

# Subtyping

(Dynamic Polymorphism)

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# References

- [PFPL](#)
  - Chapter 24 Structural Subtyping
  - Chapter 27 Inheritance
- [TAPL \(pdf\)](#)
  - Chapter 15 Subtyping
- [[Concepts in PLs](#)]

# Recap: Subtyping and Inheritance

- Interface
  - The external view of an object
- Subtyping
  - Relation between interfaces
- Implementation
  - The internal representation of an object
- Inheritance
  - Relation between implementations

# Various Object-Oriented Languages

- Pure dynamically-typed OO languages
  - Object implementation and run-time lookup
  - Class-based languages (Smalltalk)
  - Prototype-based languages (Self, JavaScript)
- Statically-typed OO languages
  - C++
    - using static typing to eliminate search
    - problems with C++ multiple inheritance
  - Java
    - using Interfaces to avoid multiple inheritance

# Smalltalk: Subtyping

- If interface **A** contains all of interface **B**, then **A** objects can also be used **B** objects.

Point	ColorPoint
x:y:	x:y:
moveDx:Dy:	moveDx:Dy:
x	x
y	y
draw	color
	draw

ColorPoint interface contains Point  
ColorPoint is a subtype of Point

# Subtyping and Inheritance

- Smalltalk/JavaScript subtyping is implicit
  - Not a part of the programming language
  - Important aspect of how systems are built
- Inheritance is explicit
  - Used to implement systems
  - No forced relationship to subtyping

# C++

- C++ is an object-oriented extension of C, Bell Labs
- Object-oriented features
  - Classes
  - Objects, with dynamic lookup of virtual functions
  - Inheritance
    - Single and multiple inheritance
    - Public and private base classes
  - Subtyping
    - Tied to **inheritance** mechanism
  - Encapsulation
    - Public, private, protected visibility

# C++: Virtual functions

- Member functions are either
  - Virtual, if explicitly declared or inherited as virtual
  - Non-virtual otherwise
- Virtual functions
  - Accessed by indirection through ptr in object
  - May be redefined in derived (sub) classes
- Non-virtual functions
  - Are called in the usual way. *Just ordinary functions.*
  - Cannot redefine in derived classes (except overloading)
- Pay overhead only if you use virtual functions



# C++ Subtyping

- Subtyping in principle
  - $A <: B$  if every  $A$  object can be used without type error whenever a  $B$  object is required
- C++:  $A <: B$  if class  $A$  has public base class  $B$ 
  - Independent classes not subtypes

# Java

- 1990-95 James Gosling and others at Sun
- Syntax similar to C++
- Object
  - has fields and methods
  - is allocated on heap, not run-time stack
  - accessible through reference (only ptr assignment)
  - garbage collected
- Dynamic lookup
  - Similar in behavior to other languages
  - Static typing => more efficient than Smalltalk
  - Dynamic linking, interfaces => slower than C++

# Inheritance

- Similar to Smalltalk, C++
- Subclass inherits from superclass
  - Single inheritance only (but Java has interfaces)
- Some additional features
  - Conventions regarding *super* in constructor and *finalize* methods
  - Final classes and methods cannot be redefined

# Interfaces vs Multiple Inheritance

- C++ multiple inheritance
  - A single class may inherit from two base classes
  - Constraints of C++ require derived class representation to resemble *all* base classes
- Java interfaces
  - A single class may implement two interfaces
  - No inheritance (of implementation) involved
  - Java implementation (discussed later) does not require similarity between class representations

# Subtyping Principles

- Subtyping judgement  $\tau' <: \tau$

-

$$\overline{\tau <: \tau}$$

$$\frac{\tau'' <: \tau' \quad \tau' <: \tau}{\tau'' <: \tau}$$

- Subsumption rule

$$\frac{\Gamma \vdash e : \tau' \quad \tau' <: \tau}{\Gamma \vdash e : \tau}$$

- Numeric types

-  $\text{int} <: \text{rat} <: \text{real}$

- Product types, Sum types

$$\frac{J \subseteq I}{\langle \tau_i \rangle_{i \in I} <: \langle \tau_j \rangle_{j \in J}}$$

$$\frac{J \subseteq I}{[\tau_i]_{i \in I} <: [\tau_j]_{j \in J}}$$

Width subtyping

(较宽积类型是较窄积类型的子类型)

# Subtyping Principles

- Variance:

- Product and sum types: Depth subtyping (Covariance)

$$\frac{\tau'_i <: \tau_i (\forall i \in I)}{\langle \tau'_i \rangle_{i \in I} <: \langle \tau_i \rangle_{i \in I}}$$

$$\frac{\tau'_i <: \tau_i (\forall i \in I)}{[\tau'_i]_{i \in I} <: [\tau_i]_{i \in I}}$$

- Partial function types

- covariant in its range.

$$\frac{\tau'_2 <: \tau_2}{\tau_1 \rightarrow \tau'_2 <: \tau_1 \rightarrow \tau_2}$$

- contravariant in its domain position

$$\frac{\tau'_1 <: \tau_1}{\tau_1 \rightarrow \tau_2 <: \tau'_1 \rightarrow \tau_2}$$

# Subtyping Principles

- Quantified Types

$$\frac{\Delta, t \text{ type} \vdash \tau' <: \tau}{\Delta \vdash \forall(t. \tau') <: \forall(t. \tau)}$$

$$\frac{\Delta, t \text{ type} \vdash \tau' <: \tau}{\Delta \vdash \exists(t. \tau') <: \exists(t. \tau)}$$

- Substitution: If  $\Delta, t \text{ type} \vdash \tau_1 <: \tau_2$ , and  $\Delta \vdash \tau \text{ type}$ , then  $\Delta \vdash [\tau/t]\tau_1 <: [\tau/t]\tau_2$