Path Sensitivity

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Most content comes from http://cs.au.dk/~amoeller/spa/

Information in Conditions

```
x = input;
y = 0;
z = 0;
while (x>0) {
  z = z+x;
  if (17>y) { y = y+1; }
  x = x-1;
}
```

The interval analysis (with widening) concludes:
x = [-∞,∞], y = [0,∞], z = [-∞,∞]

Modeling Conditions

Add artificial "assert" statements:

The statement assert(*E*) models that *E* is *true* in the current program state

- it causes a runtime error otherwise
- but we only insert it where the condition will always be true

Encoding Conditions

```
x = input;
y = 0;
z = 0;
while (x>0) {
  assert(x>0);
  z = z+x;
  if (17>y) { assert(17>y); y = y+1; }
  else { assert(!(17>y)); }
  x = x-1;
}
assert(!(x>0));
```

preserves semantics since asserts are guarded by conditions

(alternatively, we could add dataflow constraints on the CFG *edges*)

Constraints for Assert

- A trivial but sound constraint:
  ⟦v⟧= *JOIN*(v)
- A non-trivial constraint for assert(*x*>*E*):
  ⟦v⟧=*JOIN*(v)[*x*→*gt*(*JOIN*(v)(*x*),*eval*(*JOIN*(v),*E*))]
  where
  $gt([l_1,h_1],[l_2,h_2]) = [l_1,h_1] \sqcap [l_2,\infty]$
- Similar constraints are defined for the dual cases
- More tricky to define for other conditions...

Exploiting Conditions

```
x = input;
y = 0;
z = 0;
while (x>0) {
  assert(x>0);
  z = z+x;
  if (17>y) { assert(17>y); y = y+1; }
  else { assert(!(17>y)); }
  x = x-1;
}
assert(!(x>0));
```

The interval analysis now concludes:
x= [-∞,0], y= [0,17], z= [0,∞]

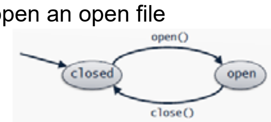
Branch Correlations

- With assert we have a simple form of *path sensitivity* (sometimes called *control sensitivity*)
- But it is insufficient to handle *correlation* of branches:

```
if (17 > x) { ... }
... // statements that do not change x
if (17 > x) { ... }
...
```


Open and Closed Files

- Built-in functions open() and close() on a file
- Requirements:
  - never close a closed file
  - never open an open file



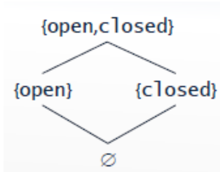
- We want a static analysis to check this...(for simplicity, let us assume there is only one file)

A Tricky Example

```
if (condition) {
  open();
  flag = 1;
} else {
  flag = 0;
}
...
if (flag) {
  close();
}
```


The Naive Analysis (1/2)

- The lattice models the status of the file:
  L = (2{open,closed},⊆)



- For every CFG node, v, we have a constraint variable ⟦v⟧ denoting the status *after* v

$$JOIN(v) = \bigcup_{w \in pred(v)} [\![w]\!]$$

The Naive Analysis(2/2)

- Constraints for interesting statements:
  ⟦*entry*⟧= {closed}
  ⟦open()⟧= {open}
  ⟦close()⟧= {closed}
- For all otherCFG nodes:
  ⟦v⟧= *JOIN*(v)
- Before the close() statement the analysis concludes that the file is {open,closed} ☹

```
if (condition) {
  open();
  flag = 1;
} else {
  flag = 0;
}
...
if (flag) {
  close();
}
```

The Slightly Less Naive Analysis

- We obviously need to keep track of the flag variable
- Our second attempt is the lattice:

$$L = (2^{\{open,closed\}} \times 2^{\{flag=0,flag\neq 0\}}, \subseteq \times \subseteq)$$

- Additionally, we add assert(...) to model conditionals
- Even so, we still only know that the file is {open,closed} and that flag is {flag=0,flag≠0} ☹

```
if (condition) {
  open();
  flag = 1;
} else {
  flag = 0;
}
...
if (flag) {
  close();
}
```

Enhanced Program

```
if (condition) {
  assert(condition);
  open();
  flag = 1;
} else {
  assert(!condition);
  flag = 0;
}
...
if (flag) {
  assert(flag);
  close();
} else {
  assert(!flag);
}
```


Relational Analysis

- We need an analysis that keeps track of *relations* between variables
- One approach is to maintain *multiple* abstract states per program point, one for each *path context*
- For the file example we need the lattice:

$L = \text{Paths} \rightarrow 2^{\{\texttt{open},\texttt{closed}\}}$ (note: isomorphic to $2^{\text{Paths}\times\{\texttt{open},\texttt{closed}\}}$)

Where Paths = {flag=0,flag≠0} is the set of path contexts

Relational Constraints(1/2)

- For the file statements:

  $[\![entry]\!] = \lambda p.\{\texttt{closed}\}$

  $[\![\texttt{open()}]\!] = \lambda p.\{\texttt{open}\}$

  $[\![\texttt{closed()}]\!] = \lambda p.\{\texttt{closed}\}$

- For `flag` assignments:

  "infeasible"

  $[\![\texttt{flag = 0}]\!] = [\texttt{flag=0} \rightarrow \bigcup_{p\in P} JOIN(v)(p),\ \texttt{flag}\neq\texttt{0} \rightarrow \varnothing]$

  $[\![\texttt{flag = } n]\!] = [\texttt{flag}\neq\texttt{0} \rightarrow \bigcup_{p\in P} JOIN(v)(p),\ \texttt{flag=0} \rightarrow \varnothing]$ where *n* is a non-0 constant number

  $[\![\texttt{flag = } E]\!] = \lambda q. \bigcup_{p\in P} JOIN(v)(p)$ for any other *E*

Relational Constraints(2/2)

- For `assert` statements:

  $[\![\texttt{assert(flag)}]\!] =$
  $[\texttt{flag}\neq\texttt{0} \rightarrow JOIN(v)(\texttt{flag}\neq\texttt{0}),\ \texttt{flag=0} \rightarrow \varnothing]$

  $[\![\texttt{assert(!flag)}]\!] =$
  $[\texttt{flag=0} \rightarrow JOIN(v)(\texttt{flag=0}),\ \texttt{flag}\neq\texttt{0} \rightarrow \varnothing]$

- For all other CFG nodes:

  $[\![v]\!] = \text{JOIN}(v) = \lambda p. \bigcup_{w\in pred(v)} [\![w]\!](p)$

Generated Constraints

$[\![entry]\!] = \lambda p.\{\texttt{closed}\}$
$[\![\texttt{condition}]\!] = [\![entry]\!]$
$[\![\texttt{assert(condition)}]\!] = [\![\texttt{condition}]\!]$
$[\![\texttt{open()}]\!] = \lambda p.\{\texttt{open}\}$
$[\![\texttt{flag = 1}]\!] = [\texttt{flag}\neq\texttt{0} \rightarrow \bigcup [\![\texttt{open()}]\!](p),\ \texttt{flag=0} \rightarrow \varnothing]$
$[\![\texttt{assert(!condition)}]\!] = [\![\texttt{condition}]\!]$
$[\![\texttt{flag = 0}]\!] = [\texttt{flag=0} \rightarrow \bigcup [\![\texttt{assert(!condition)}]\!](p),\ \texttt{flag}\neq\texttt{0} \rightarrow \varnothing]$
$[\![\texttt{...}]\!] = \lambda p.([\![\texttt{flag = 1}]\!](p) \cup [\![\texttt{flag = 0}]\!](p))$
$[\![\texttt{flag}]\!] = [\![\texttt{...}]\!]$
$[\![\texttt{assert(flag)}]\!] = [\texttt{flag}\neq\texttt{0} \rightarrow [\![\texttt{flag}]\!](\texttt{flag}\neq\texttt{0}),\ \texttt{flag=0} \rightarrow \varnothing]$
$[\![\texttt{close()}]\!] = \lambda p.\{\texttt{closed}\}$
$[\![\texttt{assert(!flag)}]\!] = [\texttt{flag=0} \rightarrow [\![\texttt{flag}]\!](\texttt{flag=0}),\ \texttt{flag}\neq\texttt{0} \rightarrow \varnothing]$
$[\![exit]\!] = \lambda p.([\![\texttt{close()}]\!](p) \cup [\![\texttt{assert(!flag)}]\!](p))$

Minimal Solution

	flag = 0	flag ≠ 0
⟦entry⟧	{closed}	{closed}
⟦condition⟧	{closed}	{closed}
⟦assert(condition)⟧	{closed}	{closed}
⟦open()⟧	{open}	{open}
⟦flag = 1⟧	∅	{open}
⟦assert(!condition)⟧	{closed}	{closed}
⟦flag = 0⟧	{closed}	∅
⟦...⟧	{closed}	{open}
⟦flag⟧	{closed}	{open}
⟦assert(flag)⟧	∅	{open}
⟦close()⟧	{closed}	{closed}
⟦assert(!flag)⟧	{closed}	∅
⟦exit⟧	{closed}	{closed}

We now know the file is `open` before `close()` ☺

Challenges

- The static analysis designer must choose Paths
  - Often as Boolean combinations of predicates from conditionals
  - iterative refinement (e.g. *counter-example guided abstraction refinement*) can be used for gradually finding relevant predicates
- Exponential blow-up:
  - for k predicates, we have 2^k different contexts
  - Redundancy often cuts this down
- Reasoning about assert:
  - how to update the lattice elements with sufficient precision?
  - Possibly involves heavy-weight theorem proving

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Improvements

- Run auxiliary analyses first, for example:
  - constant propagation
  - sign analysis

  will help in handling flag assignments
- Dead code propagation, change

  $[\![\text{open()}]\!] = \quad \text{p.}\{\text{open}\}$

  into the still sound but more precise

  $[\![\text{open()}]\!] = \lambda \text{p.if } \mathit{JOIN}(\text{v})(\text{p}) = \emptyset \text{ then } \emptyset \text{ else } \{\text{open}\}$

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