

# Static Program Analysis

## Part 7 – interprocedural analysis

<http://cs.au.dk/~amoeller/spa/>

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# Interprocedural analysis

- Analyzing the body of a single function:
  - *intra*procedural analysis
- Analyzing the whole program with function calls:
  - *inter*procedural analysis
- A naive approach:
  - analyze each function in isolation
  - be maximally pessimistic about results of function calls
  - rarely sufficient precision...

# CFG for whole programs

The idea:

- construct a CFG for each function
- then glue them together to reflect function calls and returns

We need to take care of:

- parameter passing
- return values
- values of local variables across calls  
(including recursive functions, so not enough to assume unique variable names)

# A simplifying assumption

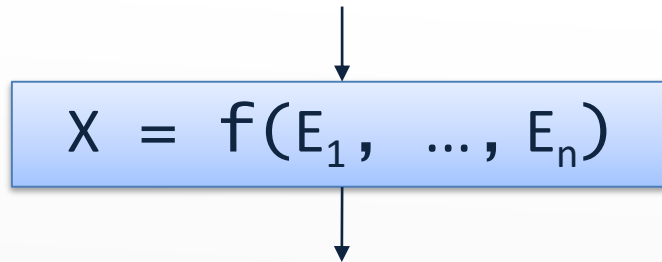
- Assume that all function calls are of the form

$$X = f(E_1, \dots, E_n);$$

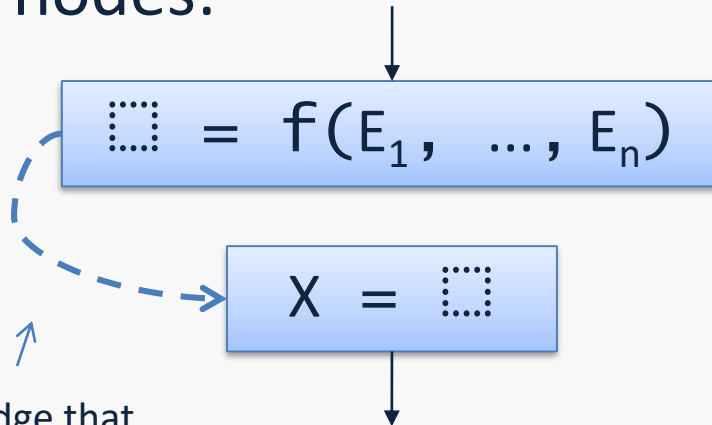
- This can always be obtained by normalization

# Interprocedural CFGs (1/3)

Split each original call node



into two nodes:



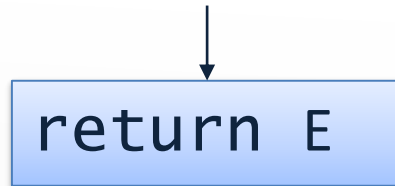
← the “call node”

← the “after-call node”

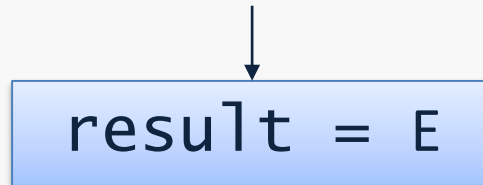
a special edge that connects the call node with its after-call node

# Interprocedural CFGs (2/3)

Change each return node



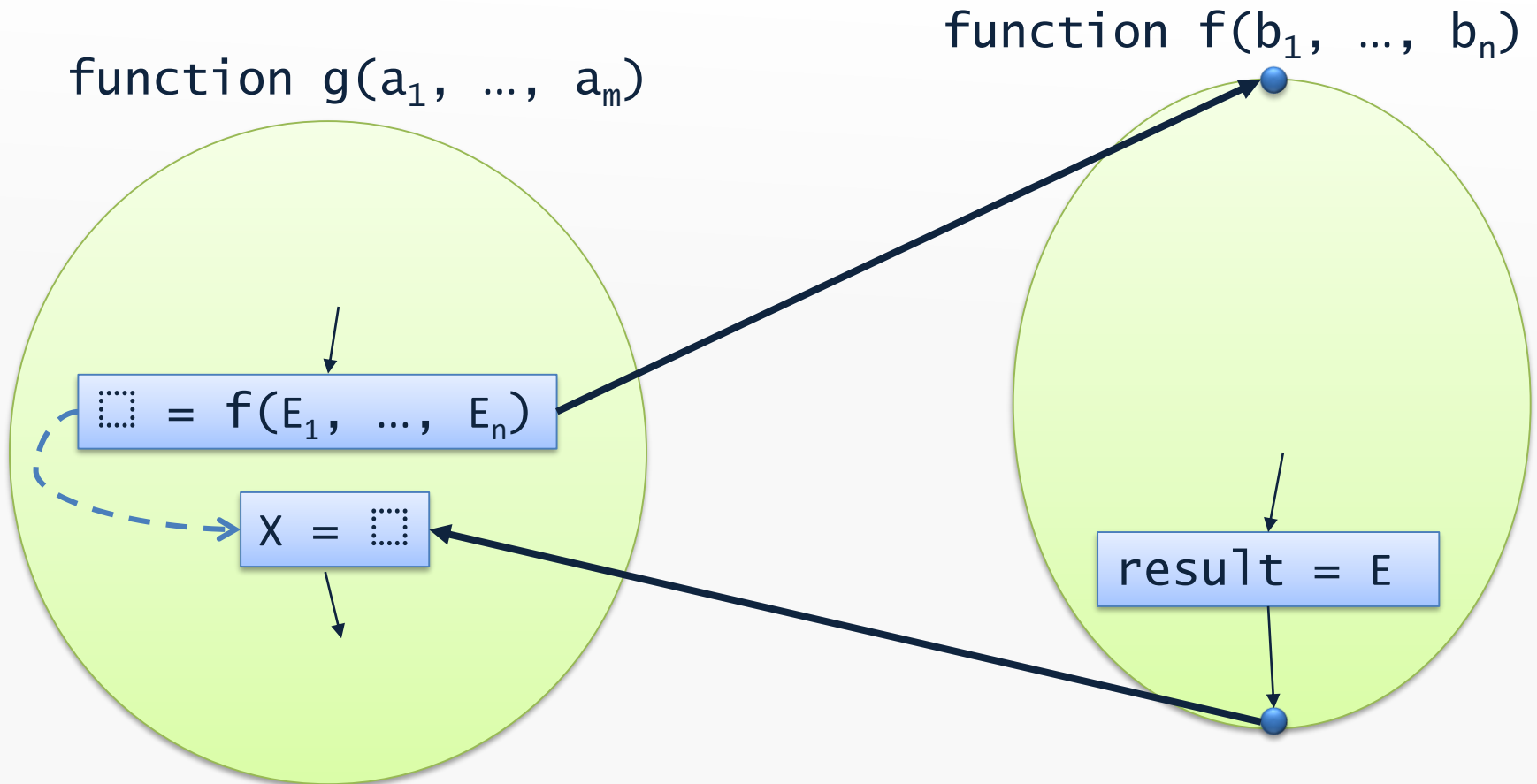
into an assignment:



(where `result` is a fresh variable)

# Interprocedural CFGs (3/3)

Add call edges and return edges:



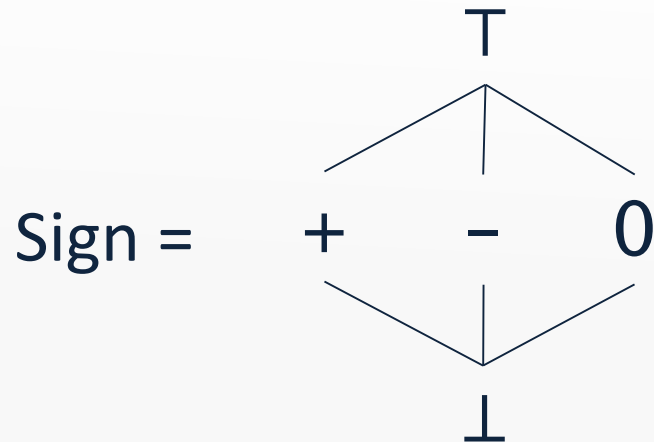
# Constraints

- For call/entry nodes:
  - be careful to model evaluation of *all* the actual parameters before binding them to the formal parameter names (otherwise, it may fail for recursive functions)
- For after-call/exit nodes:
  - like an assignment: `X = result`
  - but also restore local variables from before the call using the `call`  $\rightsquigarrow$  `after-call` edge
- The details depend on the specific analysis...



# Example: interprocedural sign analysis

- Recall the intraprocedural sign analysis...
- Lattice for abstract values:



- Lattice for abstract states:  
 $Vars \rightarrow Sign$

# Example: interprocedural sign analysis

- Constraint for entry node  $v$  of function  $f(b_1, \dots, b_n)$ :

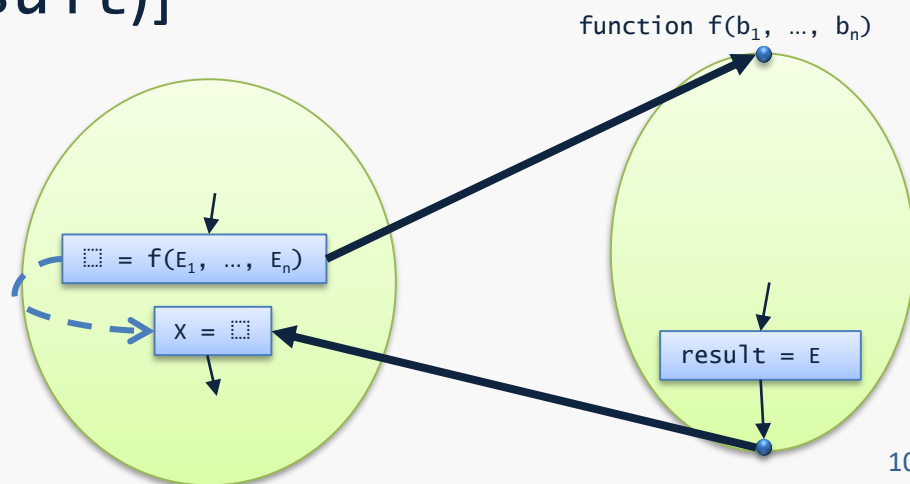
$$\llbracket v \rrbracket = \bigsqcup_{w \in \text{pred}(v)} \perp [b_1 \rightarrow \text{eval}(\llbracket w \rrbracket, E_1^w), \dots, b_n \rightarrow \text{eval}(\llbracket w \rrbracket, E_n^w)]$$

- Constraint for after-call node  $v$  labeled  $X = \square$ , with call node  $v'$ :

$$\llbracket v \rrbracket = \llbracket v' \rrbracket [X \rightarrow \llbracket w \rrbracket(\text{result})]$$

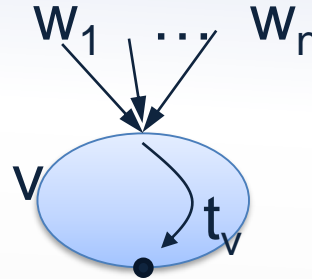
where  $w \in \text{pred}(v)$

(Recall: no global variables, no heap, and no higher-order functions)



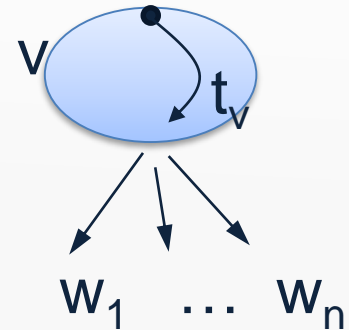
# Alternative formulations

$$1) \llbracket v \rrbracket = t_v(\bigsqcup_{w \in \text{pred}(v)} \llbracket w \rrbracket)$$



$$2) \forall w \in \text{succ}(v): t_v(\llbracket v \rrbracket) \sqsubseteq \llbracket w \rrbracket$$

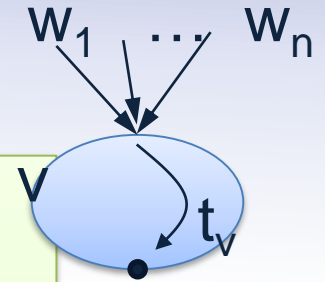
- recall "solving inequations"
- may require fewer join operations if there are many CFG edges
- more suitable for *interprocedural* flow



# The worklist algorithm (original version)

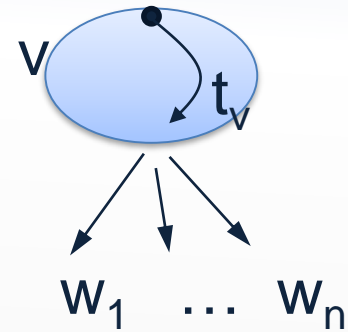
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```
x1 = ⊥; ... xn = ⊥
W = {v1, ..., vn}
while (W ≠ ∅) {
  vi = W.removeNext()
  y = fi(x1, ..., xn)
  if (y ≠ xi) {
    for (vj ∈ dep(vi)) {
      W.add(vj)
    }
    xi = y
  }
}
```



# The worklist algorithm (alternative version)

```
x1 = ⊥; ... xn = ⊥  
W = {v1, ..., vn}  
while (W ≠ ∅) {  
    vi = W.removeNext()  
    y = ti(xi)  
    for (vj ∈ dep(vi)) {  
        propagate(y, vj)  
    }  
}
```

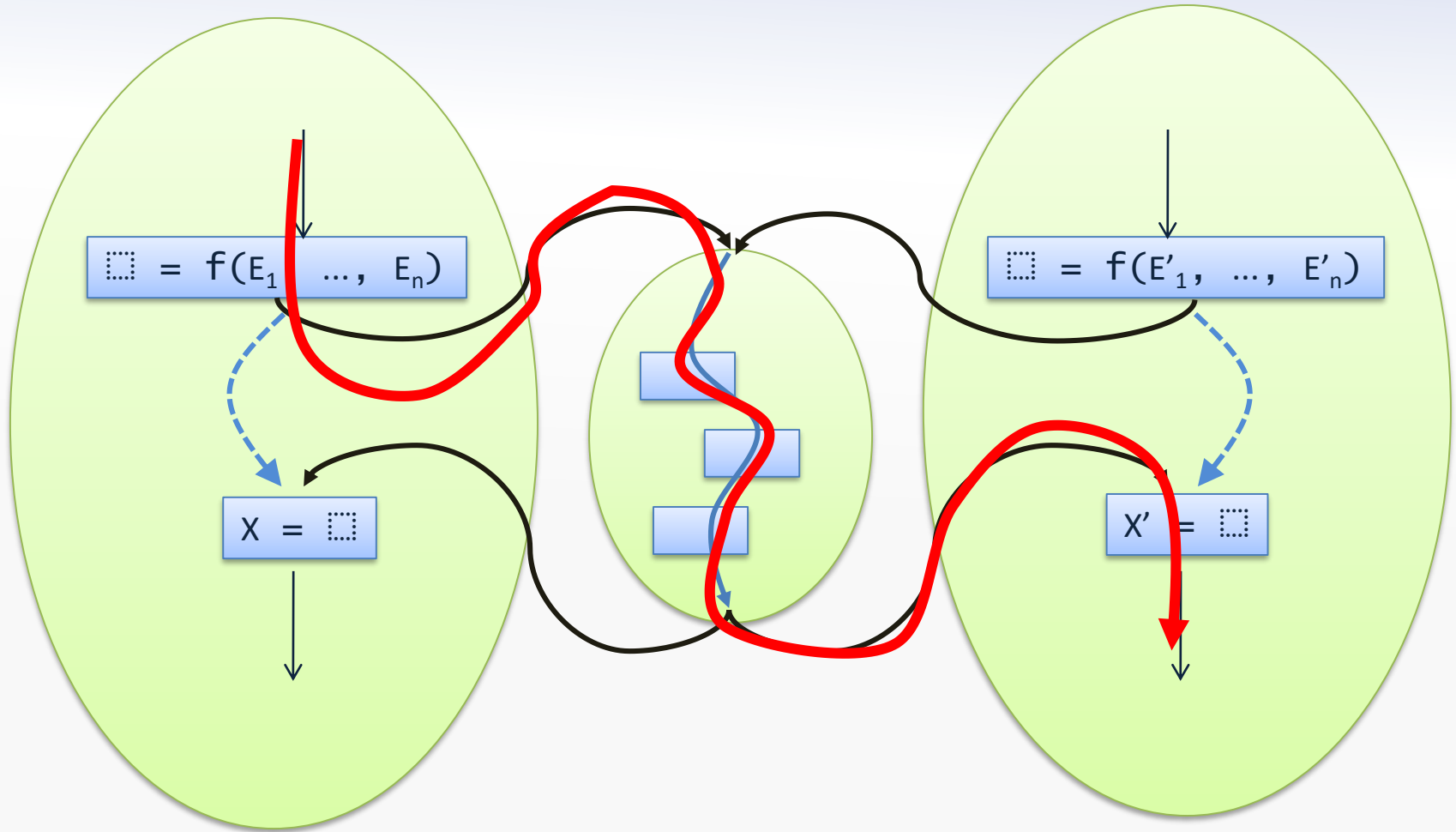


```
propagate(y, vj) {  
    z = xj ∪ y  
    if (z ≠ xj) {  
        xj = z  
        W.add(vj)  
    }  
}
```

# Agenda

- Interprocedural analysis
- **Context-sensitive  
interprocedural analysis**

# Interprocedurally invalid paths



# Example

What is the sign of the return value of g?

```
f(z) {  
    return z*42;  
}  
  
g() {  
    var x,y;  
    x = f(0);  
    y = f(87);  
    return x + y;  
}
```

Our current analysis says “T”



# Function cloning

## (alternatively, function inlining)

- Clone functions such that each function has only one callee
- Can avoid interprocedurally invalid paths 😊
- For high nesting depths, gives exponential blow-up 😐
- Doesn't work on (mutually) recursive functions 😞
- Use heuristics to determine when to apply (trade-off between CFG size and precision)

# Example, with cloning

What is the sign of the return value of g?

```
f1(z1) {  
    return z1*42;  
}
```

```
f2(z2) {  
    return z2*42;  
}
```

```
g() {  
    var x,y;  
    x = f1(0);  
    y = f2(87);  
    return x + y;  
}
```

# Context sensitive analysis

- Function cloning provides a kind of context sensitivity (also called polyvariant analysis)
- Instead of physically copying the function CFGs, do it *logically*
- Replace the lattice for abstract states, States, by

Contexts  $\rightarrow$  lift(States)

where Contexts is a set of ***call contexts***

- the contexts are abstractions of the state at function entry
  - Contexts must be finite to ensure finite height of the lattice
  - the bottom element of lift(States) represents “unreachable” contexts
- Different strategies for choosing the set Contexts...

# One-level cloning

- Let  $c_1, \dots, c_n$  be the call nodes in the program
- Define  $\text{Contexts} = \{c_1, \dots, c_n\} \cup \{\varepsilon\}$ 
  - each call node now defines its own “call context”  
(using  $\varepsilon$  to represent the call context at the main function)
  - the context is then like the return address of the top-most stack frame in the call stack
- Same effect as one-level cloning, but without actually copying the function CFGs
- Usually straightforward to generalize the constraints for a context insensitive analysis to this lattice
- (Example: context-sensitive sign analysis – later...)

# The call string approach

- Let  $c_1, \dots, c_n$  be the call nodes in the program
- Define Contexts as the set of strings over  $\{c_1, \dots, c_n\}$  of length  $\leq k$ 
  - such a string represents the top-most  $k$  call locations on the call stack
  - the empty string  $\varepsilon$  again represents the call context at the main function
- For  $k=1$  this amounts to one-level cloning

# Example:

## interprocedural sign analysis with call strings ( $k=1$ )

Lattice for abstract states:  $\text{Contexts} \rightarrow \text{lift}(\text{Vars} \rightarrow \text{Sign})$   
where  $\text{Contexts} = \{\varepsilon, c_1, c_2\}$

```
f(z) {  
  var t1, t2;  
  t1 = z*6;  
  t2 = t1*7;  
  return t2;  
}
```

...

```
x = f(0); // c1
```

```
y = f(87); // c2
```

...

$[\varepsilon \mapsto \text{unreachable},$   
 $c_1 \mapsto \perp[z \mapsto 0, t_1 \mapsto 0, t_2 \mapsto 0],$   
 $c_2 \mapsto \perp[z \mapsto +, t_1 \mapsto +, t_2 \mapsto +]]$

What is an example program  
that requires  $k=2$   
to avoid loss of precision?

# Context sensitivity with call strings

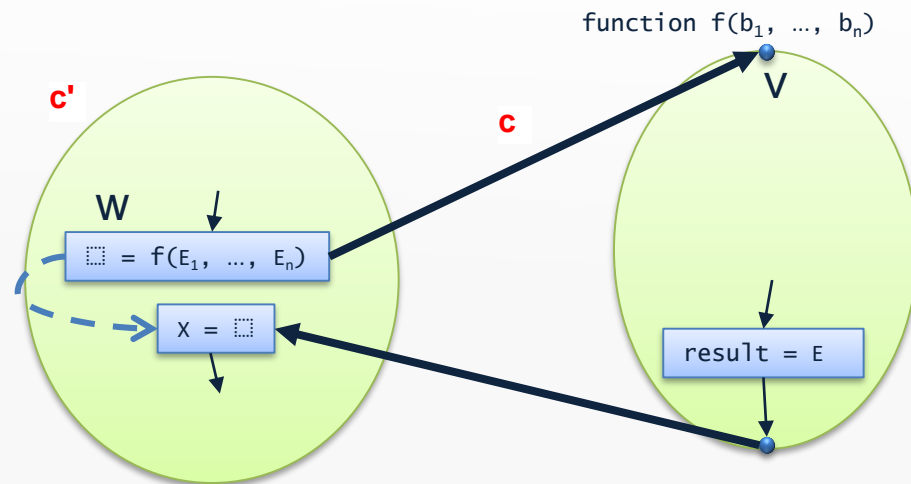
## function entry nodes, for k=1

Constraint for entry node  $v$  of function  $f(b_1, \dots, b_n)$ :

(if not 'main')

$c$ : context  
 $c'$ : context at the call node

$$\llbracket v \rrbracket(c) = \bigsqcup_{\substack{w \in \text{pred}(v) \wedge \\ c = w \wedge \\ c' \in \text{Contexts} \wedge \\ \llbracket w \rrbracket(c') \neq \text{unreachable}}} S_w^{c'}$$



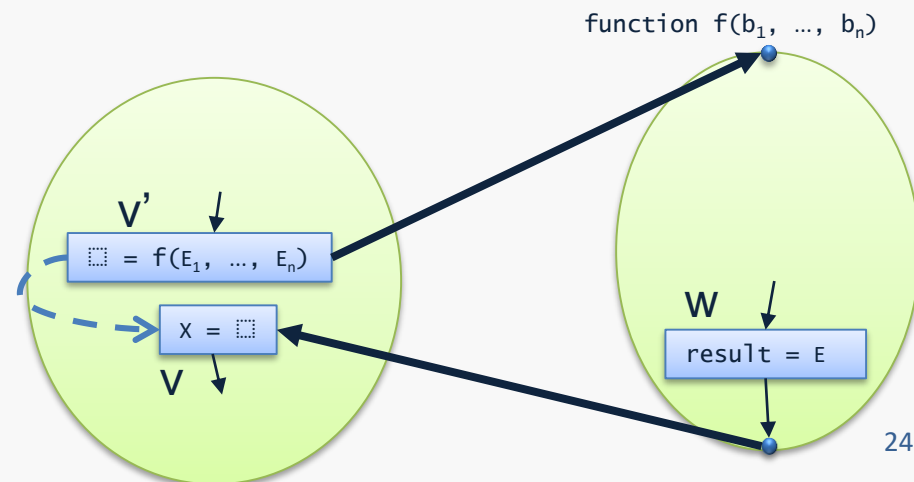
$$s_w^{c'} = \perp [b_1 \rightarrow \text{eval}(\llbracket w \rrbracket(c'), E_1^w), \dots, b_n \rightarrow \text{eval}(\llbracket w \rrbracket(c'), E_n^w)]$$

# Context sensitivity with call strings after-call nodes, for $k=1$

Constraint for after-call node  $v$  labeled  $X = \square$ ,  
with call node  $v'$  and exit node  $w \in \text{pred}(v)$ :

$$\llbracket v \rrbracket(c) = \llbracket v' \rrbracket(c)[X \rightarrow \llbracket w \rrbracket(v')(\text{result})]$$

if  $\llbracket v' \rrbracket(c) \neq \text{unreachable} \wedge \llbracket w \rrbracket(v') \neq \text{unreachable}$





# The functional approach

- The call string approach considers control flow
  - but why distinguish between two different call sites if their abstract states are the same? e.g. 符号分析例中, f(42), f(87) 的抽象状态都一样
- The functional approach instead considers data
- In the most general form, choose
$$\text{Contexts} = \text{States}$$
(requires States to be finite)
- Each element of the lattice  $\text{States} \rightarrow \text{lift}(\text{States})$  is now a map  $m$  that provides an element  $m(x)$  from States (or “unreachable”) for each possible  $x$  where  $x$  describes the state at function entry

# Example:

## interprocedural sign analysis with the functional approach

Lattice for abstract states:  $\text{Contexts} \rightarrow \text{lift}(\text{Vars} \rightarrow \text{Sign})$

where  $\text{Contexts} = \text{Vars} \rightarrow \text{Sign}$

```
f(z) {  
  var t1, t2;  
  t1 = z*6;  
  t2 = t1*7;  
  return t2;  
}
```

...

```
x = f(0);
```

```
y = f(87);
```

...

$\perp[z \mapsto 0] \mapsto \perp[z \mapsto 0, t1 \mapsto 0, t2 \mapsto 0],$   
 $\perp[z \mapsto +] \mapsto \perp[z \mapsto +, t1 \mapsto +, t2 \mapsto +],$   
all other contexts  $\mapsto$  unreachable ]

# The functional approach

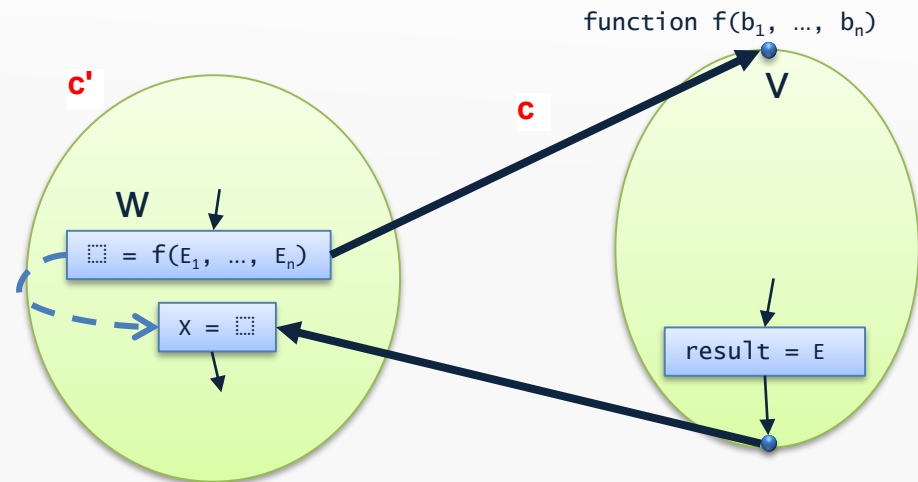
- The lattice element for a function exit node is thus a *function summary* that maps abstract function input to abstract function output
- This can be exploited at call nodes!
- When entering a function with abstract state  $x$ :
  - consider the function summary  $s$  for that function
  - if  $s(x)$  already has been computed, use that to model the entire function body, then proceed directly to the after-call node
- Avoids the problem with interprocedurally invalid paths!
- ...but may be expensive if States is large

# Context sensitivity with the functional approach

## function entry nodes

Constraint for entry node  $v$  of function  $f(b_1, \dots, b_n)$ :  
(if not 'main')

$$\begin{aligned} \llbracket v \rrbracket(c) = \bigsqcup_{\substack{w \in \text{pred}(v) \wedge \\ c = s_w^{c'} \wedge \\ c' \in \text{Contexts} \wedge \\ \llbracket w \rrbracket(c') \neq \text{unreachable}}} s_w^{c'} \end{aligned}$$



$$s_w^{c'} = \perp [b_1 \rightarrow \text{eval}(\llbracket w \rrbracket(c'), E_1^w), \dots, b_n \rightarrow \text{eval}(\llbracket w \rrbracket(c'), E_n^w)]$$

# Context sensitivity with the functional approach after-call nodes

Constraint for after-call node  $v$  labeled  $X = \square$ ,  
with call node  $v'$  and exit node  $w \in \text{pred}(v)$ :

$$\llbracket v \rrbracket(c) = \llbracket v' \rrbracket(c)[X \rightarrow \llbracket w \rrbracket(s_{v'}^c)(\text{result})]$$

if  $\llbracket v' \rrbracket(c) \neq \text{unreachable} \wedge \llbracket w \rrbracket(s_{v'}^c) \neq \text{unreachable}$

