

## Pointer Analysis

Yu Zhang

Most content comes from <http://cs.au.dk/~amoeller/spa/>

1

## Agenda

- Introduction to points-to analysis
- Andersen's analysis
- Steensgaard's analysis
- Interprocedural points-to analysis
- Null pointer analysis
- Flow-sensitive points-to analysis

2

## Analyzing Programs with Pointers

How do we perform e.g. constant propagation analysis when the programming language has pointers? (or object references?)

```
...
*x = 42;
*y = -87;
z = *x;
// is z 42 or -87?
```

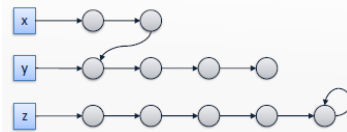
```
E → &X
   | alloc E
   | *E
   | null
   | ...
```

```
S → *X = E;
   | ...
```

3

## Heap Pointers

- For simplicity, we ignore records
  - alloc then only allocates a single cell
  - only linear structures can be built in the heap



- Let's at first also ignore functions as values
- We still have many interesting analysis challenges...

4

## Pointer Targets

- The fundamental question about pointers: *What cells can they point to?*
- We need a suitable abstraction
- The set of (abstract) cells, *Cells*, contains
  - alloc-*i* for each allocation site with index *i*
  - *X* for each program variable named *X*
- This is called **allocation site abstraction**
- Each abstract cell may correspond to many concrete memory cells at runtime

5

## Points-to Analysis

- Determine for each pointer variable *X* the set  $pt(X)$  of the cells *X* may point to
- A *conservative* ("may points-to") analysis:
  - the set may be too large
  - can show absence of aliasing:  $pt(X) \cap pt(Y) = \emptyset$
- We'll focus on *flow-insensitive* analyses:
  - Take place on the AST
  - Before or together with the control-flow analysis

```
...
*x = 42;
*y = -87;
z = *x;
// is z 42 or -87?
```

6

## Obtaining Points-to Information

- An almost-trivial analysis (called *address-taken*):
  - include all `alloc-i` cells 注: 为程序正文中的分配点
  - Include the `X` cell if the expression `&X` occurs in the program
- Improvement for a typed language:
  - Eliminate those cells whose types do not match
  - This is sometimes good enough
  - and clearly very fast to compute

7

## Pointer Normalization

- Assume that all pointer usage is normalized:
  - `X=alloc P` where `P` is null or an integer constant
  - `X=&Y`
  - `X=Y`
  - `X=*Y`
  - `*X=Y`
  - `X=null`
- Simply introduce lots of temporary variables...
- All sub-expressions are now named
- We choose to ignore the fact that the cells created at variable declarations are uninitialized

8

## Agenda

- Introduction to points-to analysis
- **Andersen's analysis**
- Steensgaards's analysis
- Interprocedural points-to analysis
- Null pointer analysis
- Flow-sensitive points-to analysis

9

## Andersen's Analysis (1/2)

- For every cell `c`, introduce a constraint variable  $\llbracket c \rrbracket$  ranging over sets of cells, i.e.  $\llbracket \cdot \rrbracket : Cells \rightarrow 2^{Cells}$
- Generate constraints:
  - `X=alloc P`:  $alloc-i \in \llbracket X \rrbracket$
  - `X=&Y`:  $Y \in \llbracket X \rrbracket$
  - `X=Y`:  $\llbracket Y \rrbracket \subseteq \llbracket X \rrbracket$
  - `X=*Y`:  $c \in \llbracket Y \rrbracket \Rightarrow \llbracket c \rrbracket \subseteq \llbracket X \rrbracket$  for each  $c \in Cells$
  - `*X=Y`:  $c \in \llbracket X \rrbracket \Rightarrow \llbracket Y \rrbracket \subseteq \llbracket c \rrbracket$  for each  $c \in Cells$
  - `X=null`: (no constraints)

基于集合的包含关系

10

## Andersen's Analysis (2/2)

- The points-to map is defined as:
 
$$pt(X) = \llbracket X \rrbracket$$
- The constraints fit into the cubic framework ☺
- Unique minimal solution in time  $O(n^3)$
- In practice, for Java:  $O(n^2)$
- The analysis is flow-insensitive but *directional*
  - models the direction of the flow of values in assignments

11

## Example Program

```
var p,q,x,y,z;
p = alloc null;
x = y;
x = z;
*p = z;
p = q;
q = &y;
x = *p;
p = &z;
```

$Cells = \{p, q, x, y, z, alloc-1\}$

12

### Applying Andersen

```
var p,q,x,y,z;
p = alloc null;
x = y;
x = z;
*p = z;
p = q;
q = &y;
x = *p;
p = &z;
```

```
alloc-1 ∈ [p]
[[y] ⊆ [x]
[[z] ⊆ [x]
c ∈ [p] ⇒ [z] ⊆ [α] for each c ∈ Cells
[[q] ⊆ [p]
y ∈ [q]
c ∈ [p] ⇒ [α] ⊆ [x] for each c ∈ Cells
z ∈ [p]
```

Smallest solution:  
 $pt(p) = \{ alloc-1, y, z \}$   
 $pt(q) = \{ y \}$

### Agenda

- Introduction to points-to analysis
- Andersen's analysis
- **Steensgaard's analysis**
- Interprocedural points-to analysis
- Null pointer analysis
- Flow-sensitive points-to analysis

### Steensgaard's Analysis

- View assignments as being bidirectional
- Generate constraints:
  - $X = alloc\ P:$   $alloc-i \in [X]$
  - $X = \&Y:$   $Y \in [X]$
  - $X = Y:$   $[X] = [Y]$
  - $X = *Y:$   $c \in [Y] \Rightarrow [c] = [X]$  for each  $c \in Cells$
  - $*X = Y:$   $c \in [X] \Rightarrow [Y] = [c]$  for each  $c \in Cells$
- Extra constraints:
  - $c_1, c_2 \in [c] \Rightarrow [c_1] = [c_2]$  and  $[c_1] \cap [c_2] \neq \emptyset \Rightarrow [c_1] = [c_2]$   
 (whenever a cell may point to two cells, they are essentially merged into one)
- Steensgaard's original formulation uses conditional unification for  $X = Y:$   
 $c \in [Y] \Rightarrow [X] = [Y]$  for each  $c \in Cells$  (avoids unifying if  $Y$  is never a pointer)

基于类型及其等价关系

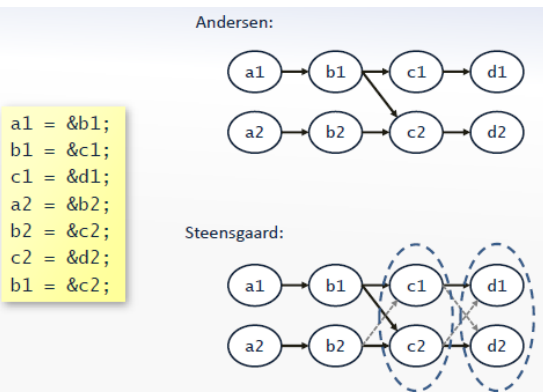
### Applying Steensgaard

```
var p,q,x,y,z;
p = alloc null;
x = y;
x = z;
*p = z;
p = q;
q = &y;
x = *p;
p = &z;
```

```
alloc-1 ∈ [p]
[[y] = [x]
[[z] = [x]
α ∈ [p] ⇒ [z] = [α]
[[q] = [p]
y ∈ [q]
α ∈ [p] ⇒ [α] = [x]
z ∈ [p]
+ the extra constraints
```

Smallest solution:  
 $pt(p) = \{ alloc-1, y, z \}$   
 $pt(q) = \{ alloc-1, y, z \}$

### Another Example



### Recall Our Type Analysis...

- Focusing on pointers...
- Constraints:
  - $X = alloc\ P:$   $[X] = \&[P]$
  - $X = \&Y:$   $[X] = \&[Y]$
  - $X = Y:$   $[X] = [Y]$
  - $X = *Y:$   $\&[X] = [Y]$
  - $*X = Y:$   $[X] = \&[Y]$
- Implicit extra constraint for term equality:  
 $\&t_1 = \&t_2 \Rightarrow t_1 = t_2$
- Assuming the program type checks, is the solution for pointers the same as for Steensgaard's analysis?

## Agenda

- Introduction to points-to analysis
- Andersen's analysis
- Steensgaard's analysis
- **Interprocedural points-to analysis**
- Null pointer analysis
- Flow-sensitive points-to analysis

19

## Interprocedural Points-to Analysis

- In TIP, function values and pointers may be mixed together:

$(\mathbf{**x})(1,2,3)$

- In this case the CFA and the points-to analysis must happen *simultaneously!*
- The idea: Treat function values as a kind of pointers

20

## Function Call Normalization

- Assume that all function calls are of the form  
 $x = y(a_1, \dots, a_n)$
- $y$  may be a variable whose value is a function pointer
- Assume that all return statements are of the form  
**return  $z$ ;**
- As usual, simply introduce lots of temporary variables...
- Include all function names in *Cells*

21

## CFA with Andersen

- For the function call  
 $x = y(a_1, \dots, a_n)$   
and every occurrence of  
 $f(x_1, \dots, x_n) \{ \dots \text{return } z; \}$   
add these constraints:

*Andersen's analysis is already closely connected to control-flow analysis!*

$$f \in \llbracket f \rrbracket$$

$$f \in \llbracket y \rrbracket \Rightarrow (\llbracket a_i \rrbracket \subseteq \llbracket x_i \rrbracket \text{ for } i=1, \dots, n \wedge \llbracket z \rrbracket \subseteq \llbracket x \rrbracket)$$

- (Similarly for simple function calls)
- Fits directly into the cubic framework!

22

## CFA with Steensgaard

- For the function call  
 $x = y(a_1, \dots, a_n)$   
and every occurrence of  
 $f(x_1, \dots, x_n) \{ \dots \text{return } z; \}$   
add these constraints:
- $$f \in \llbracket f \rrbracket$$
- $$f \in \llbracket y \rrbracket \Rightarrow (\llbracket a_i \rrbracket = \llbracket x_i \rrbracket \text{ for } i=1, \dots, n \wedge \llbracket z \rrbracket = \llbracket x \rrbracket)$$
- (Similarly for simple function calls)
  - Fits into the unification framework, but requires a generalization of the ordinary union-find solver

23

## Context-sensitive Pointer Analysis

- Generalize the abstract domain  $Cells \rightarrow 2^{Cells}$  to  
 $Contexts \rightarrow Cells \rightarrow 2^{Cells}$   
(or equivalently:  $Cells \times Contexts \rightarrow 2^{Cells}$ )  
where *Contexts* is a (finite) set of call contexts
- As usual, many possible choices of *Contexts*  
– recall the call string approach and the functional approach
- We can also track the set of reachable contexts for each function (like the use of lifted lattices earlier):  
 $Contexts \rightarrow \text{lift}(Cells \rightarrow 2^{Cells})$
- Does this still fit into the cubic solver?

24

## Context-sensitive Pointer Analysis

```
foo(a) {
  return *a;
}

bar() {
  ...
  x = alloc null; // alloc-1
  y = alloc null; // alloc-2
  *x = alloc null; // alloc-3
  *y = alloc null; // alloc-4
  ...
  q = foo(x);
  w = foo(y);
  ...
}
```

Are q and w aliases?

25

## Context-sensitive Pointer Analysis

```
mk() {
  return alloc null; // alloc-1
}

baz() {
  var x,y;
  x = mk();
  y = mk();
  ...
}
```

Are x and y aliases?

26

## Context-sensitive Pointer Analysis

- We can go one step further and introduce *context-sensitive heap* (a.k.a. *heap cloning*)
- Let each abstract cell be a pair of
  - alloc-*i* (the alloc with index *i*) or *X* (a program variable)
  - a **heap context from a (finite) set *HeapContexts***
- This allows abstract cells to be named by the source code allocation site **and (information from) the current context**
- One choice:
  - set *HeapContexts* = *Contexts*
  - at alloc, use the entire current call context as heap context

27

## Agenda

- Introduction to points-to analysis
- Andersen's analysis
- Steensgaard's analysis
- Interprocedural points-to analysis
- **Null pointer analysis**
- Flow-sensitive points-to analysis

28

## Null Pointer Analysis

- Decide for every dereference *\*p*, is *p* different from null?
- (Why not just treat null as a special cell in an Andersen or Steensgaard-style analysis?)
- Use the monotone framework
  - Assuming that a points-to map *pt* has been computed
- Let us consider an intraprocedural analysis (i.e. we ignore function calls)

29

## A Lattice for null Analysis

- Define the simple lattice *Null*:

$$\begin{array}{c} ? \\ | \\ NN \end{array}$$

where **NN** represents "definitely not null" and **?** represents "maybe null"

- Use for every program point the map lattice:

$$Cells \rightarrow Null$$

30

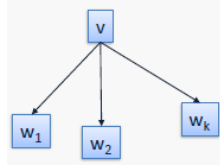
## Setting Up

- For every CFG node,  $v$ , we have a variable  $\llbracket v \rrbracket$ :
  - a map giving abstract values for all cells at the program point *after*  $v$

- Auxiliary definition:

$$JOIN(v) = \sqcup_{w \in pred(v)} \llbracket w \rrbracket$$

(i.e. we make a *forward* analysis)



31

## Null Analysis Constraints

- For operations involving pointers:
    - $X = \text{alloc } P$ :  $\llbracket v \rrbracket = ???$
    - $X = \&Y$ :  $\llbracket v \rrbracket = ???$
    - $X = Y$ :  $\llbracket v \rrbracket = ???$
    - $X = *Y$ :  $\llbracket v \rrbracket = ???$
    - $*X = Y$ :  $\llbracket v \rrbracket = ???$
    - $X = \text{null}$ :  $\llbracket v \rrbracket = ???$
- where  $P$  is null or an integer constant
- For all other CFG nodes:
    - $\llbracket v \rrbracket = JOIN(v)$

32

## Null Analysis Constraints

- For a heap store operation  $*X = Y$  we need to model the change of whatever  $X$  points to
- That may be *multiple* abstract cells (i.e. the cells  $pt(X)$ )
- With the present abstraction, each abstract heap cell  $\text{alloc-}i$  may describe *multiple* concrete cells
- So we settle for **weak** update:

$$*X = Y: \quad \llbracket v \rrbracket = \text{store}(JOIN(v), X, Y)$$

where  $\text{store}(\sigma, X, Y) = \sigma[\alpha \mapsto \sigma(\alpha) \sqcup \sigma(Y)]_{\alpha \in pt(X)}$

33

## Null Analysis Constraints

- For a heap load operation  $X = *Y$  we need to model the change of the program variable  $X$
- Our abstraction has a *single* abstract cell for  $X$
- That abstract cell represents a *single* concrete cell
- So we can use **strong** update:

$$X = *Y: \quad \llbracket v \rrbracket = \text{load}(JOIN(v), X, Y)$$

where  $\text{load}(\sigma, X, Y) = \sigma[X \mapsto \sqcup_{\alpha \in pt(Y)} \sigma(\alpha)]$

34

## Strong and Weak Updates

```

mk() {
  return alloc null; // alloc-1
}

...
a = mk();
b = mk();
*a = alloc null; // alloc-2
n = null;
*b = n; // strong update here would be unsound!
c = *a;
  
```

is C null here?

The abstract cell  $\text{alloc-}1$  corresponds to *multiple* concrete cells

35

## Strong and Weak Updates

```

a = alloc null; // alloc-1
b = alloc null; // alloc-2
*a = alloc null; // alloc-3
*b = alloc null; // alloc-4
if (...) {
  x = a;
} else {
  x = b;
}
n = null;
*x = n; // strong update here would be unsound!
c = *x;
  
```

is C null here?

The points-to set for  $x$  contains *multiple* abstract cells

36

### Null Analysis Constraints

- $X = \text{alloc } P$ :  $\llbracket v \rrbracket = \text{JOIN}(v)[X \mapsto \text{NN}, \text{alloc-}i \mapsto ?]$
- $X = \&Y$ :  $\llbracket v \rrbracket = \text{JOIN}(v)[X \mapsto \text{NN}]$
- $X = Y$ :  $\llbracket v \rrbracket = \text{JOIN}(v)[X \mapsto \text{JOIN}(v)(Y)]$
- $X = \text{null}$ :  $\llbracket v \rrbracket = \text{JOIN}(v)[X \mapsto ?]$

could be improved...

- In each case, the assignment modifies a program variable
- So we can use strong updates, as for heap load operations

37

### Strong and Weak Updates, Revisited

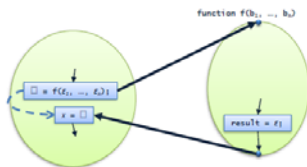
- Strong update:  $\sigma[c \mapsto \text{new-value}]$ 
  - possible if  $c$  is known to refer to a **single** concrete cell
  - works for assignments to local variables (as long as TIP doesn't have e.g. nested functions)
- Weak update:  $\sigma[c \mapsto \sigma(c) \sqcup \text{new-value}]$ 
  - necessary if  $c$  may refer to **multiple** concrete cells
  - bad for precision, we lose some of the power of flow-sensitivity
  - required for assignments to heap cells (unless we extend the analysis abstraction!)

38

### Interprocedural Null Analysis

- Context insensitive or context sensitive, as usual...
  - at the after-call node, use the heap from the callee
- But be careful! *Pointers to local variables may escape to the callee*
  - the abstract state at the after-call node cannot simply copy the abstract values for local variables from the abstract state

Escape Analysis  
逃逸分析: 分析对象是否逃逸出一个函数



39

### Using the Null Analysis

- The pointer dereference  $*p$  is "safe" at entry of  $v$  if  $\text{JOIN}(v)(p) = \text{NN}$
- The quality of the null analysis depends on the quality of the underlying points-to analysis

40

### Example Program & Constraints

```
p = alloc null;
q = &p;
n = null;
*q = n;
*p = n;
```

Andersen generates:  
 $pt(p) = \{\text{alloc-}1\}$   
 $pt(q) = \{p\}$   
 $pt(n) = \emptyset$

```
\llbracket p=alloc null \rrbracket = \perp[p \mapsto \text{NN}, \text{alloc-}1 \mapsto ?]
\llbracket q=\&p \rrbracket = \llbracket p=alloc null \rrbracket[q \mapsto \text{NN}]
\llbracket n=null \rrbracket = \llbracket q=\&p \rrbracket[n \mapsto ?]
\llbracket *q=n \rrbracket = \llbracket n=null \rrbracket[p \mapsto \llbracket n=null \rrbracket(p) \sqcup \llbracket n=null \rrbracket(n)]
\llbracket *p=n \rrbracket = \llbracket *q=n \rrbracket[\text{alloc-}1 \mapsto \llbracket *q=n \rrbracket(\text{alloc-}1) \sqcup \llbracket *q=n \rrbracket(n)]
```

41

### Solution

```
\llbracket p=alloc null \rrbracket = [p \mapsto \text{NN}, q \mapsto \text{NN}, n \mapsto \text{NN}, \text{alloc-}1 \mapsto ?]
\llbracket q=\&p \rrbracket = [p \mapsto \text{NN}, q \mapsto \text{NN}, n \mapsto \text{NN}, \text{alloc-}1 \mapsto ?]
\llbracket n=null \rrbracket = [p \mapsto \text{NN}, q \mapsto \text{NN}, n \mapsto ?, \text{alloc-}1 \mapsto ?]
\llbracket *q=n \rrbracket = [p \mapsto ?, q \mapsto \text{NN}, n \mapsto ?, \text{alloc-}1 \mapsto ?]
\llbracket *p=n \rrbracket = [p \mapsto ?, q \mapsto \text{NN}, n \mapsto ?, \text{alloc-}1 \mapsto ?]
```

- At the program point before the statement  $*q=n$  the analysis now knows that  $q$  is definitely non-null
- ... and before  $*p=n$ , the pointer  $p$  is may be null
- Due to the weak updates for all heap store operations, precision is bad for alloc- $i$  cells

42

## Agenda

- Introduction to points-to analysis
- Andersen's analysis
- Steensgaard's analysis
- Interprocedural points-to analysis
- Null pointer analysis
- **Flow-sensitive points-to analysis**

43

## Points-to Graphs

- Graphs that describe possible heaps:
  - nodes are abstract cells
  - edges are possible pointers between the cells
- The lattice of points-to graphs is  $2^{Cells \times Cells}$  ordered under subset inclusion (or alternatively,  $Cells \rightarrow 2^{Cells}$ )
- For every CFG node,  $v$ , we introduce a constraint variable  $\llbracket v \rrbracket$  describing the state *after*  $v$
- Intraprocedural analysis (i.e. ignore function calls)

44

## Constraints

- For pointer operations:
  - $X = \text{alloc } P$ :  $\llbracket v \rrbracket = JOIN(v) \downarrow X \cup \{(X, \text{alloc-}i)\}$
  - $X = \&Y$ :  $\llbracket v \rrbracket = JOIN(v) \downarrow X \cup \{(X, Y)\}$
  - $X = Y$ :  $\llbracket v \rrbracket = JOIN(v) \downarrow X \cup \{(X, t) \mid (Y, t) \in JOIN(v)\}$
  - $X = *Y$ :  $\llbracket v \rrbracket = JOIN(v) \downarrow X \cup \{(X, t) \mid (Y, s) \in \sigma, (s, t) \in JOIN(v)\}$
  - $*X = Y$ :  $\llbracket v \rrbracket = JOIN(v) \cup \{(s, t) \mid (X, s) \in JOIN(v), (Y, t) \in JOIN(v)\}$
  - $X = \text{null}$ :  $\llbracket v \rrbracket = JOIN(v) \downarrow X$  note: weak update!

where  $\sigma \downarrow X = \{(s, t) \in \sigma \mid s \neq X\}$

$JOIN(v) = \bigcup_{w \in pred(v)} \llbracket w \rrbracket$
- For all other CFG nodes:
  - $\llbracket v \rrbracket = JOIN(v)$

45

## Example Program

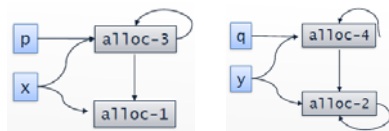
```

var x,y,n,p,q;
x = alloc null; y = alloc null;
*x = null; *y = y;
n = input;
while (n>0) {
    p = alloc null; q = alloc null;
    *p = x; *q = y;
    x = p; y = q;
    n = n-1;
}
    
```

46

## Result of Analysis

- After the loop we have this points-to graph:



- We conclude that  $x$  and  $y$  will always be disjoint

```

var x,y,n,p,q;
x = alloc null; y = alloc null;
*x = null; *y = y;
n = input;
while (n>0) {
    p = alloc null; q = alloc null;
    *p = x; *q = y;
    x = p; y = q;
    n = n-1;
}
    
```

48

## Points-to Maps from Points-to Graphs

- A points-to map for each program point  $v$ :

$$pt(X) = \{ t \mid (X, t) \in \llbracket v \rrbracket \}$$

- More expensive, but more precise:

- Andersen:  $pt(x) = \{ y, z \}$
- flow-sensitive:  $pt(x) = \{ z \}$

```

x = &y;
x = &z;
    
```

48



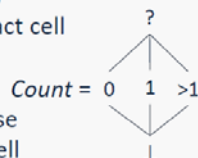
## Improving Precision with Abstract Counting

- The points-to graph is missing information:
  - alloc-2 nodes always form a self-loop in the example

- We need a more detailed lattice:

$$2^{Cell \times Cell} \times (Cell \rightarrow Count)$$

where we for each cell keep track of how many concrete cells that abstract cell describes

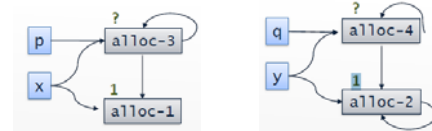


- This permits **strong updates** on those that describe precisely 1 concrete cell

49

## Constraints and Better Results

- $X = \text{alloc } P: \dots$
- $*X = Y: \dots$
- ...
- After the loop we have this extended points-to graph:



- Thus, alloc-2 nodes form a self-loop

50

## Escape Analysis

- Perform a points-to analysis
- Look at return expression
- Check reachability in the points-to graph to arguments or variables defined in the function itself
- None of those
  - ↓
  - no escaping stack cells

```
baz() {
  var x;
  return &x;
}

main() {
  var p;
  p=baz();
  *p=1;
  return *p;
}
```

51