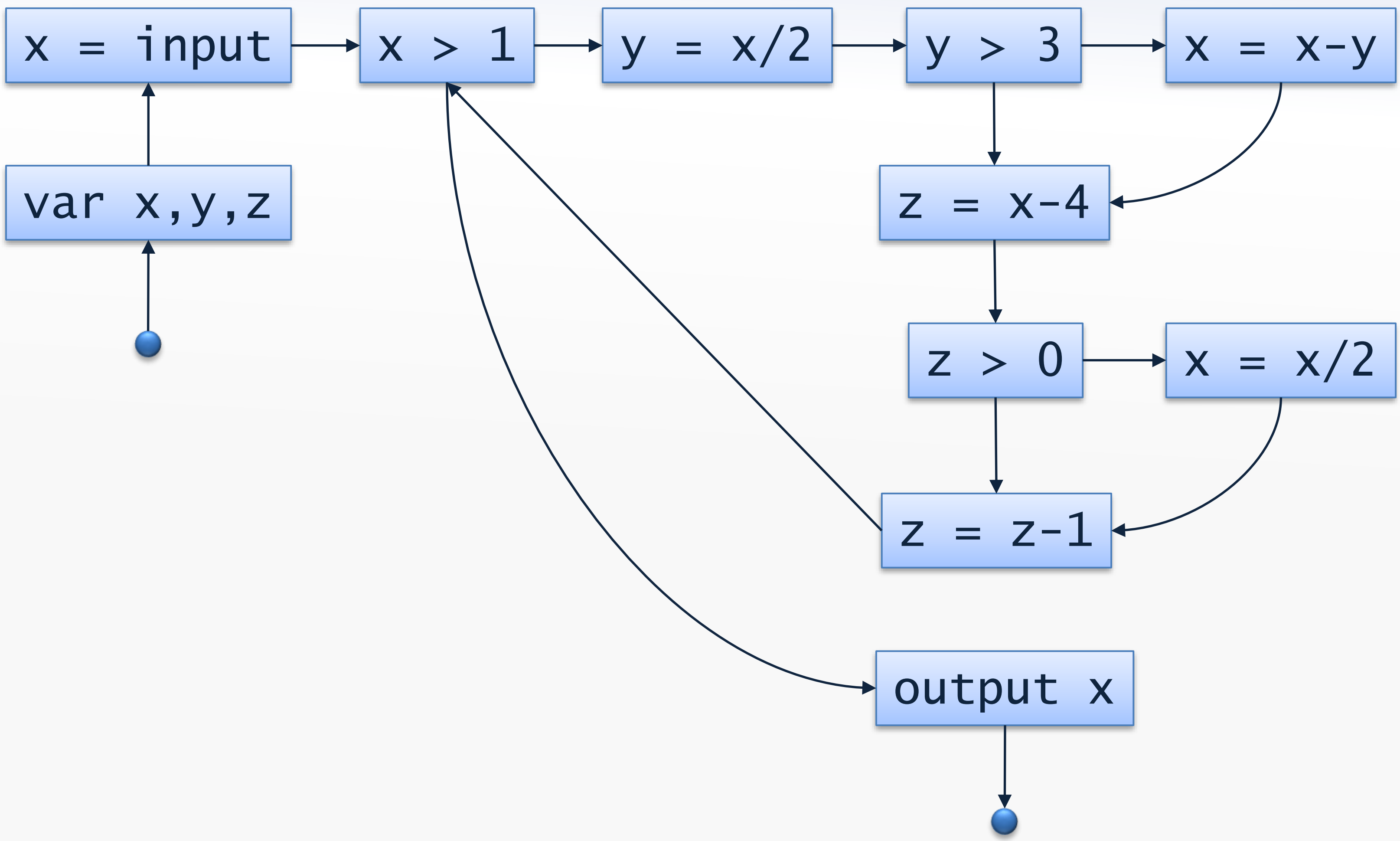
A subset lattice of program variables

```
var x,y,z;
x = input;
while (x>1) {
  y = x/2;
  if (y>3) x = x-y;
  z = x-4;
  if (z>0) x = x/2;
  z = z-1;
}
output x;
```

$L = (2^{\{x,y,z\}}, \subseteq)$



The control flow graph



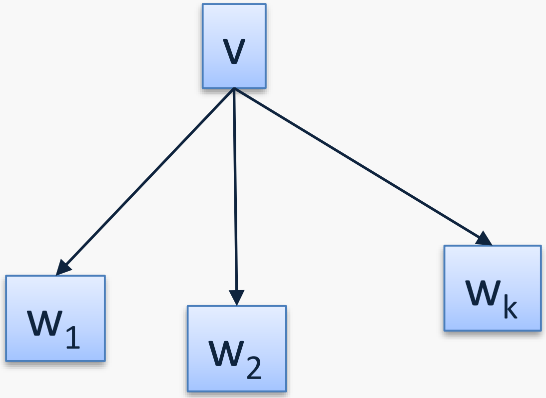
Setting up

- For every CFG node, v, we have a variable $[\![v]\!]$:
  - the subset of program variables that are live at the program point *before* v

- Since the analysis is conservative, the computed sets may be *too large*

- Auxiliary definition:

$$JOIN(v) = \bigcup_{w \in succ(v)} [\![w]\!]$$



Liveness constraints

$vars(E)$ = variables occurring in E

- For the exit node:

  $[\![exit]\!] = \varnothing$

- For conditions and output:

  $[\![\ \texttt{if (}E\texttt{)}\ ]\!] = [\![\ \texttt{output}\ E\ ]\!] = JOIN(\mathrm{v}) \cup vars(E)$

- For assignments:

  $[\![\ x = E\ ]\!] = JOIN(\mathrm{v}) \setminus \{x\} \cup vars(E)$

- For variable declarations:

  $[\![\ \texttt{var}\ x_1\texttt{,}\ \ldots\texttt{,}\ x_n\ ]\!] = JOIN(\mathrm{v}) \setminus \{x_1, \ldots, x_n\}$

- For all other nodes:

  $[\![\mathrm{v}]\!] = JOIN(\mathrm{v})$

right-hand sides are monotone since *JOIN* is monotone, and …

Generated constraints

$[\![\texttt{var x,y,z}]\!] = [\![\texttt{z=input}]\!] \setminus \{x,y,z\}$

$[\![\texttt{x=input}]\!] = [\![\texttt{x>1}]\!] \setminus \{x\}$

$[\![\texttt{x>1}]\!] = ([\![\texttt{y=x/2}]\!] \cup [\![\texttt{output x}]\!]) \cup \{x\}$

$[\![\texttt{y=x/2}]\!] = ([\![\texttt{y>3}]\!] \setminus \{y\}) \cup \{x\}$

$[\![\texttt{y>3}]\!] = [\![\texttt{x=x-y}]\!] \cup [\![\texttt{z=x-4}]\!] \cup \{y\}$

$[\![\texttt{x=x-y}]\!] = ([\![\texttt{z=x-4}]\!] \setminus \{x\}) \cup \{x\}$

$[\![\texttt{z>0}]\!] = [\![\texttt{x=x/2}]\!] \cup [\![\texttt{z=z-1}]\!] \cup \{z\}$

$[\![\texttt{x=x/2}]\!] = ([\![\texttt{z=z-1}]\!] \setminus \{x\}) \cup \{z\}$

$[\![\texttt{output x}]\!] = [\![\mathit{exit}]\!] \cup \{x\}$

$[\![\mathit{exit}]\!] = \varnothing$

Least solution

$[\![\mathit{entry}]\!] = \emptyset$

$[\![$`var x,y,z`$]\!] = \emptyset$

$[\![$`x=input`$]\!] = \emptyset$

$[\![$`x>1`$]\!] = \{x\}$

$[\![$`y=x/2`$]\!] = \{x\}$

$[\![$`y>3`$]\!] = \{x,y\}$

$[\![$`x=x-y`$]\!] = \{x,y\}$

$[\![$`z=x-4`$]\!] = \{x\}$

$[\![$`z>0`$]\!] = \{x,z\}$

$[\![$`x=x/2`$]\!] = \{x,z\}$

$[\![$`z=z-1`$]\!] = \{x,z\}$

$[\![$`output x`$]\!] = \{x\}$

$[\![\mathit{exit}]\!] = \emptyset$

Many non-trivial answers!

Optimizations

- Variables `y` and `z` are never simultaneously live

  ⇒ they can share the same variable location

- The value assigned in `z=z-1` is never read

  ⇒ the assignment can be skipped

```
var x,yz;
x = input;
while (x>1) {
  yz = x/2;
  if (yz>3) x = x-yz;
  yz = x-4;
  if (yz>0) x = x/2;
}
output x;
```

- better register allocation
- a few clock cycles saved

Time complexity (for the naive algorithm)

- With n CFG nodes and k variables:
  - the lattice L^n has height $k \cdot n$
  - so there are at most $k \cdot n$ iterations
- Subsets of Vars (the variables in the program) can be represented as bitvectors:
  - each element has size k
  - each $\cup$, $\setminus$, $=$ operation takes time $O(k)$
- Each iteration uses $O(n)$ bitvector operations:
  - so each iteration takes time $O(k \cdot n)$
- Total time complexity: $O(k^2 n^2)$

- Exercise: what is the complexity for the worklist algorithm?

Agenda

- Live variables analysis
- **Available expressions analysis**
- Very busy expressions analysis
- Reaching definitions analysis
- Constant propagation analysis

Available expressions analysis

- A (nontrivial) expression is *available* at a program point if its current value has already been computed earlier in the execution

- The approximation generally includes *too few* expressions
  – the analysis can only report *"available"* if the expression is definitely available
  – no need to re-compute available expressions (e.g. common subexpression elimination)

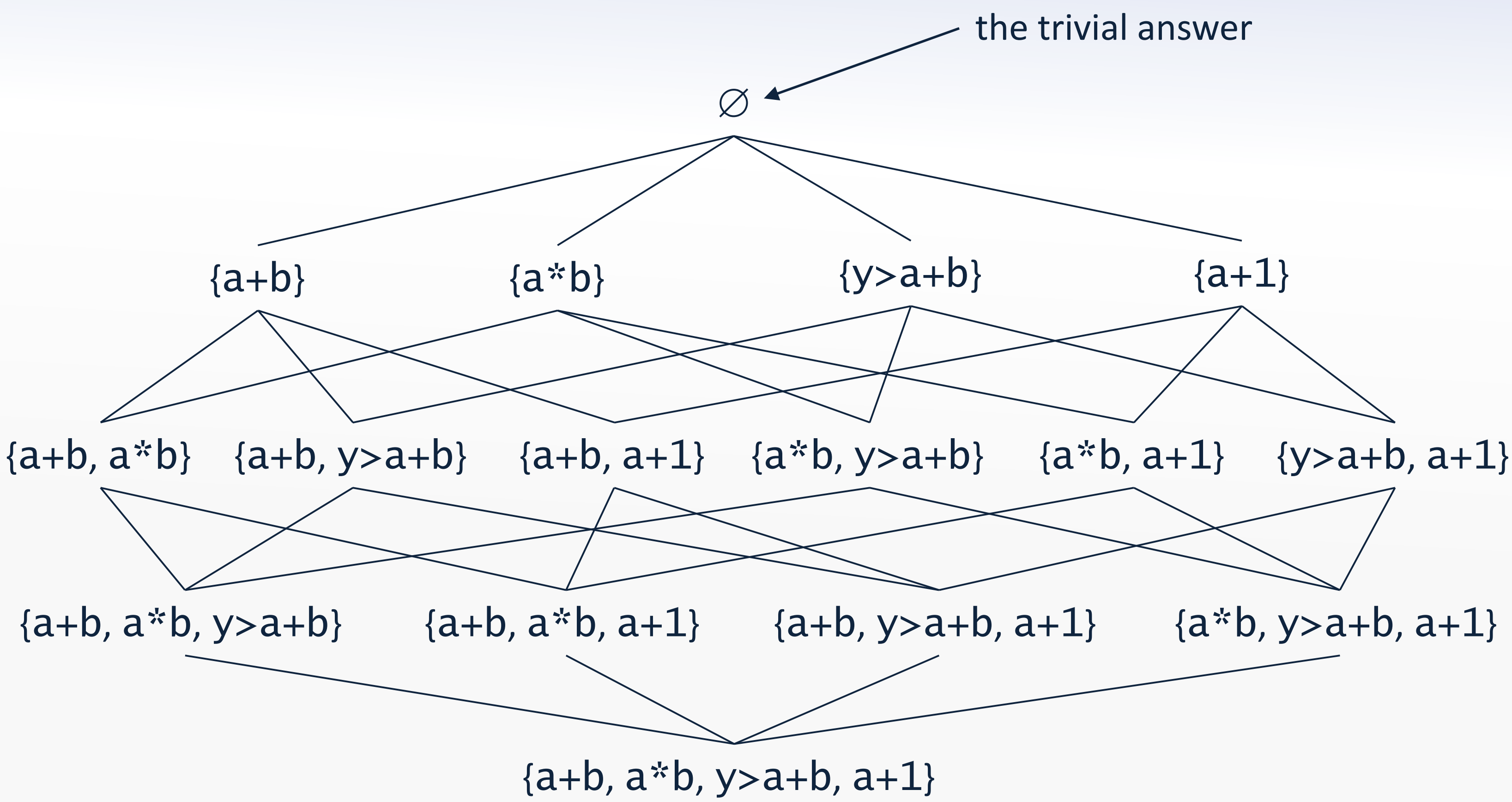
A lattice for available expressions

A reverse subset-lattice of nontrivial expressions

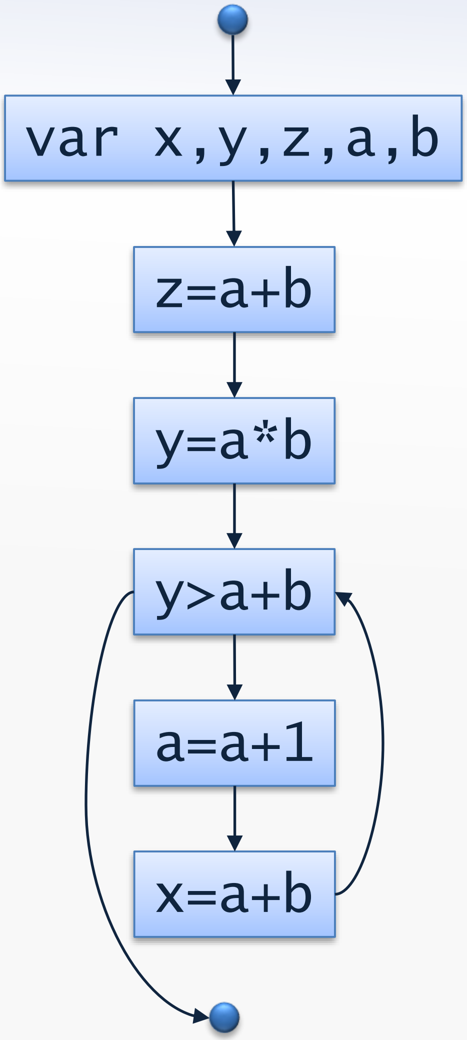
```
var x,y,z,a,b;
z = a+b;
y = a*b;
while (y > a+b) {
  a = a+1;
  x = a+b;
}
```

$$L = (2^{\{a+b,\ a*b,\ y>a+b,\ a+1\}}, \supseteq)$$

Reverse subset lattice



The flow graph



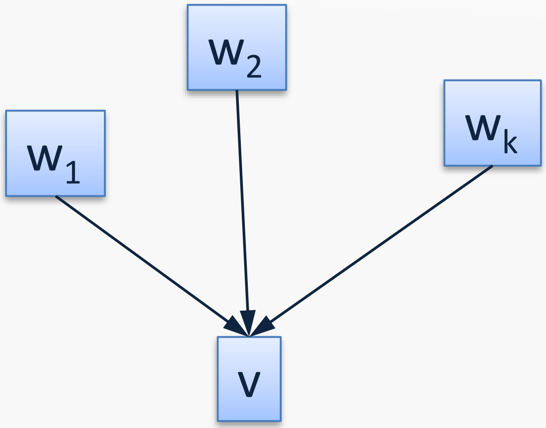
Setting up

- For every CFG node, v, we have a variable ⟦v⟧:
  - the subset of program variables that are available at the program point *after* v

- Since the analysis is conservative, the computed sets may be *too small*

- Auxiliary definition:

$$\mathit{JOIN}(v) = \bigcap_{w \in \mathit{pred}(v)} [\![w]\!]$$



Auxiliary functions

- The function $X{\downarrow}x$ removes all expressions from X that contain a reference to the variable *x*

- The function *exps*(*E*) is defined as:
  - $exps(intconst) = \varnothing$
  - $exps(x) = \varnothing$
  - $exps(\texttt{input}) = \varnothing$
  - $exps(E_1\ op\ E_2) = \{E_1\ op\ E_2\} \cup exps(E_1) \cup exps(E_2)$
    but don't include expressions containing `input`

Availability constraints

- For the *entry* node:

  $[\![entry]\!] = \emptyset$

- For conditions and output:

  $[\![\texttt{if (}E\texttt{)}]\!] = [\![\texttt{output } E]\!] = JOIN(\mathrm{v}) \cup exps(E)$

- For assignments:

  $[\![x = E]\!] = (JOIN(\mathrm{v}) \cup exps(E))\downarrow x$

- For any other node v:

  $[\![\mathrm{v}]\!] = JOIN(\mathrm{v})$

Generated constraints

$[\![\mathit{entry}]\!] = \emptyset$

$[\![\texttt{var x,y,z,a,b}]\!] = [\![\mathit{entry}]\!]$

$[\![\texttt{z=a+b}]\!] = \mathit{exps}(\texttt{a+b})\downarrow\texttt{z}$

$[\![\texttt{y=a*b}]\!] = ([\![\texttt{z=a+b}]\!] \cup \mathit{exps}(\texttt{a*b}))\downarrow\texttt{y}$

$[\![\texttt{y>a+b}]\!] = ([\![\texttt{y=a*b}]\!] \cap [\![\texttt{x=a+b}]\!]) \cup \mathit{exps}(\texttt{y>a+b})$

$[\![\texttt{a=a+1}]\!] = ([\![\texttt{y>a+b}]\!] \cup \mathit{exps}(\texttt{a+1}))\downarrow\texttt{a}$

$[\![\texttt{x=a+b}]\!] = ([\![\texttt{a=a+1}]\!] \cup \mathit{exps}(\texttt{a+b}))\downarrow\texttt{x}$

$[\![\mathit{exit}]\!] = [\![\texttt{y>a+b}]\!]$

Least solution

$[\![\textit{entry}]\!] = \emptyset$

$[\![$`var x,y,z,a,b`$]\!] = \emptyset$

$[\![$`z=a+b`$]\!] = \{$`a+b`$\}$

$[\![$`y=a*b`$]\!] = \{$`a+b`, `a*b`$\}$

$[\![$`y>a+b`$]\!] = \{$`a+b`, `y>a+b`$\}$

$[\![$`a=a+1`$]\!] = \emptyset$

$[\![$`x=a+b`$]\!] = \{$`a+b`$\}$

$[\![\textit{exit}]\!] = \{$`a+b`$\}$

Again, many nontrivial answers!

Optimizations

- We notice that a+b is available before the loop
- The program can be optimized (slightly):

```
var x,y,x,a,b,aplusb;
aplusb = a+b;
z = aplusb;
y = a*b;
while (y > aplusb) {
  a = a+1;
  aplusb = a+b;
  x = aplusb;
}
```

Agenda

- Live variables analysis
- Available expressions analysis
- **Very busy expressions analysis**
- Reaching definitions analysis
- Constant propagation analysis

Very busy expressions analysis

- A (nontrivial) expression is *very busy* if it will definitely be evaluated before its value changes

- The approximation generally includes *too few* expressions
  - the answer "*very busy*" must be the true one
  - very busy expressions may be pre-computed (e.g. loop hoisting)

- Same lattice as for available expressions

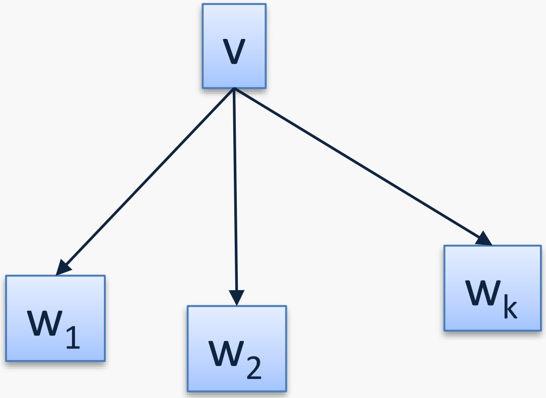
Setting up

- For every CFG node, v, we have a variable $[\![v]\!]$:
  - the subset of program variables that are very busy at the program point *before* v

- Since the analysis is conservative, the computed sets may be *too small*

- Auxiliary definition:

$$JOIN(v) = \bigcap_{w \in succ(v)} [\![w]\!]$$



Very busy constraints

- For the *exit* node:

  $[\![exit]\!] = \emptyset$

- For conditions and output:

  $[\![\texttt{if (}E\texttt{)}]\!] = [\![\texttt{output}\ E]\!] = JOIN(\mathrm{v}) \cup exps(E)$

- For assignments:

  $[\![x = E]\!] = JOIN(\mathrm{v}){\downarrow}x \cup exps(E)$

- For all other nodes:

  $[\![\mathrm{v}]\!] = JOIN(\mathrm{v})$

An example program

```
var x,a,b;
x = input;
a = x-1;
b = x-2;
while (x > 0) {
  output a*b-x;
  x = x-1;
}
output a*b;
```

The analysis shows that a*b is very busy

Code hoisting

```
var x,a,b;
x = input;
a = x-1;
b = x-2;
while (x > 0) {
  output a*b-x;
  x = x-1;
}
output a*b;
```

```
var x,a,b,atimesb;
x = input;
a = x-1;
b = x-2;
atimesb = a*b;
while (x > 0) {
  output atimesb-x;
  x = x-1;
}
output atimesb;
```

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- Live variables analysis
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- **Reaching definitions analysis**
- Constant propagation analysis

Reaching definitions analysis

- The *reaching definitions* for a program point are those assignments that may define the current values of variables

- The conservative approximation may include *too many* possible assignments

A lattice for reaching definitions

The subset lattice of assignments

$L = (2^{\{\texttt{x=input}, \texttt{y=x/2}, \texttt{x=x-y}, \texttt{z=x-4}, \texttt{x=x/2}, \texttt{z=z-1}\}}, \subseteq)$

```
var x,y,z;
x = input;
while (x > 1) {
  y = x/2;
  if (y>3) x = x-y;
  z = x-4;
  if (z>0) x = x/2;
  z = z-1;
}
output x;
```

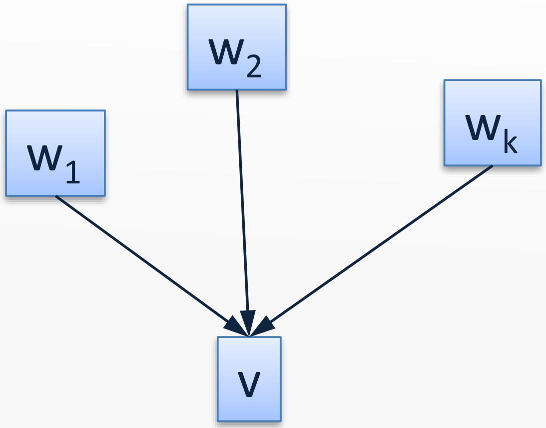
Reaching definitions constraints

- For assignments:

$$[\![x = E]\!] = \mathit{JOIN}(\mathrm{v}){\downarrow}x \cup \{ x = E \}$$

- For all other nodes:

$$[\![\mathrm{v}]\!] = \mathit{JOIN}(\mathrm{v})$$



- Auxiliary definition:

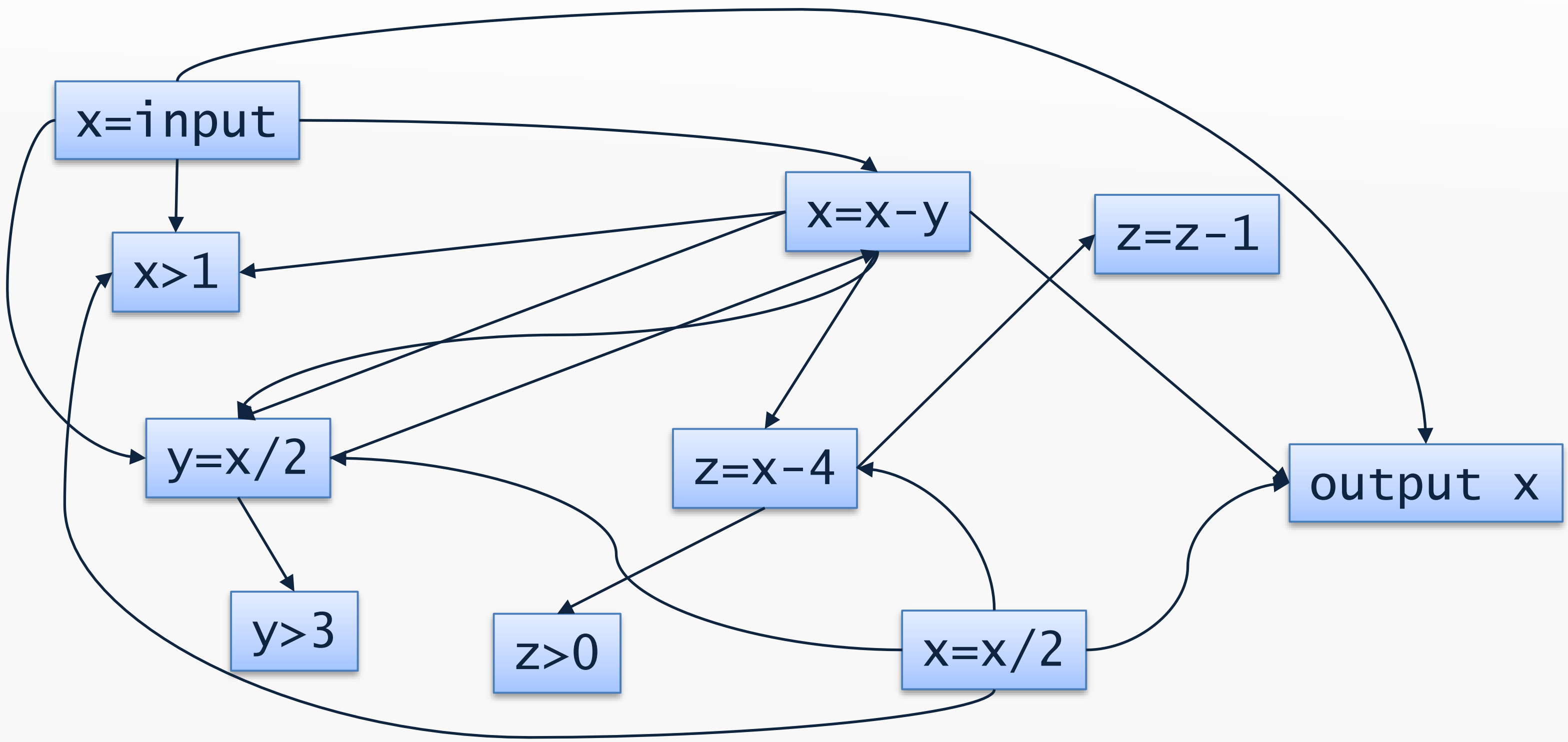
$$\mathit{JOIN}(\mathrm{v}) = \bigcup_{\mathrm{w} \in \mathit{pred}(\mathrm{v})} [\![\mathrm{w}]\!]$$

- The function $\mathrm{X}{\downarrow}x$ removes assignments to x from X

Def-use graph

Reaching definitions define the def-use graph:

- like a CFG but with edges from *def* to *use* nodes
- basis for *dead code elimination* and *code motion*



Forward vs. backward

- A *forward* analysis:
  - computes information about the *past* behavior
  - examples: available expressions, reaching definitions

- A *backward* analysis:
  - computes information about the *future* behavior
  - examples: liveness, very busy expressions

May vs. must

- A *may* analysis:
  - describes information that is *possibly* true
  - an *over*-approximation
  - examples: liveness, reaching definitions

- A *must* analysis:
  - describes information that is *definitely* true
  - an *under*-approximation
  - examples: available expressions, very busy expressions

Classifying analyses

	forward	backward
may	example: reaching definitions $[\![v]\!]$ describes state after v $\text{JOIN}(v) = \bigsqcup_{w \in pred(v)} [\![w]\!] = \bigcup_{w \in pred(v)} [\![w]\!]$	example: liveness $[\![v]\!]$ describes state before v $\text{JOIN}(v) = \bigsqcup_{w \in succ(v)} [\![w]\!] = \bigcup_{w \in succ(v)} [\![w]\!]$
must	example: available expressions $[\![v]\!]$ describes state after v $\text{JOIN}(v) = \bigsqcup_{w \in pred(v)} [\![w]\!] = \bigcap_{w \in pred(v)} [\![w]\!]$	example: very busy expressions $[\![v]\!]$ describes state before v $\text{JOIN}(v) = \bigsqcup_{w \in succ(v)} [\![w]\!] = \bigcap_{w \in succ(v)} [\![w]\!]$

Initialized variables analysis

- Compute for each program point those variables that have *definitely* been initialized in the *past*
- (Called *definite assignment* analysis in Java and C#)
- $\Rightarrow$ *forward must analysis*
- Reverse subset lattice of all variables

$$JOIN(v) = \bigcap_{w \in pred(v)} [\![w]\!]$$

- For assignments: $[\![\, x = E \,]\!] = JOIN(v) \cup \{x\}$
- For all others: $[\![v]\!] = JOIN(v)$

Agenda

- Live variables analysis
- Available expressions analysis
- Very busy expressions analysis
- Reaching definitions analysis
- **Constant propagation analysis**

Constant propagation optimization

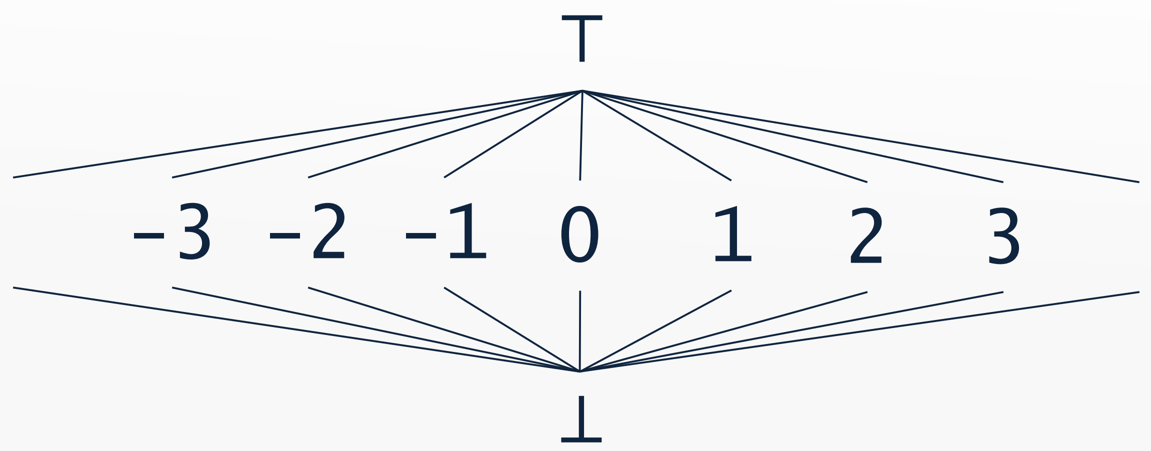
```
var x,y,z;
x = 27;
y = input,
z = 2*x+y;
if (x<0) { y=z-3; } else { y=12 }
output y;
```

```
var x,y,z;
x = 27;
y = input;
z = 54+y;
if (0) { y=z-3; } else { y=12 }
output y;
```

```
var y;
y = input;
output 12;
```

Constant propagation analysis

- Determine variables with a constant value
- Flat lattice:



Constraints for constant propagation

- Essentially as for the Sign analysis…
- Abstract operator for addition:

$$\overline{+}(n,m) = \begin{cases} \bot & \text{if } n=\bot \lor m=\bot \\ \top & \text{else if } n=\top \lor m=\top \\ n+m & \text{otherwise} \end{cases}$$