第1章 电力与电场

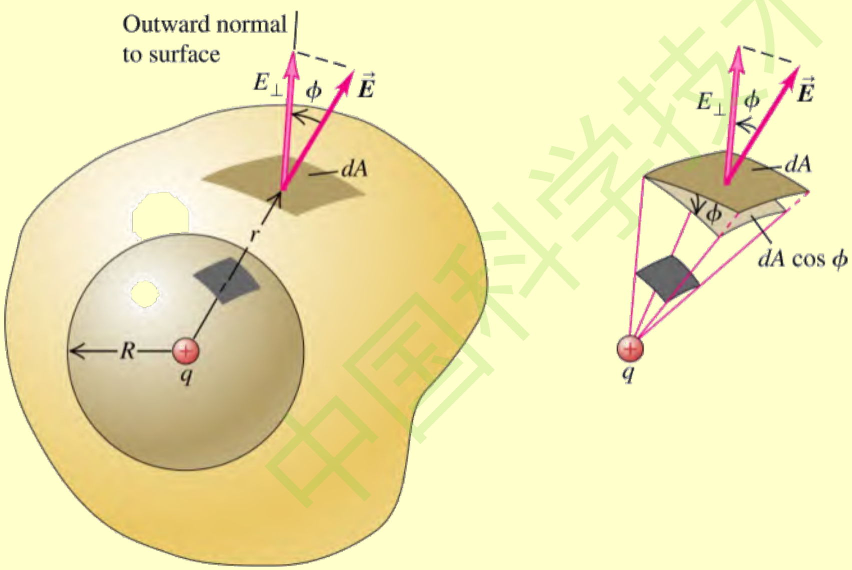
§1.1 电力起源
§1.2 库仑定律
§1.3 电场强度
§1.4 高斯定理
§1.5 环路定理

高斯定理 VS. 环路定理

高斯定理

$$\oiint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\iiint_V \rho dV$$

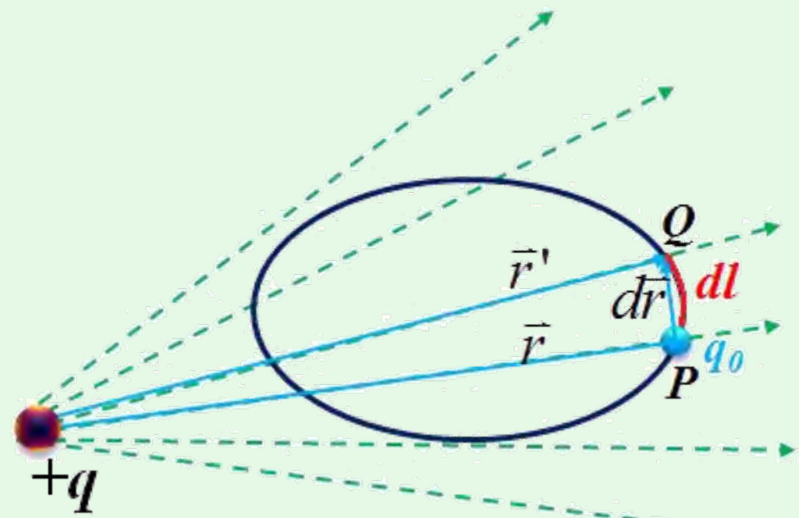
$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$



环路定理

$$\oint_L \vec{E}\cdot d\vec{l} = 0$$

$$\nabla\times\vec{E} = 0$$

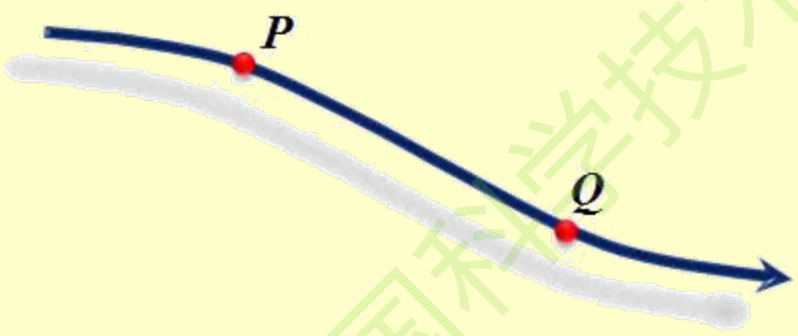


电势与电场

电场 → 电势

$$\vec{E}\,(\vec{r}) = \int d\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{(\vec{r}-\vec{r}\,')dq}{|\vec{r}-\vec{r}\,'|^3}$$

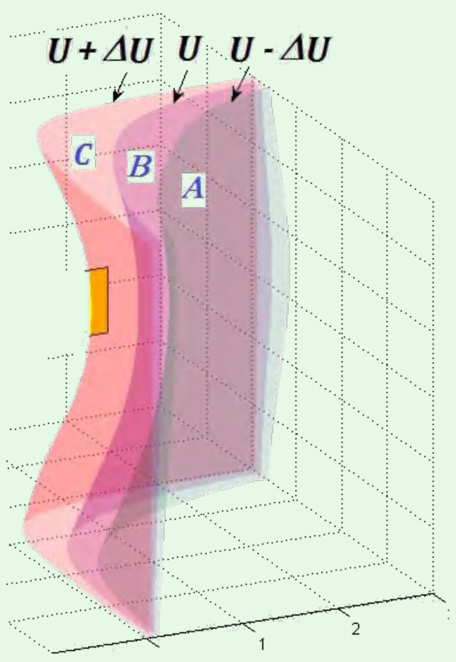
$$\boldsymbol{U_{PQ} = \int_P^Q \vec{E}\cdot d\vec{l}}$$



电势 → 电场

$$U\,(\vec{r}) = \int dU = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{|\vec{r}-\vec{r}\,'|}$$

$$\boldsymbol{\vec{E} = -\nabla U}$$



$$\vec{E} = -\frac{\Delta U}{\Delta n}\vec{n} = -\nabla U$$

静电场中任何一点的电场强度，其大小等于该点电势梯度的大小，方向与电势梯度的方向相反，即电势减小的方向

直角坐标：

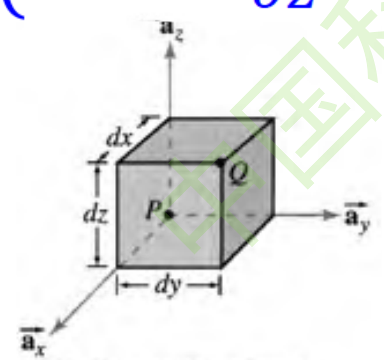
$$\begin{cases} E_x = -\dfrac{\partial U}{\partial x} \\ E_y = -\dfrac{\partial U}{\partial y} \\ E_z = -\dfrac{\partial U}{\partial z} \end{cases}$$

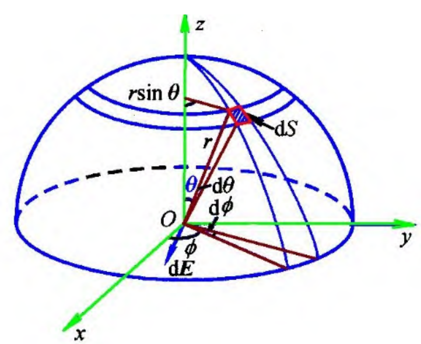
球坐标：

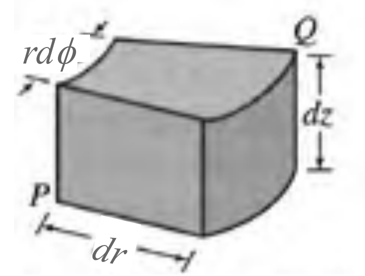
$$\begin{cases} E_r = -\dfrac{\partial U}{\partial r} \\ E_\theta = -\dfrac{1}{r}\dfrac{\partial U}{\partial \theta} \\ E_\varphi = -\dfrac{1}{r\sin\theta}\dfrac{\partial U}{\partial \varphi} \end{cases}$$

柱坐标：

$$\begin{cases} E_r = -\dfrac{\partial U}{\partial r} \\ E_\varphi = -\dfrac{1}{r}\dfrac{\partial U}{\partial \varphi} \\ E_z = -\dfrac{\partial U}{\partial z} \end{cases}$$

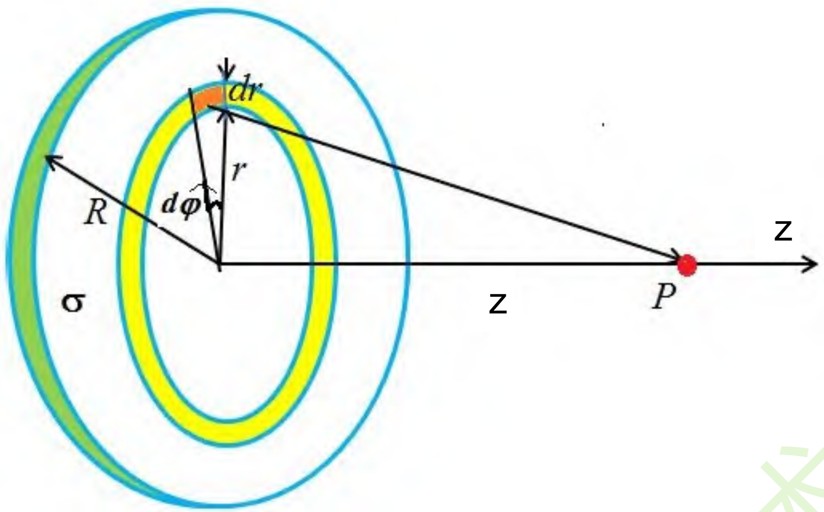






电势与电场求解例(2)

求均匀带电圆盘轴线上的电场分布



$$U(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iint_{S'} \frac{\sigma(\vec{r}')dS'}{|\vec{r}-\vec{r}'|}$$

$$= \frac{1}{4\pi\varepsilon_0}\iint_{S} \frac{\sigma dS}{\sqrt{z^2+r^2}}$$

$$dS = rdrd\varphi$$

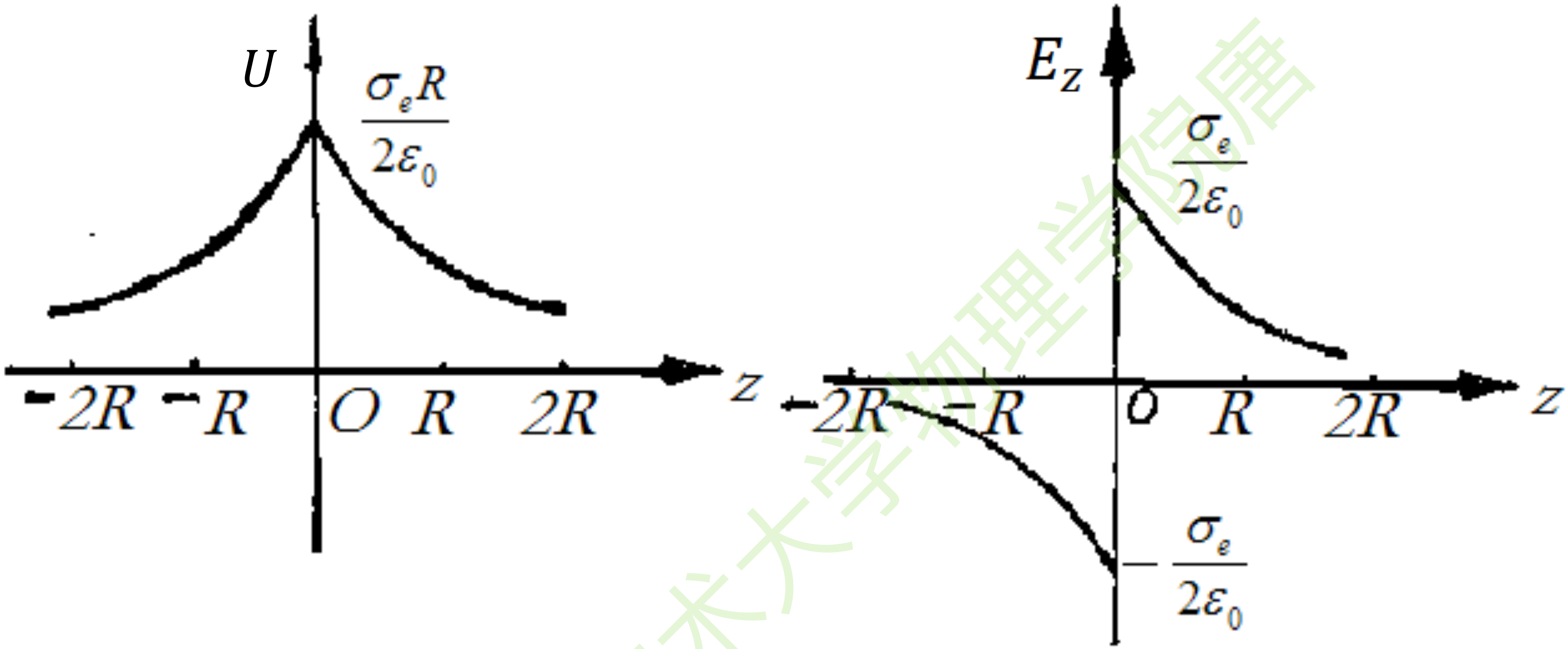
$$U(\vec{r}) = \frac{\sigma}{4\pi\varepsilon_0}\iint_{S} \frac{dS}{\sqrt{z^2+r^2}} = \frac{\sigma}{4\pi\varepsilon_0}\int_0^R \frac{rdr}{\sqrt{z^2+r^2}}\int_0^{2\pi} d\varphi$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2}-|z|\right)$$

$$U(\vec{r}) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - |z|\right) = \frac{\sigma}{2\varepsilon_0}\begin{cases}\sqrt{z^2+R^2} - z & z>0\\ \sqrt{z^2+R^2} + z & z<0\end{cases}$$

$$\begin{cases} E_r = -\dfrac{\partial U}{\partial r} = 0 \\ E_\varphi = -\dfrac{1}{r}\dfrac{\partial U}{\partial \varphi} = 0 \\ E_z = -\dfrac{\partial U}{\partial z} = \begin{cases} \dfrac{\sigma}{2\varepsilon_0}\left(1 - \dfrac{z}{\sqrt{z^2+R^2}}\right) & z>0 \\ -\dfrac{\sigma}{2\varepsilon_0}\left(1 + \dfrac{z}{\sqrt{z^2+R^2}}\right) & z<0 \end{cases} \end{cases}$$

$$E_z \xrightarrow{R\to\infty} \begin{cases} \dfrac{\sigma}{2\varepsilon_0} & z>0 \\ -\dfrac{\sigma}{2\varepsilon_0} & z<0 \end{cases} \qquad E_z \xrightarrow{z\to 0} \begin{cases} \dfrac{\sigma}{2\varepsilon_0} & z>0 \\ -\dfrac{\sigma}{2\varepsilon_0} & z<0 \end{cases}$$

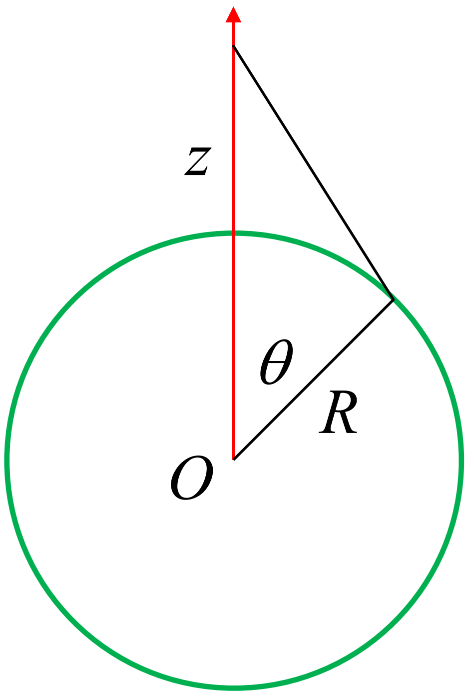


$$\frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2}-|z|\right) \qquad \pm\frac{\sigma}{2\varepsilon_0}\left(1-\frac{|z|}{\sqrt{R^2+z^2}}\right)$$

均匀带电圆盘轴线上的电势和电场强度的分布

电势与电场求解例(3)

求均匀带电球壳层产生的电势分布



$$U(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iint_{S'} \frac{\sigma(\vec{r}')dS'}{|\vec{r}-\vec{r}'|}$$

$$U = \frac{\sigma}{4\pi\varepsilon_0}\iint_S \frac{R^2 \sin\theta\, d\theta d\phi}{\sqrt{R^2+z^2-2Rz\cos\theta}}$$

$$= \frac{\sigma R^2}{2\varepsilon_0}\int_0^\pi \frac{\sin\theta\, d\theta}{\sqrt{R^2+z^2-2Rz\cos\theta}}$$

$$= \frac{\sigma R^2}{2\varepsilon_0}\frac{\sqrt{R^2+z^2-2Rz\cos\theta}}{Rz}\Bigg|_{\theta=0}^{\theta=\pi}$$

$$U=\frac{\sigma R^2}{2\varepsilon_0}\frac{\sqrt{R^2+z^2-2Rz\cos\theta}}{Rz}\Bigg|_{\theta=0}^{\theta=\pi}$$

$$=\frac{\sigma R^2}{2\varepsilon_0}\frac{|R+z|-|R-z|}{Rz}$$

$$=\begin{cases}\dfrac{\sigma R^2}{\varepsilon_0 z} & z>R\\ \dfrac{\sigma R}{\varepsilon_0} & z\le R\end{cases}\xrightarrow{r\equiv z,\sigma=\frac{Q}{4\pi R^2}}\begin{cases}\dfrac{Q}{4\pi\varepsilon_0 r} & r>R\\ \dfrac{Q}{4\pi\varepsilon_0 R} & r\le R\end{cases}$$

$$E_r=-\frac{\partial U}{\partial r}=\begin{cases}\dfrac{Q}{4\pi\varepsilon_0 r^2} & r>R\\ 0 & r\le R\end{cases}\qquad E_\theta=E_\phi=0$$

解II：

由高斯定理可得：

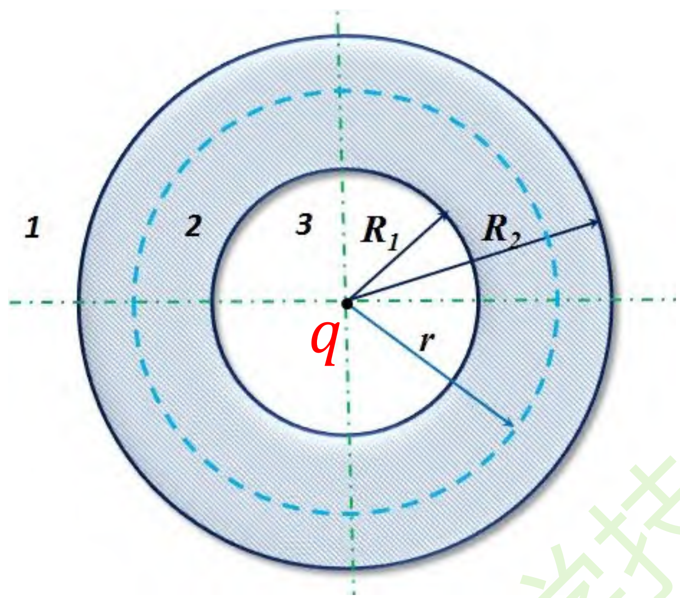
$$E_r = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2} & r > R \\ 0 & r \leq R \end{cases} \qquad E_\theta = E_\phi = 0$$

$$U(r) = \int_r^\infty \vec{E} \cdot d\vec{l} = \int_\infty^r E_r dr$$

$$= \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r} & r > R \\ \dfrac{Q}{4\pi\varepsilon_0 R} & r \leq R \end{cases}$$

电势与电场求解例(4)

一个导体球壳，内外半径分别为R_1和R_2，在球心放置电量为q的点电荷，求其电势和电场分布



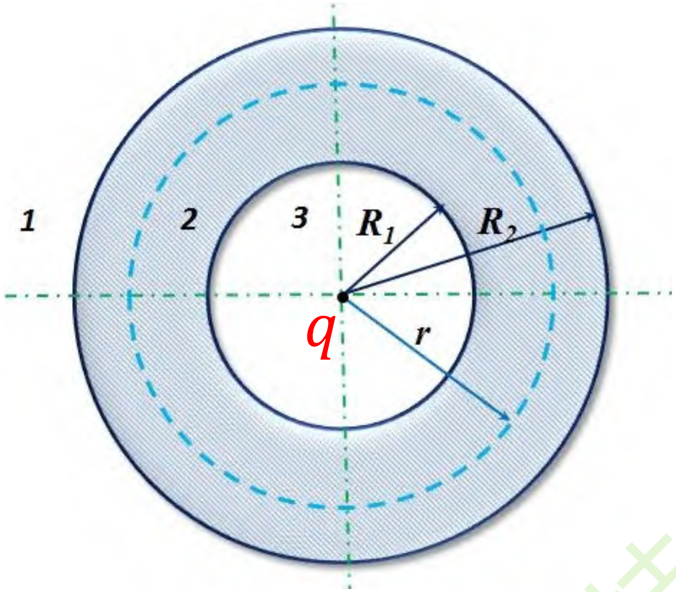
先用高斯定理求电场

$$\vec{E}=\begin{cases}\dfrac{q}{4\pi\varepsilon_0 r^2} & r<R_1\\ 0 & R_1<r<R_2\\ \dfrac{q}{4\pi\varepsilon_0 r^2} & r>R_2\end{cases}$$

取无穷远处为电势零点

$$U=\begin{cases}\dfrac{q}{4\pi\varepsilon_0}\left(\dfrac{1}{r}-\dfrac{1}{R_1}+\dfrac{1}{R_2}\right) & r<R_1\\ \dfrac{q}{4\pi\varepsilon_0 R_2} & R_1<r<R_2\\ \dfrac{q}{4\pi\varepsilon_0 r} & r>R_2\end{cases}$$

一个导体球壳，内外半径分别为R_1和R_2，在球心放置电量为q的点电荷，求其电势和电场分布



点电荷产生的电势为：

$$U_1(r) = \frac{q}{4\pi\varepsilon_0 r}$$

导体球壳不带电，产生的电势为：

$$U_2(r) = 0$$

根据电势叠加原理：$U(r) = U_1(r) + U_2(r) = \frac{q}{4\pi\varepsilon_0 r}$

与上页结果不符，为何?

求电场、电势分布

1. 从库仑定律出发

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0|\vec{r}-\vec{r}'|^3}(\vec{r}-\vec{r}')$$

$$\vec{E} = \int d\vec{E}$$

$$U_{PQ} = \int_P^Q \vec{E}\cdot d\vec{l}$$

2. 从高斯定理出发 （常用于有对称性的带电体）

$$\oiint_S \vec{E}\cdot d\vec{S} = \iiint_V \nabla\cdot\vec{E}\,dV = \frac{1}{\varepsilon_0}\iiint_V \rho\,dV$$

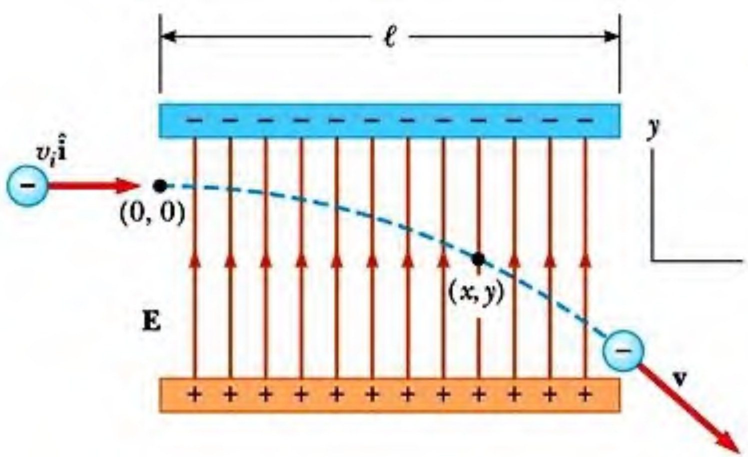
$$U_{PQ} = \int_P^Q \vec{E}\cdot d\vec{l}$$

3. 从电势出发（常用于点电荷体系或不太对称的带电体）

$$U(\vec{r}) = \int dU = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{|\vec{r}-\vec{r}'|}$$

$$\vec{E} = -\nabla U$$

§ 1.5.6 带电粒子在电场中的运动



$$m\frac{d\vec{v}(\vec{r})}{dt}=q\vec{E}(\vec{r})$$

匀强电场:

$$m\frac{d^2\vec{x}(\vec{r})}{dt^2}=qE$$

当$x(0)=0$， $v(0)=0$

$$x=\frac{qE}{2m}t^2$$

任意电场:

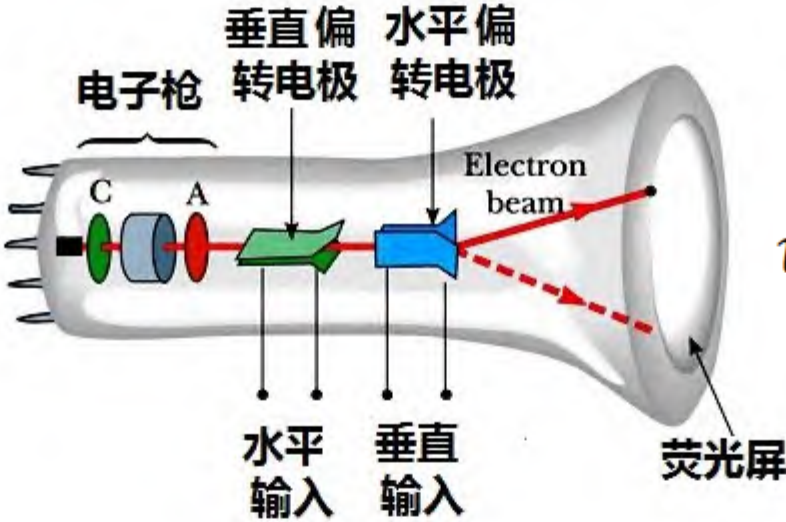
$$m\frac{d\vec{v}(\vec{r})}{dt}\cdot d\vec{l}=q\vec{E}(\vec{r})\cdot d\vec{l}$$

$$md\vec{v}(\vec{r})\cdot\frac{d\vec{l}}{dt}=q\vec{E}(\vec{r})\cdot d\vec{l}$$

$$\Delta E_k=-q\Delta U$$

$$v_2=\sqrt{v_1^2+\frac{2}{m}q(U_1-U_2)}$$

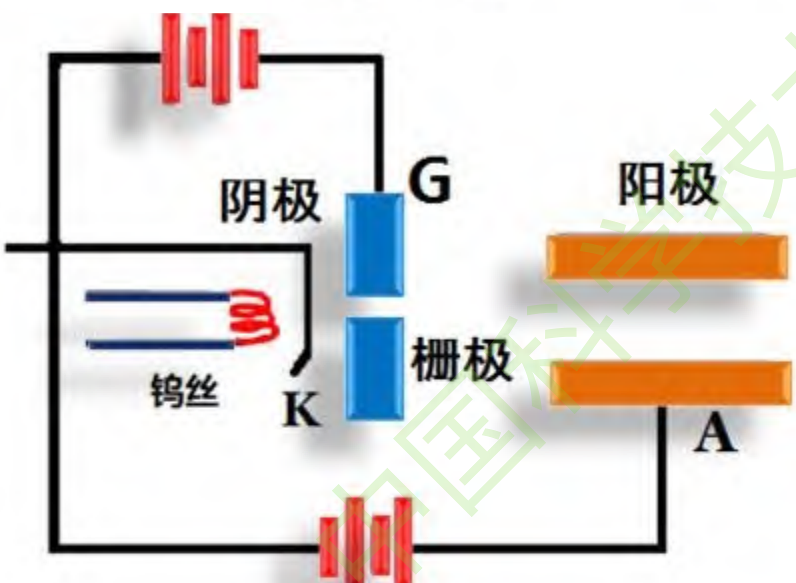
显像管原理



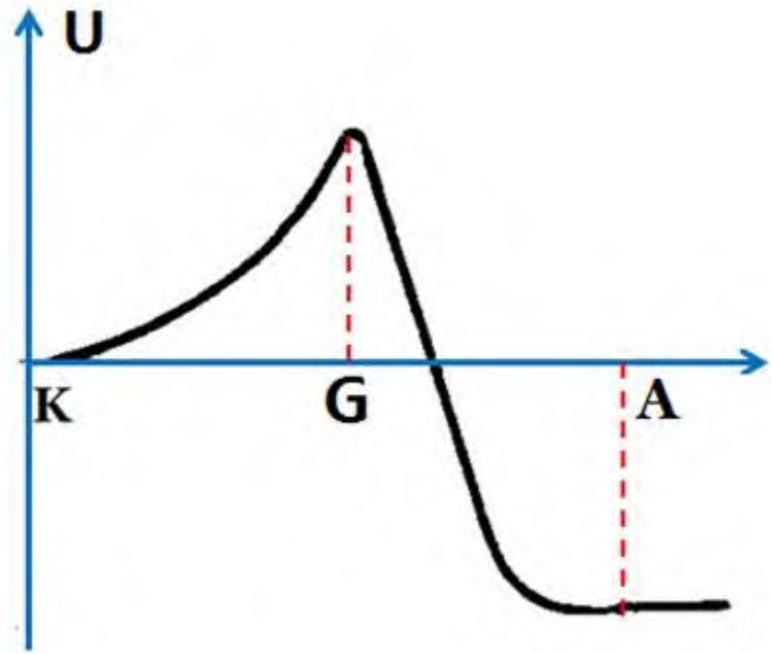
电子枪：小型加速器

$$v_0 = \sqrt{\frac{2e}{m} U_A} \xrightarrow{U_A=10000V} 6 \times 10^7 \mathrm{m/s}$$

0.2倍光速！

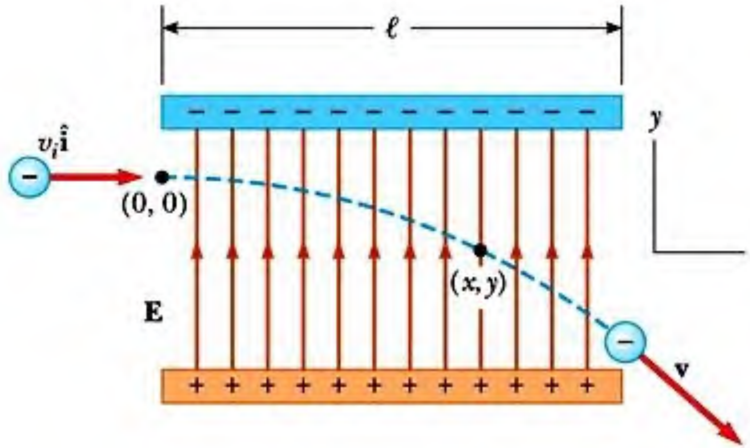


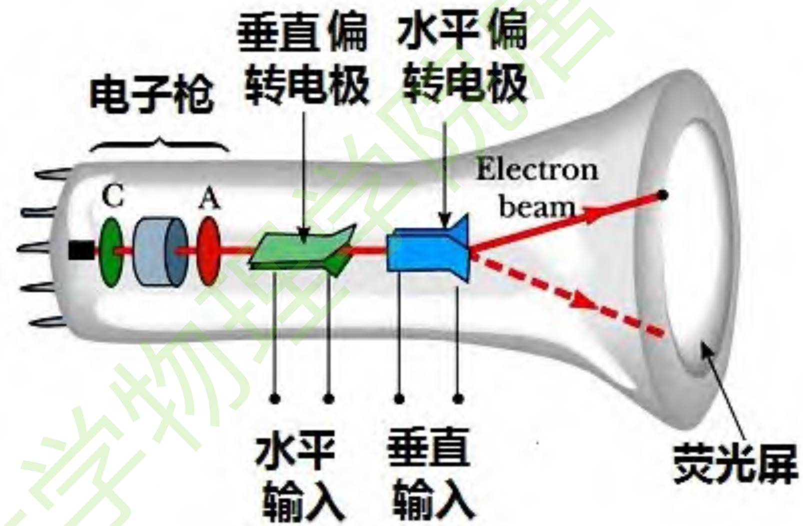
(a)



(b)

偏转电极





- 电子枪加速电子到一定速度v_0（垂直屏幕方向）
- 两个偏转电极控制电子束落在显示屏的不同位置

$$x = \frac{qE_x}{2m}t^2$$

$$y = \frac{qE_y}{2m}t^2$$

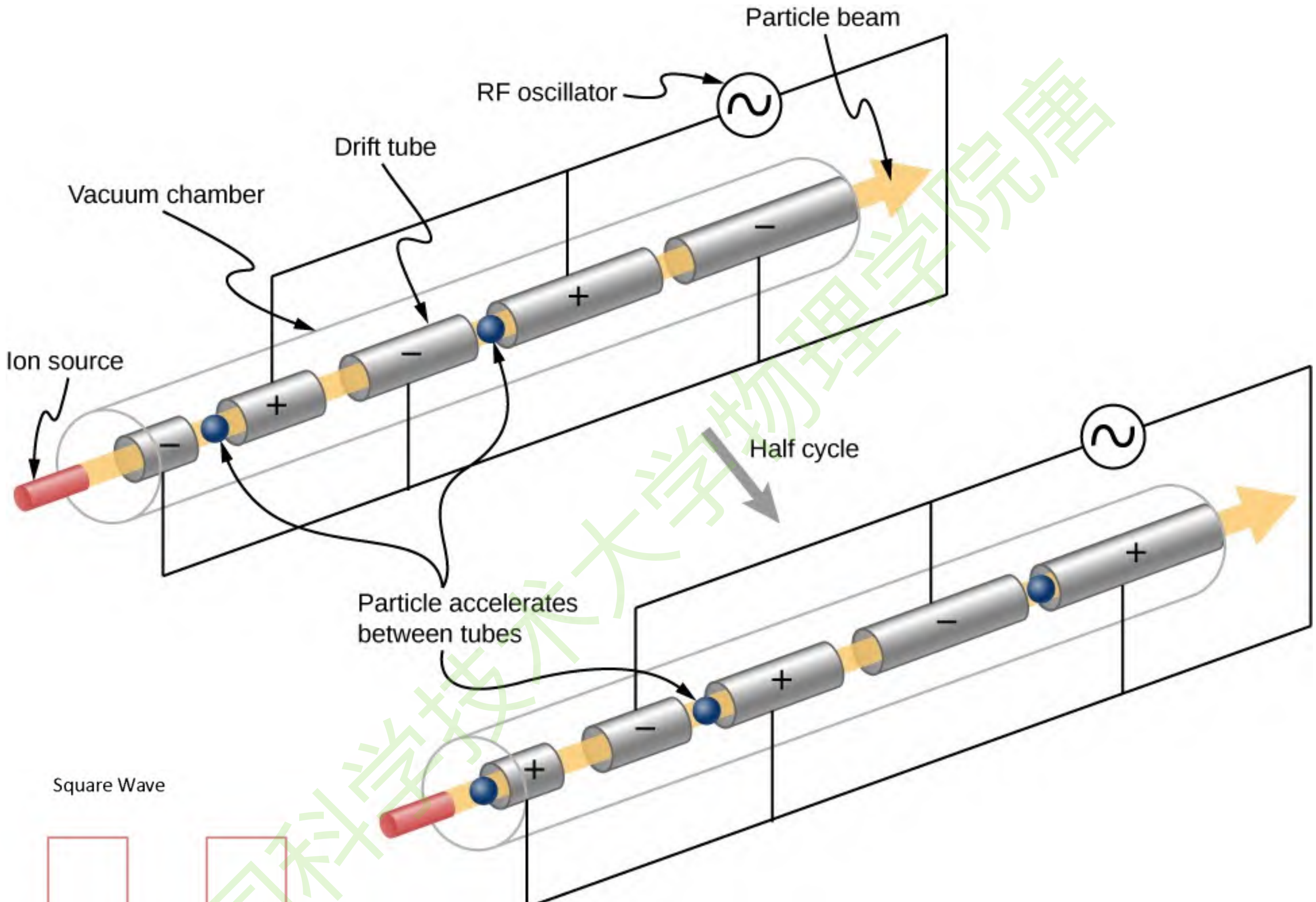
$$t = \frac{z}{v_0}$$

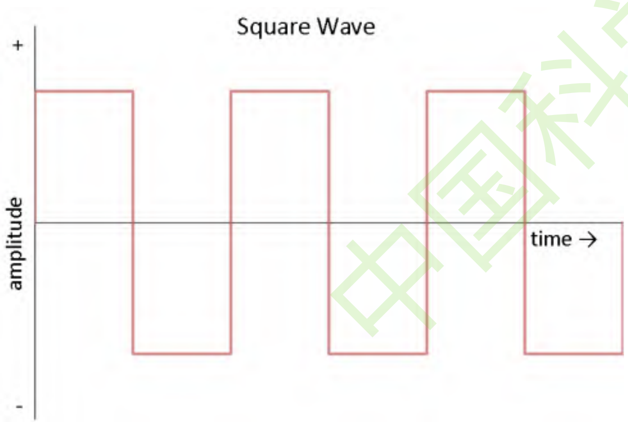
$$\begin{cases} x = \dfrac{qz^2}{2mv_0^2}E_x \\ y = \dfrac{qz^2}{2mv_0^2}E_y \end{cases}$$

直线加速器

- 通过高电压来加速带电粒子，能力是有限的
  - 很高的电压是难以获得的
  - 电压太高会导致放电

- 1924年，Gustav Ising提出一种利用交流电压的新的加速方法
- 1927年，挪威的一名研究生实现了这种办法
- 随着二战中射频技术的飞速发展，直线加速器技术得到了跨越式发展。


Particle beam
RF oscillator
Drift tube
Vacuum chamber
Ion source
Half cycle
Particle accelerates between tubes
+
−


Square Wave
amplitude
time →
+
-