

第1章 电力与电场

§ 1.1 电力起源

§ 1.2 库仑定律

§ 1.3 电场强度

§ 1.4 高斯定理

§ 1.5 环路定理

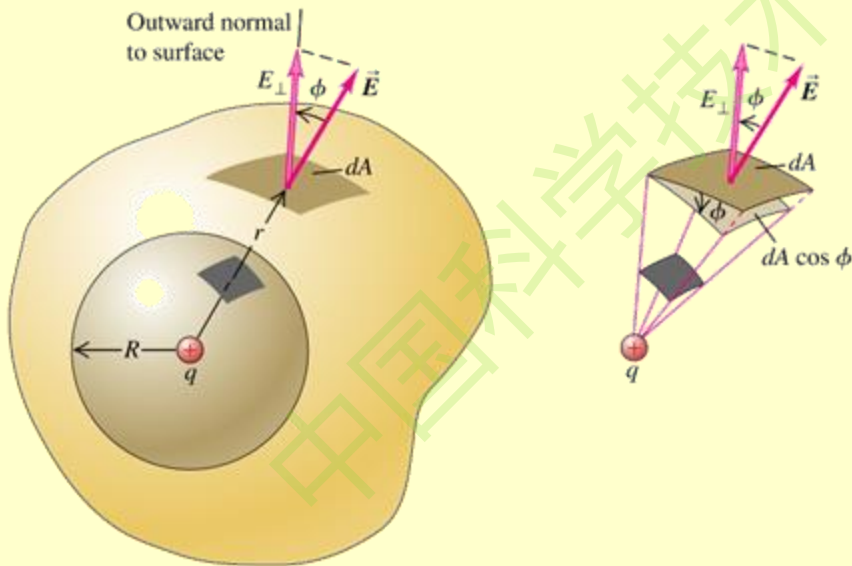
中国科学技术大学物理学院唐

高斯定理 VS. 环路定理

高斯定理

$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \iiint_V \rho dV$$

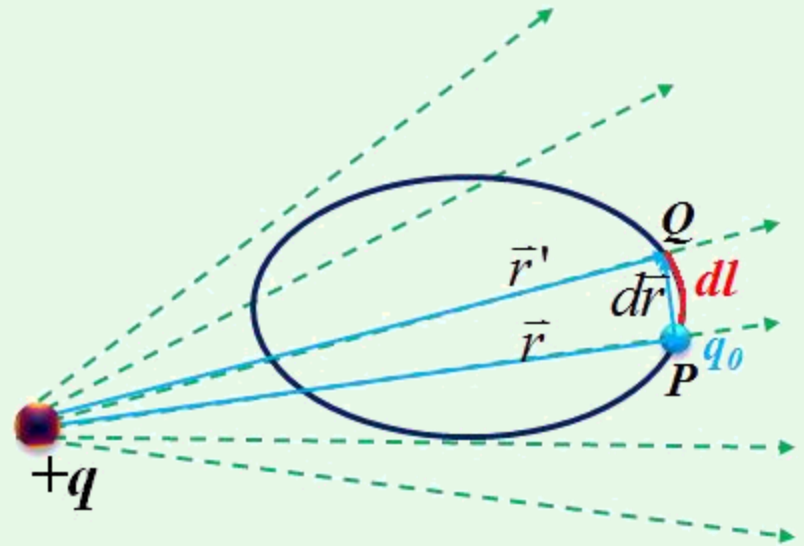
$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$



环路定理

$$\oint_L \vec{E} \cdot d\vec{l} = 0$$

$$\nabla \times \vec{E} = 0$$

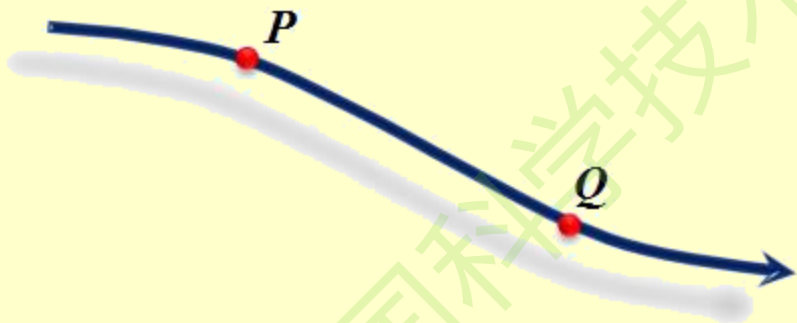


电势与电场

电场 \rightarrow 电势

$$\vec{E}(\vec{r}) = \int d\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{(\vec{r} - \vec{r}')dq}{|\vec{r} - \vec{r}'|^3}$$

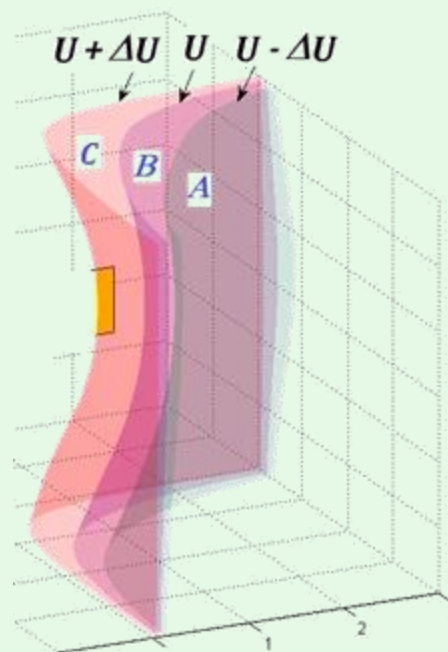
$$U_{PQ} = \int_P^Q \vec{E} \cdot d\vec{l}$$



电势 \rightarrow 电场

$$U(\vec{r}) = \int dU = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{|\vec{r} - \vec{r}'|}$$

$$\vec{E} = -\nabla U$$

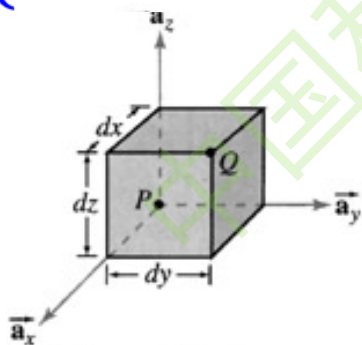


$$\vec{E} = -\frac{\Delta U}{\Delta n} \vec{n} = -\nabla U$$

静电场中任何一点的电场强度，其大小等于该点电势梯度的大小，方向与电势梯度的方向相反，即电势减小的方向

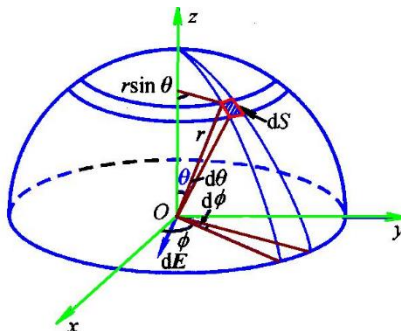
直角坐标：

$$\begin{cases} E_x = -\frac{\partial U}{\partial x} \\ E_y = -\frac{\partial U}{\partial y} \\ E_z = -\frac{\partial U}{\partial z} \end{cases}$$



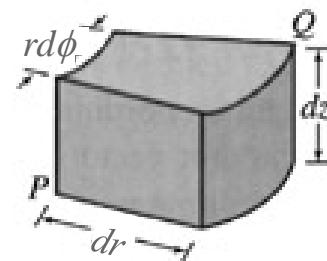
球坐标：

$$\begin{cases} E_r = -\frac{\partial U}{\partial r} \\ E_\theta = -\frac{1}{r} \frac{\partial U}{\partial \theta} \\ E_\phi = -\frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} \end{cases}$$



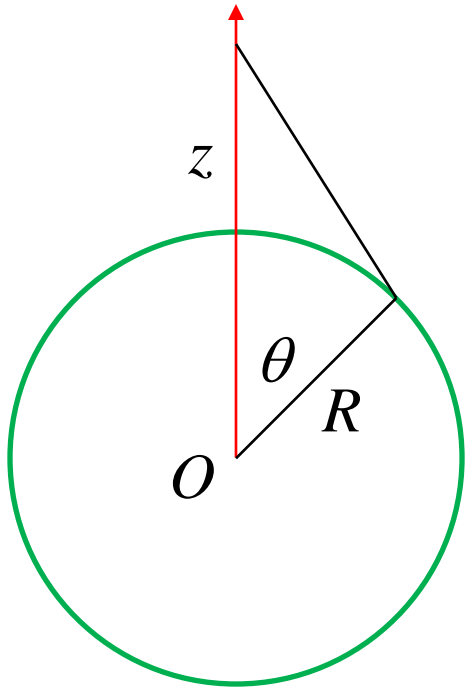
柱坐标：

$$\begin{cases} E_r = -\frac{\partial U}{\partial r} \\ E_\phi = -\frac{1}{r} \frac{\partial U}{\partial \phi} \\ E_z = -\frac{\partial U}{\partial z} \end{cases}$$



电势与电场求解例(3)

求均匀带电球壳层产生的电势分布



$$U(\vec{r}) = \frac{1}{4\pi\epsilon_0} \iint_{S'} \frac{\sigma(\vec{r}') dS'}{|\vec{r} - \vec{r}'|}$$

$$\begin{aligned} U &= \frac{\sigma}{4\pi\epsilon_0} \iint_S \frac{R^2 \sin\theta d\theta d\phi}{\sqrt{R^2 + z^2 - 2Rz \cos\theta}} \\ &= \frac{\sigma R^2}{2\epsilon_0} \int_0^\pi \frac{\sin\theta d\theta}{\sqrt{R^2 + z^2 - 2Rz \cos\theta}} \\ &= \frac{\sigma R^2}{2\epsilon_0} \frac{\sqrt{R^2 + z^2 - 2Rz \cos\theta}}{Rz} \Bigg|_{\theta=0}^{\theta=\pi} \end{aligned}$$

$$U = \frac{\sigma R^2}{2\varepsilon_0} \frac{\sqrt{R^2 + z^2 - 2Rz \cos \theta}}{Rz} \left. \begin{array}{l} \theta = \pi \\ \theta = 0 \end{array} \right\}$$

$$= \frac{\sigma R^2}{2\varepsilon_0} \frac{|R + z| - |R - z|}{Rz}$$

$$= \begin{cases} \frac{\sigma R^2}{\varepsilon_0 z} & z > R \\ \frac{\sigma R}{\varepsilon_0} & z \leq R \end{cases} \xrightarrow{r \equiv z, \sigma = \frac{Q}{4\pi R^2}} \begin{cases} \frac{Q}{4\pi\varepsilon_0 r} & r > R \\ \frac{Q}{4\pi\varepsilon_0 R} & r \leq R \end{cases}$$

$$E_r = -\frac{\partial U}{\partial r} = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r^2} & r > R \\ 0 & r \leq R \end{cases} \quad E_\theta = E_\phi = 0$$

解II:

由高斯定理可得:

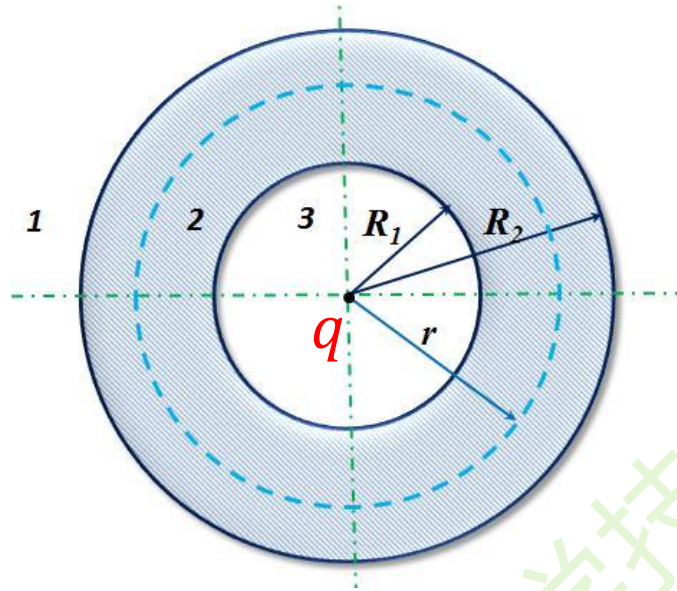
$$E_r = \begin{cases} \frac{Q}{4\pi\epsilon_0 r^2} & r > R \\ 0 & r \leq R \end{cases} \quad E_\theta = E_\phi = 0$$

$$U(r) = \int_r^\infty \vec{E} \cdot d\vec{l} = - \int_\infty^r E_r dr$$

$$= \begin{cases} \frac{Q}{4\pi\epsilon_0 r} & r > R \\ \frac{Q}{4\pi\epsilon_0 R} & r \leq R \end{cases}$$

电势与电场求解例(4)

一个导体球壳，内外半径分别为 R_1 和 R_2 ，在球心放置电量为 q 的点电荷，求其电势和电场分布



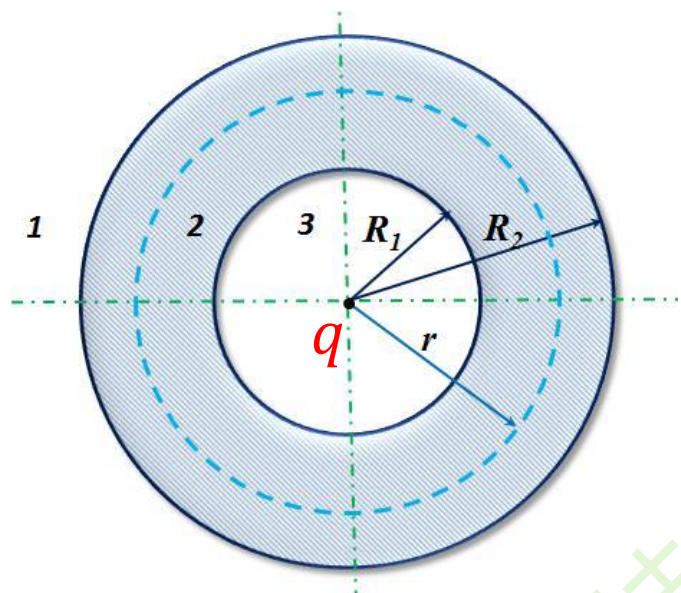
先用高斯定理求电场

$$\vec{E} = \begin{cases} \frac{q}{4\pi\epsilon_0 r^2} & r < R_1 \\ 0 & R_1 < r < R_2 \\ \frac{q}{4\pi\epsilon_0 r^2} & r > R_2 \end{cases}$$

取无穷远处为电势零点

$$U = \begin{cases} \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{R_1} + \frac{1}{R_2} \right) & r < R_1 \\ \frac{q}{4\pi\epsilon_0 R_2} & R_1 < r < R_2 \\ \frac{q}{4\pi\epsilon_0 r} & r > R_2 \end{cases}$$

一个导体球壳，内外半径分别为 R_1 和 R_2 ，在球心放置电量为 q 的点电荷，求其电势和电场分布



点电荷产生的电势为：

$$U_1(r) = \frac{q}{4\pi\epsilon_0 r}$$

导体球壳不带电，产生的电势为：

$$U_2(r) = 0$$

根据电势叠加原理： $U(r) = U_1(r) + U_2(r) = \frac{q}{4\pi\epsilon_0 r}$

与上页结果不符，为何？

求电场、电势分布

1. 从库仑定律出发

$$d\vec{E} = \frac{dq}{4\pi\epsilon_0|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

$$\vec{E} = \int d\vec{E}$$

$$U_{PQ} = \int_P^Q \vec{E} \cdot d\vec{l}$$

2. 从高斯定理出发 (常用于有对称性的带电体)

$$\oiint_S \vec{E} \cdot d\vec{S} = \iiint_V \nabla \cdot \vec{E} dV = \frac{1}{\epsilon_0} \iiint_V \rho dV$$

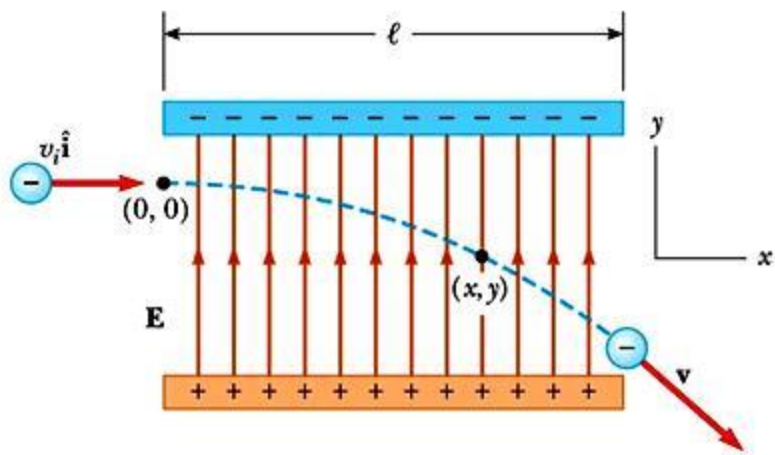
$$U_{PQ} = \int_P^Q \vec{E} \cdot d\vec{l}$$

3. 从电势出发 (常用于点电荷体系或不太对称的带电体)

$$U(\vec{r}) = \int dU = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{|\vec{r} - \vec{r}'|}$$

$$\vec{E} = -\nabla U$$

§ 1.5.6 带电粒子在电场中的运动



$$m \frac{d\vec{v}(\vec{r})}{dt} = q\vec{E}(\vec{r})$$

任意电场:

$$m \frac{d\vec{v}(\vec{r})}{dt} \cdot d\vec{l} = q\vec{E}(\vec{r}) \cdot d\vec{l}$$

$$m d\vec{v}(\vec{r}) \cdot \frac{d\vec{l}}{dt} = q\vec{E}(\vec{r}) \cdot d\vec{l}$$

$$\Delta E_k = -q\Delta U$$

$$v_2 = \sqrt{v_1^2 + \frac{2}{m} q(U_1 - U_2)}$$

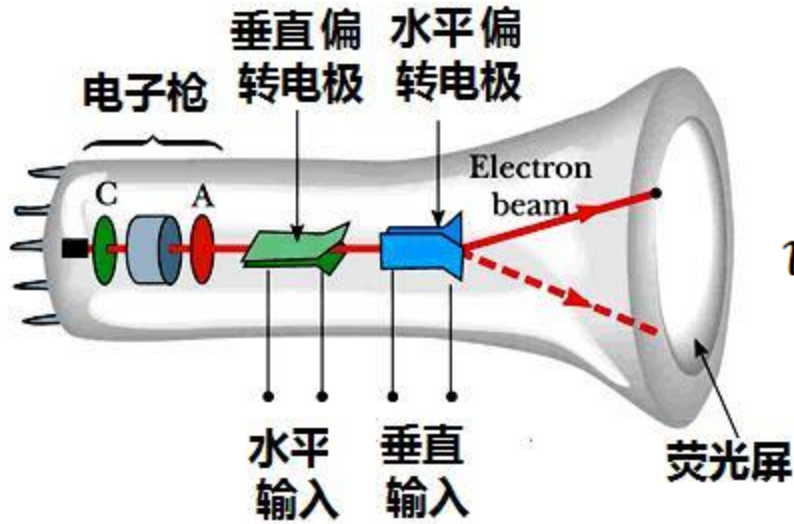
匀强电场:

$$m \frac{d^2 \vec{x}(\vec{r})}{dt^2} = qE$$

当 $x(0) = 0, v(0) = 0$

$$x = \frac{qE}{2m} t^2$$

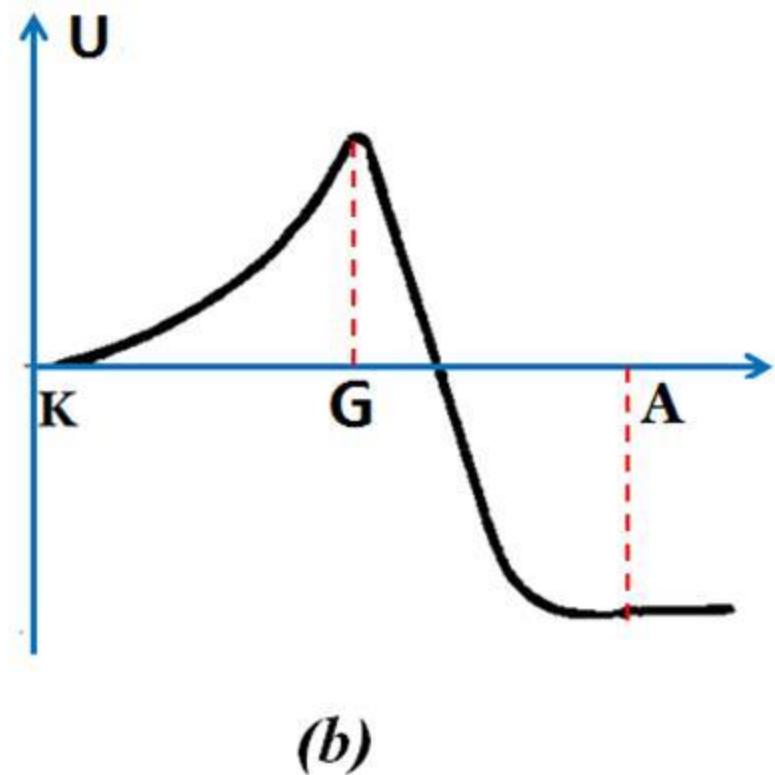
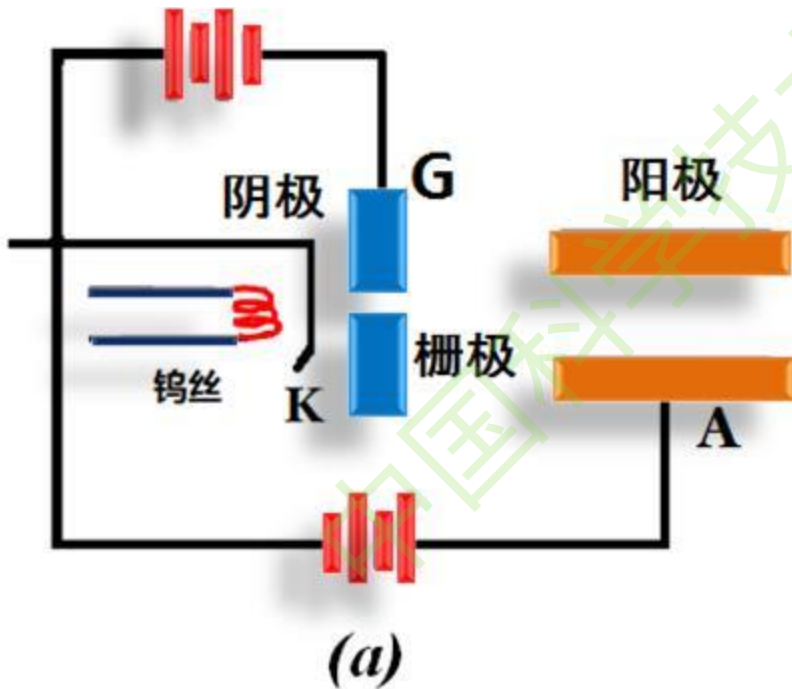
显像管原理



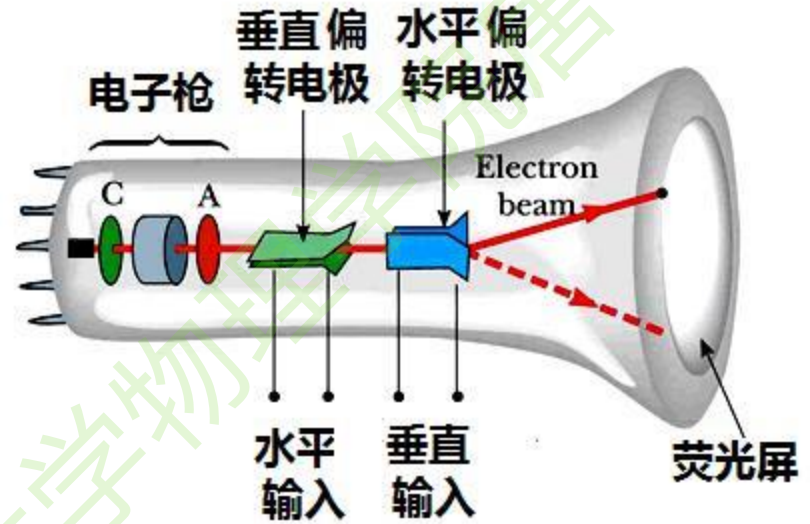
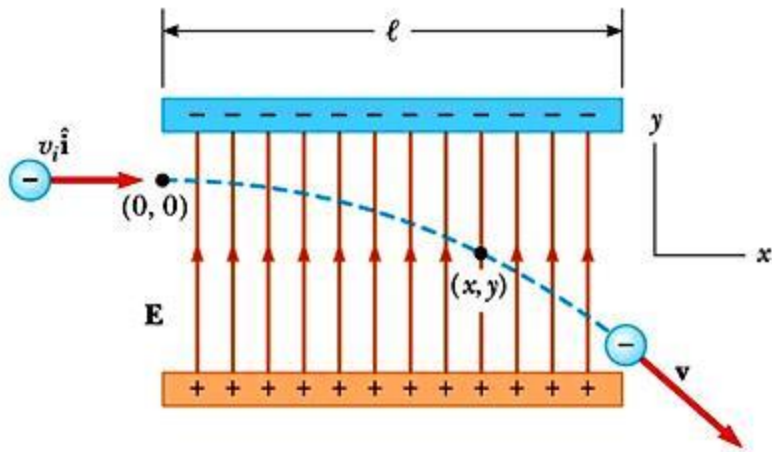
电子枪：小型加速器

$$v_0 = \sqrt{\frac{2e}{m} U_A} \xrightarrow{U_A=10000V} 6 \times 10^7 \text{ m/s}$$

0.2倍光速！



偏转电极



- 电子枪加速电子到一定速度 v_0 （垂直屏幕方向）
- 两个偏转电极控制电子束落在显示屏的不同位置

$$x = \frac{qE_x}{2m} t^2$$

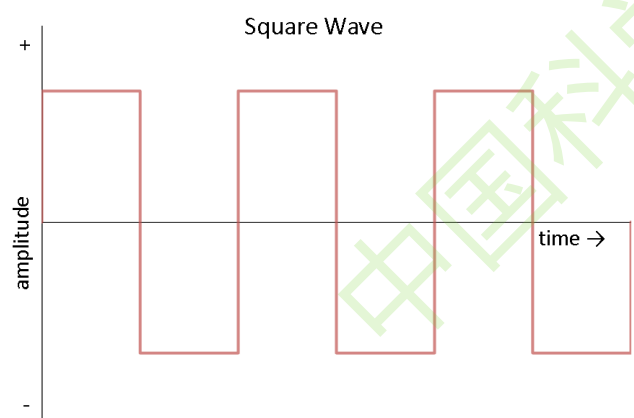
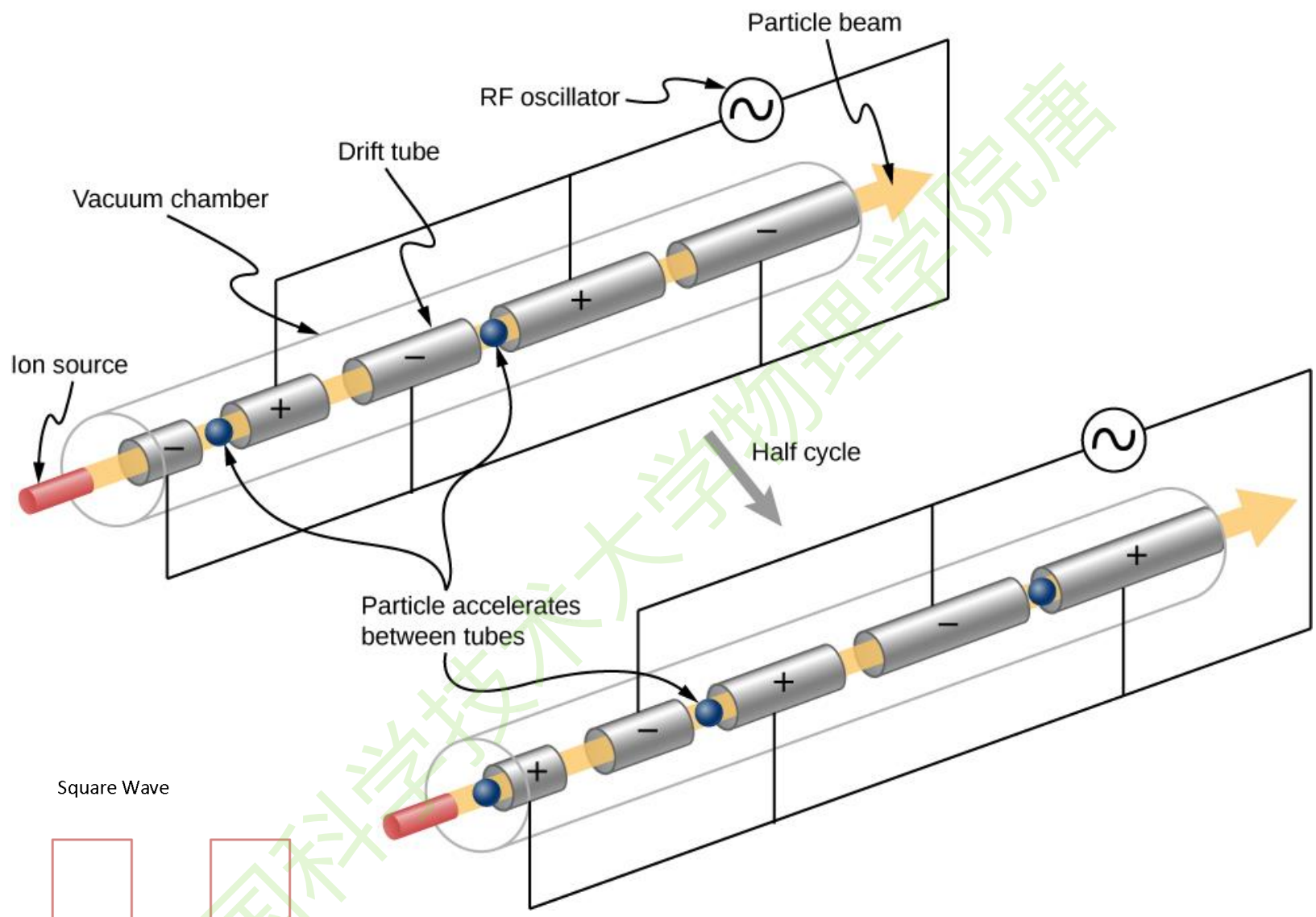
$$y = \frac{qE_y}{2m} t^2$$

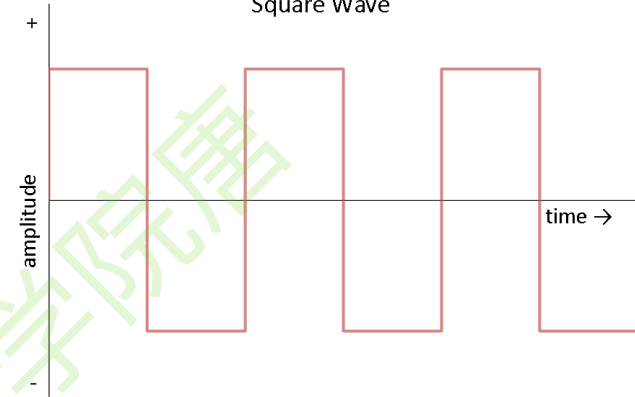
$$t = \frac{z}{v_0}$$

$$\left\{ \begin{array}{l} x = \frac{qz^2}{2mv_0^2} E_x \\ y = \frac{qz^2}{2mv_0^2} E_y \end{array} \right.$$

直线加速器

- 通过高电压来加速带电粒子，能力是有限的
 - 很高的电压是难以获得的
 - 电压太高会导致放电
- 1924年，Gustav Ising提出一种利用交流电压的新的加速方法
- 1927年，挪威的一名研究生实现了这种办法
- 随着二战中射频技术的飞速发展，直线加速器技术得到了跨越式发展。





- 第一个半周期：粒子加速

$$E_1 = \frac{1}{2} m v_1^2 = qU \quad v_1 = \sqrt{\frac{2qU}{m}}$$

- 第二个半周期：粒子在漂移管中匀速运动

$$l_1 = v_1 \times \frac{1}{2} T = \frac{v_1}{2f} = \frac{1}{2f} \sqrt{\frac{2qU}{m}}$$

- 第三个半周期：粒子继续加速

$$E_2 - E_1 = qU \quad E_2 = 2qU \quad v_2 = \sqrt{\frac{4qU}{m}}$$

- 第四个半周期：粒子在漂移管中匀速运动

$$l_2 = \frac{1}{2f} \sqrt{\frac{4qU}{m}}$$

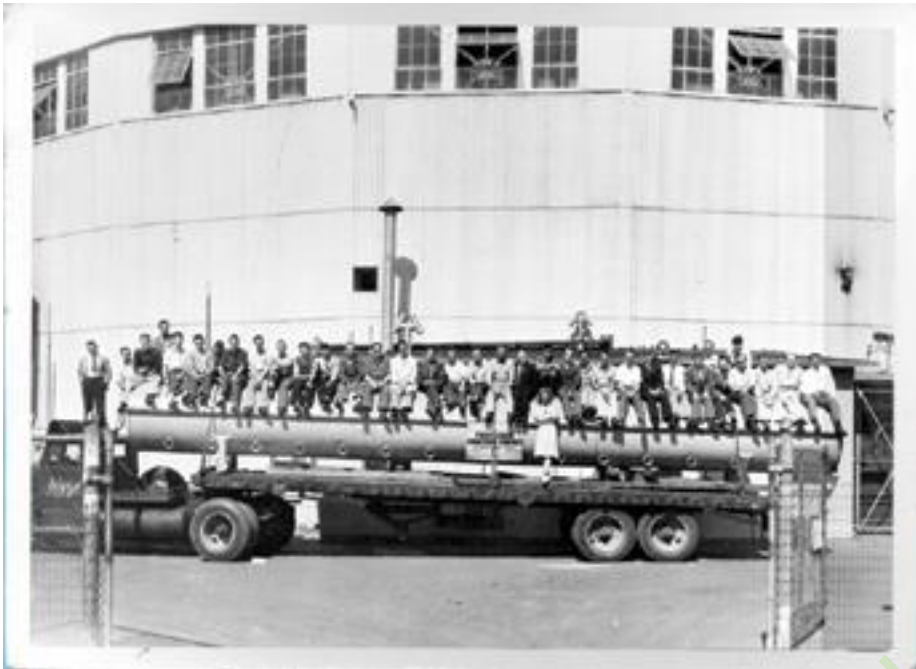
- 第n个周期的前半周期：粒子继续加速

$$E_n = nqU \quad v_2 = \sqrt{\frac{2nqU}{m}}$$

- 第n个周期的后半周期：粒子在漂移管中匀速运动

$$l_n = \frac{1}{2f} \sqrt{\frac{2nqU}{m}}$$

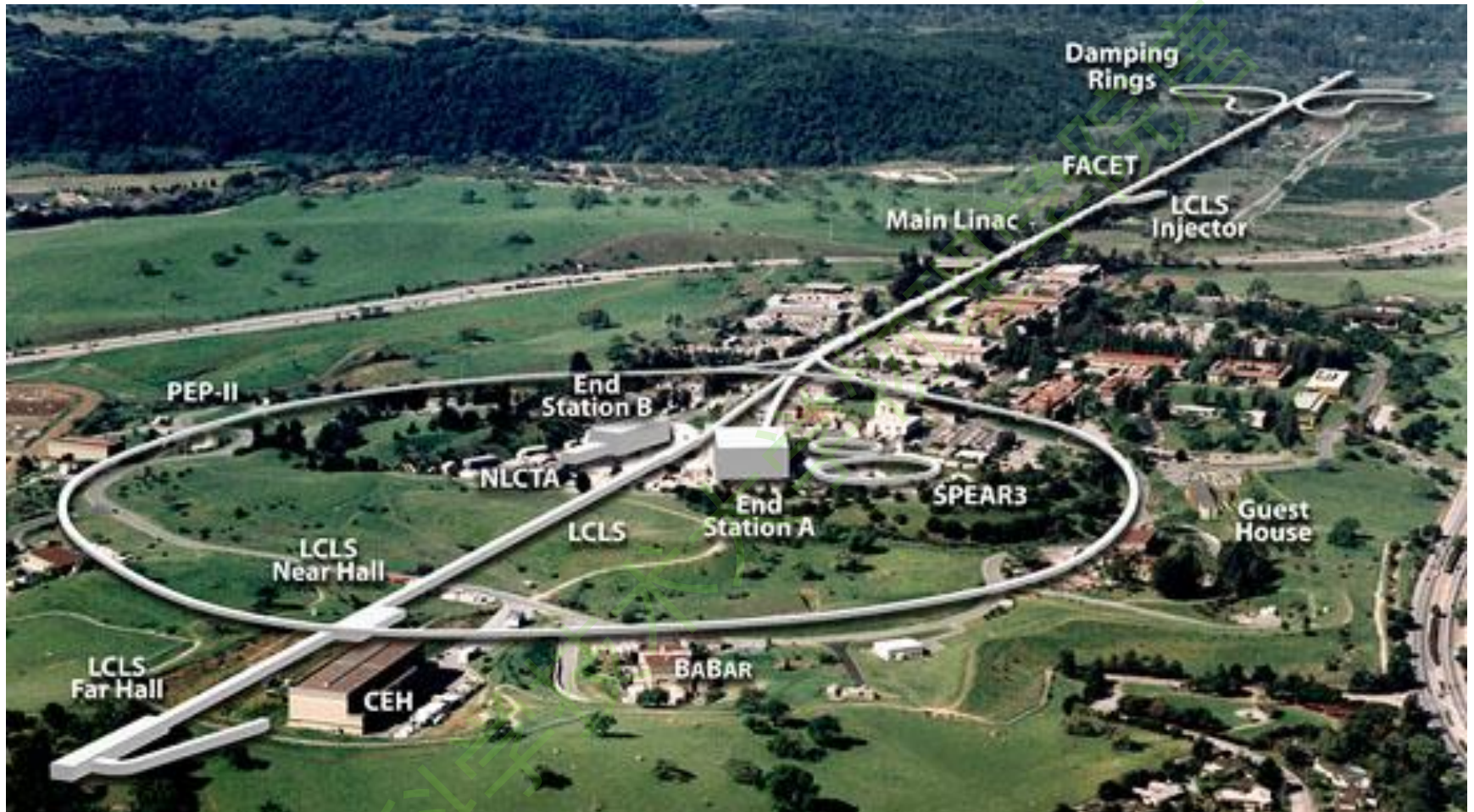
- 级数越多，粒子的能量越大
- 但是，漂移管的长度也越长
 - 射频信号的频率越大，漂移管长度则越小



1947年，美国伯克利国家实验室Luis Alvarez设计了一个12米长的**质子直线加速器**，可将质子能量加速到32 MeV



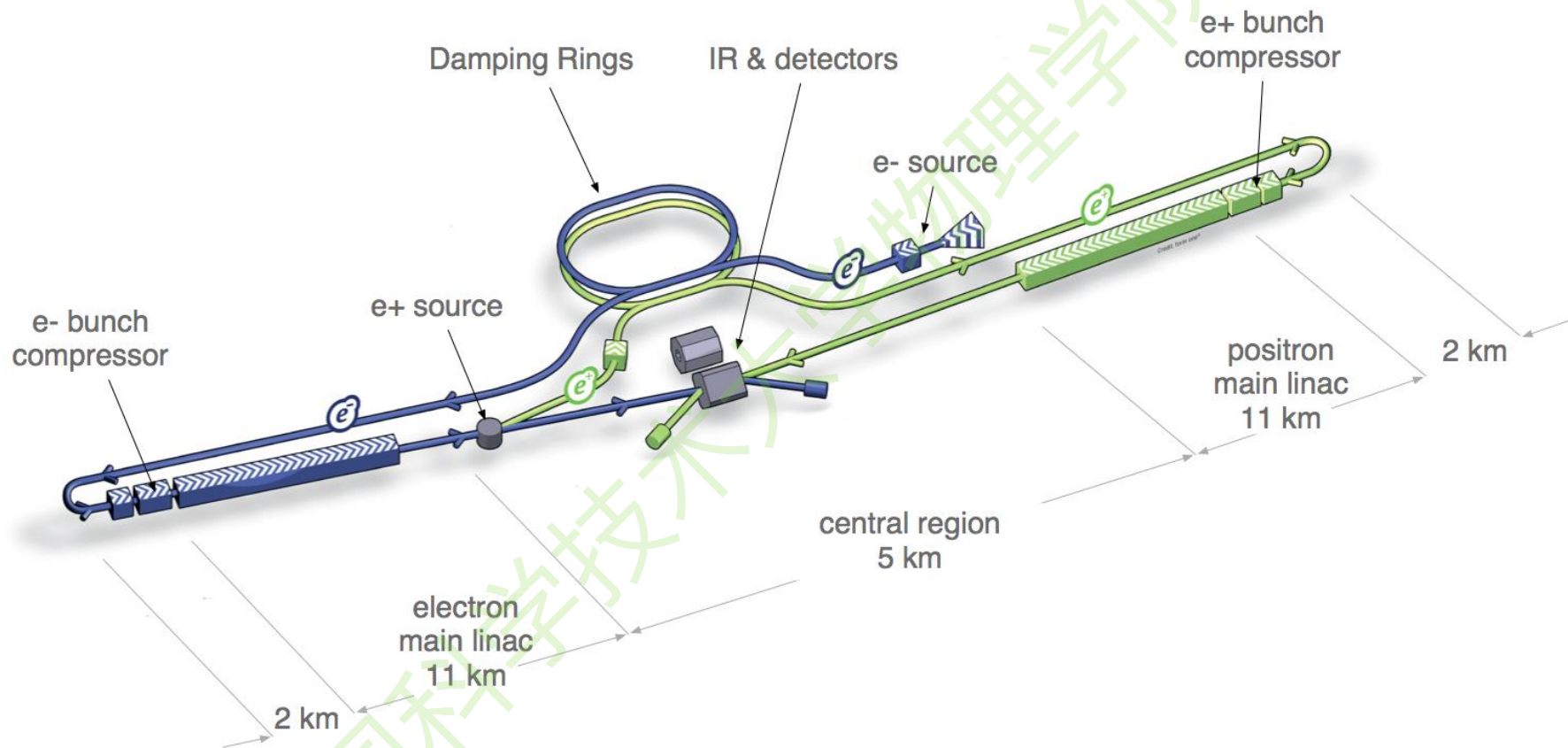
1949年，美国斯坦福大学William Hansen设计建造了一个3.6米长的**电子直线加速器**，可将电子能量加速到6 MeV



3.2 km长电子直线加速器，加速电子到 5×10^{10} eV

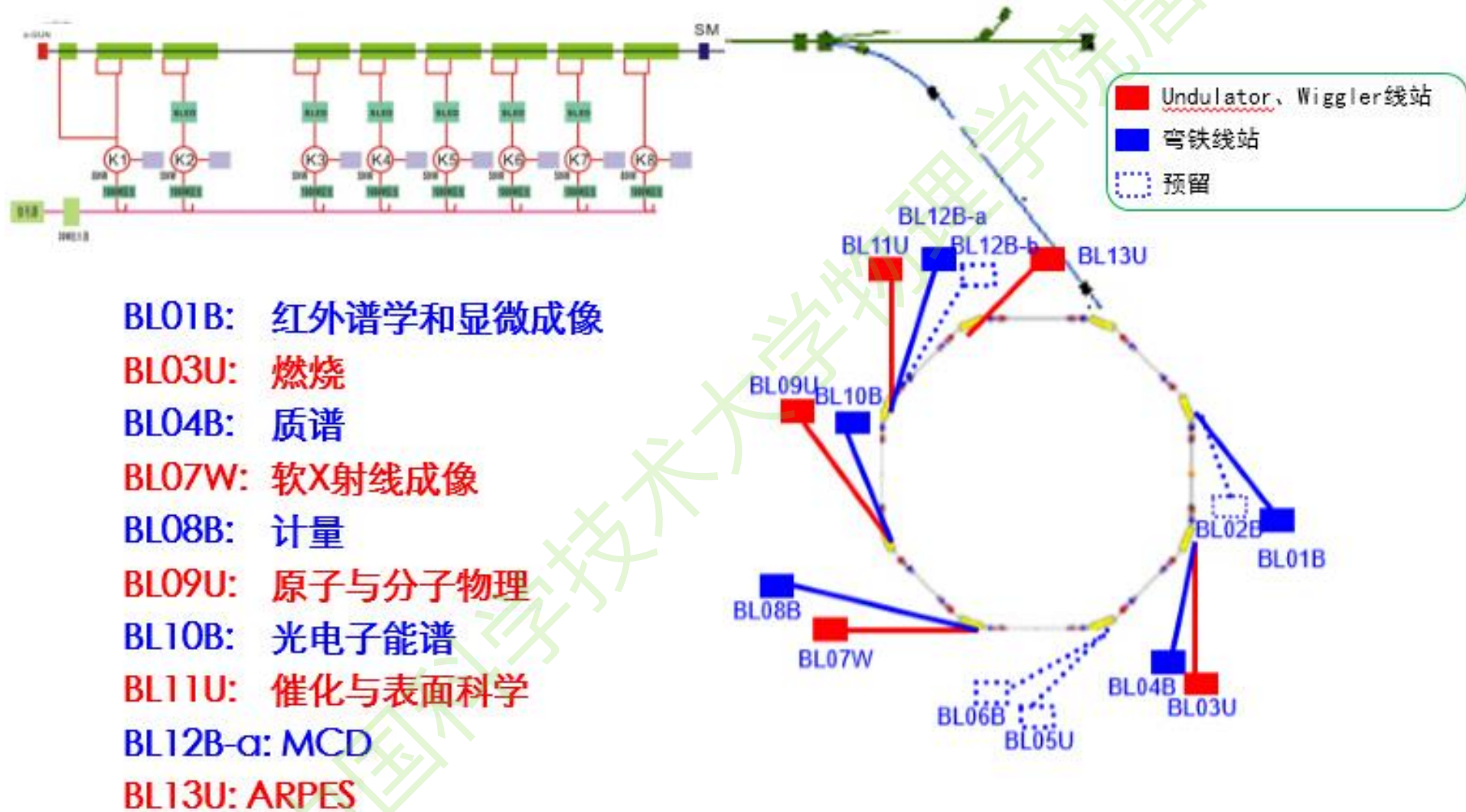
国际直线对撞机 (The International Linear Collider)

下一代直线加速器

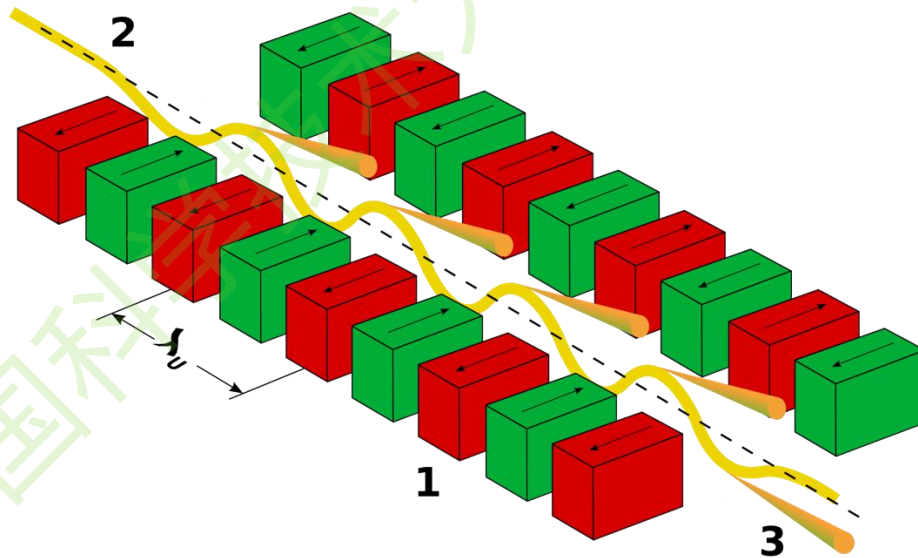
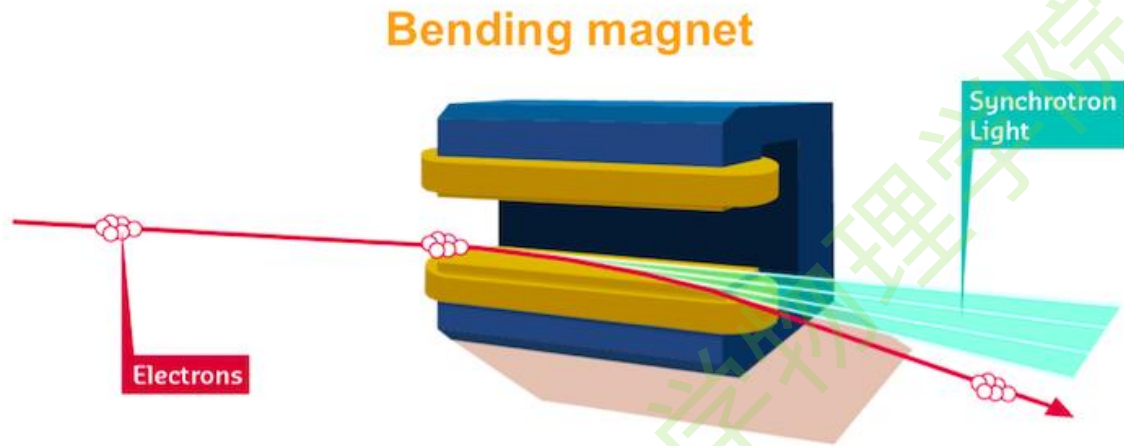


能量达到 10^{11-12} eV, 研究Higgs (上帝粒子) 的特性等

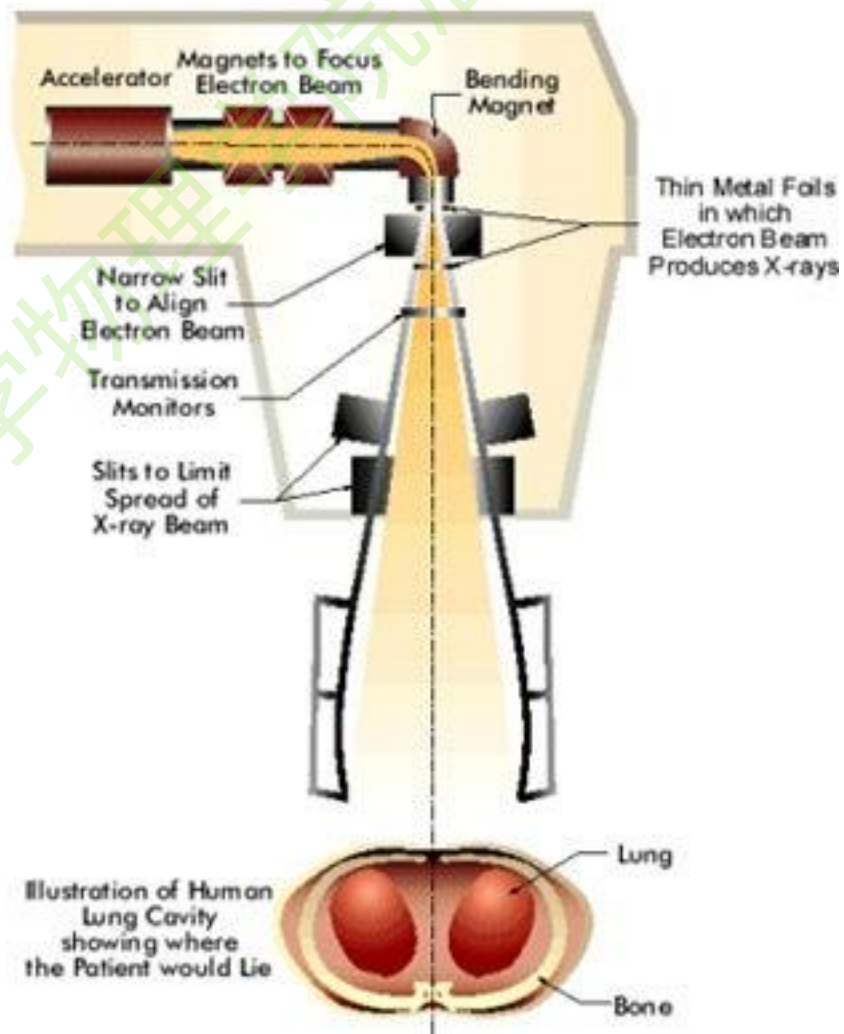
合肥光源@合肥同步辐射国家实验室



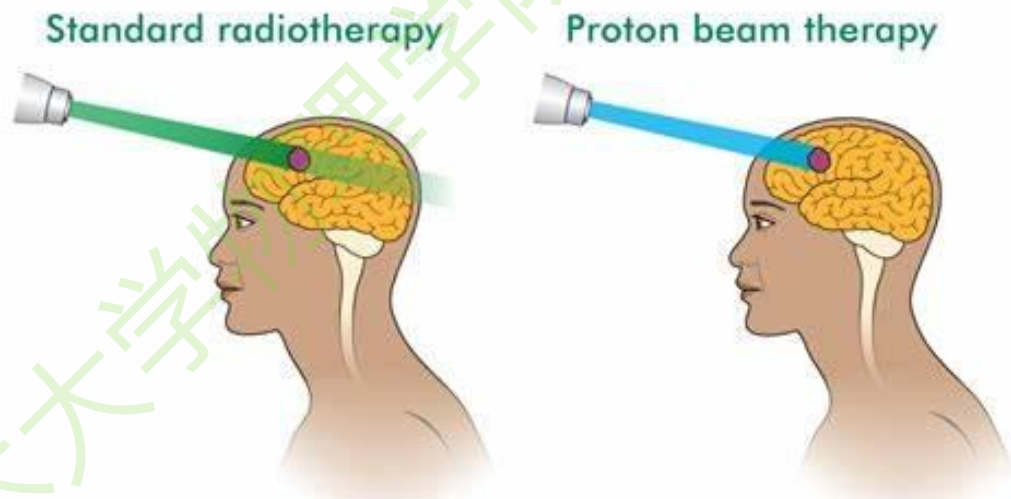
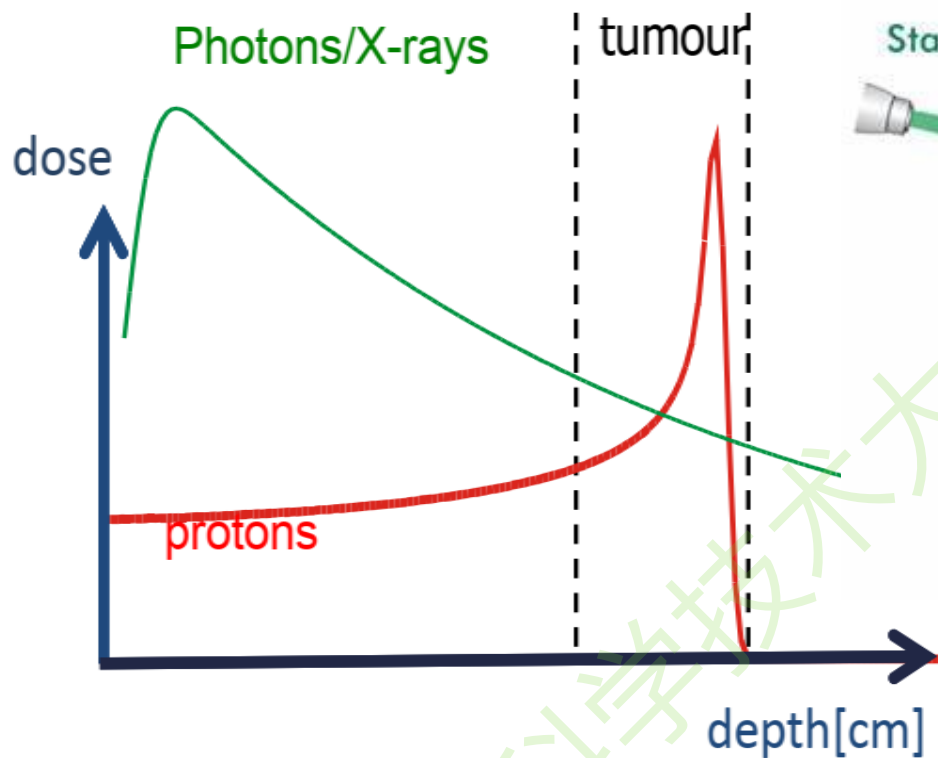
同步辐射的产生



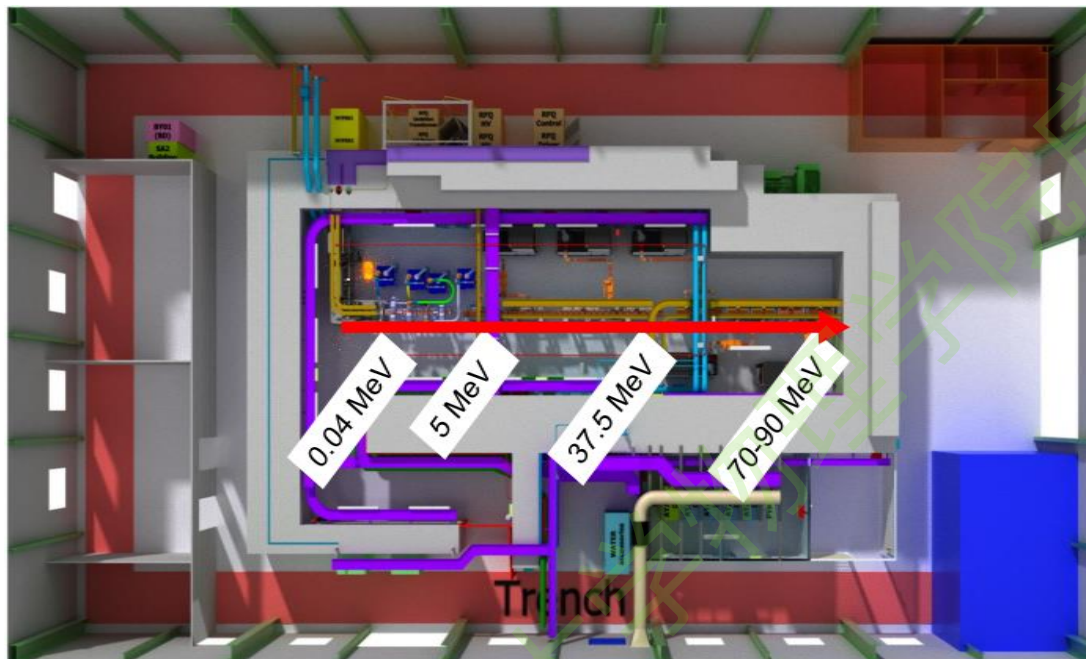
医院中的电子加速器 —— 放疗



医院中的质子加速器——放疗



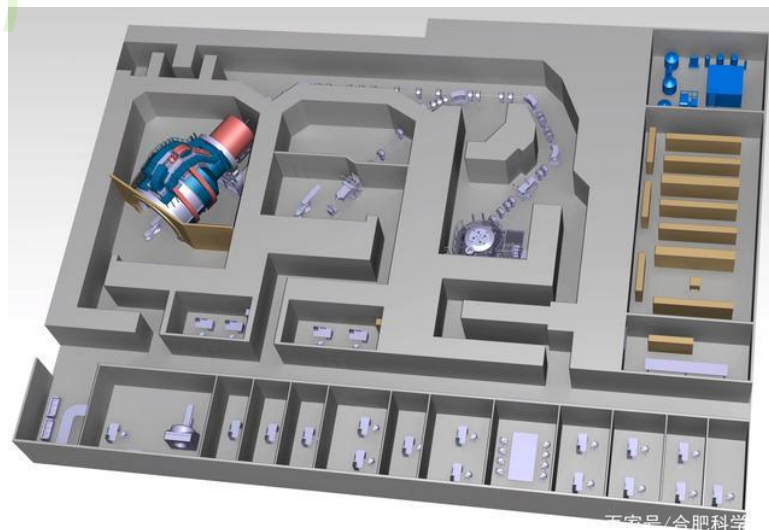
中国科学院大学



CERN
LIGHT



百家号/合肥科学港



百家号/合肥科学

合肥离子医学中心

静电场中的“折射”现象

- 水平方向不受力

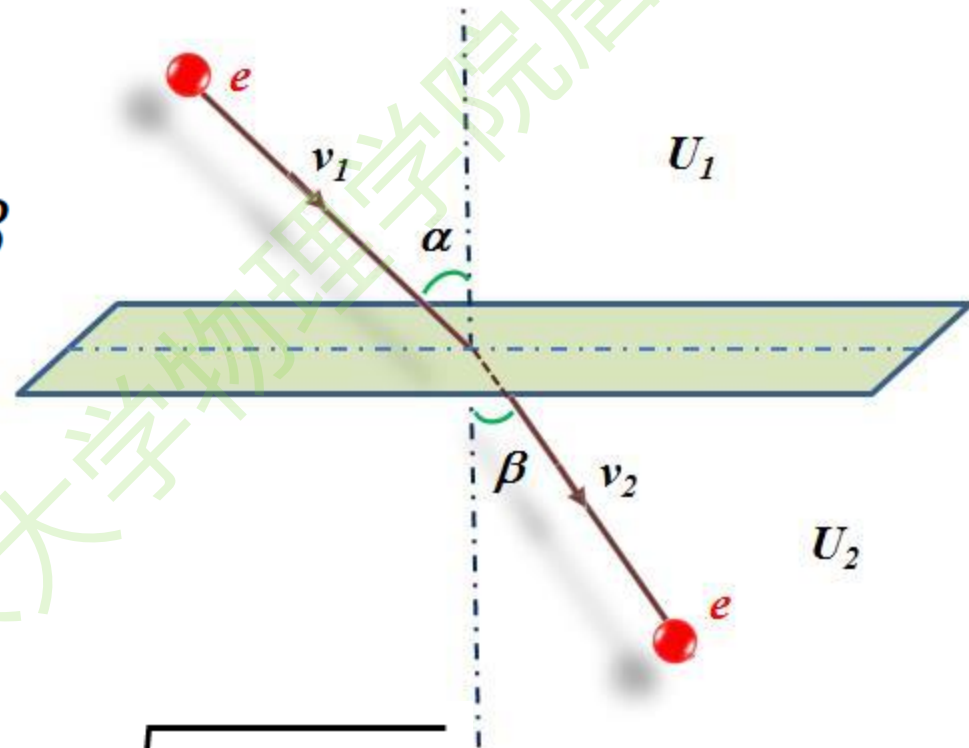
$$mv_1 \sin \alpha = mv_2 \sin \beta$$

$$\frac{\sin \alpha}{\sin \beta} = \frac{v_2}{v_1}$$

- 若粒子从静止被加速

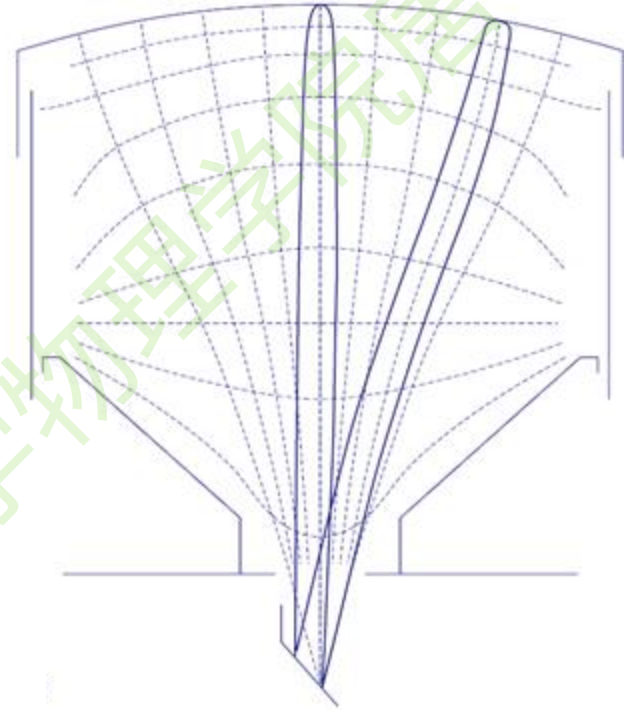
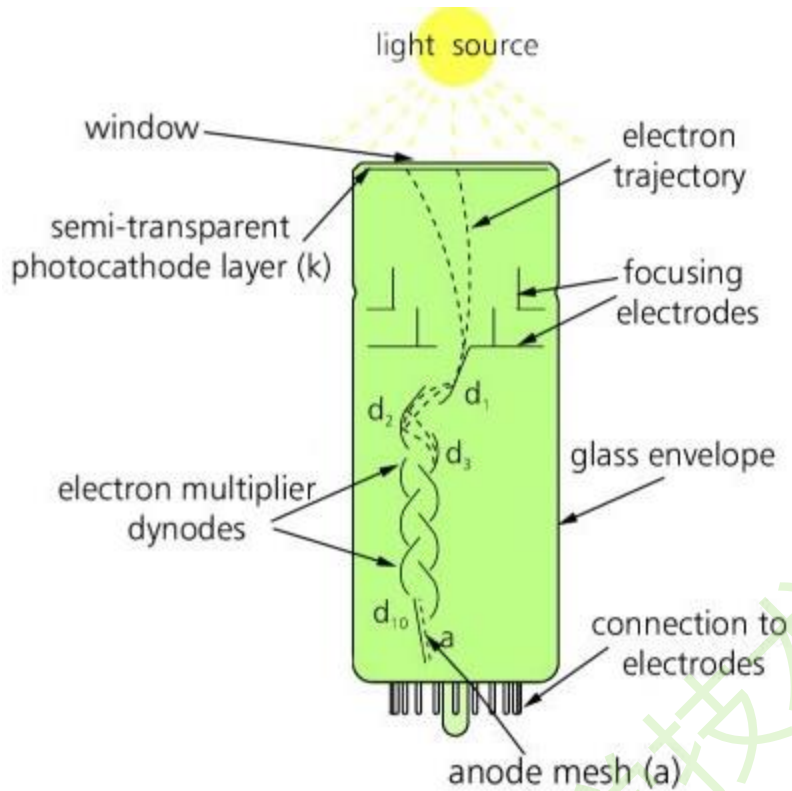
$$v_1 = \sqrt{2eU_1/m} \quad v_2 = \sqrt{2eU_2/m}$$

$$\frac{\sin \alpha}{\sin \beta} = \frac{v_2}{v_1} = \sqrt{\frac{U_2}{U_1}}$$



电势变化越快，折射越大

光电倍增管



$$4^{10} = 10^6$$

Photomultiplier tube (PMT)




The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2015 to

Nobelpriset i fysik 2015 The Nobel Prize in Physics 2015

Nobelpriset i fysik 2015



KUNGL. VETENSKAP AKADEMIE
THE ROYAL SWEDISH ACADEMY OF SCIENCES



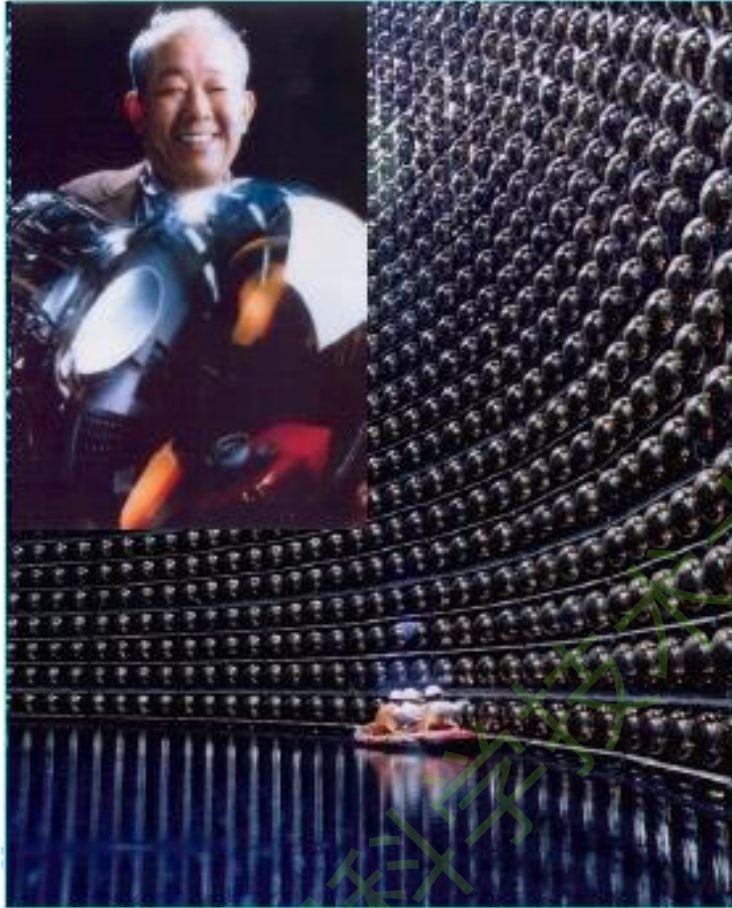
Takaaki Kajita
Super-Kamiokande Collaboration
University of Tokyo, Kashiwa, Japan

Arthur B. McDonald
Sudbury Neutrino Observatory Collaboration
Queen's University, Kingston, Canada

"för upptäckten av neutrinooscillationer, som visar att neutriner har massa"
the discovery of neutrino oscillations, which shows that neutrinos have mass

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

宇宙中微子探测：2002诺贝尔物理奖



Masatoshi Koshihira (小柴昌俊)

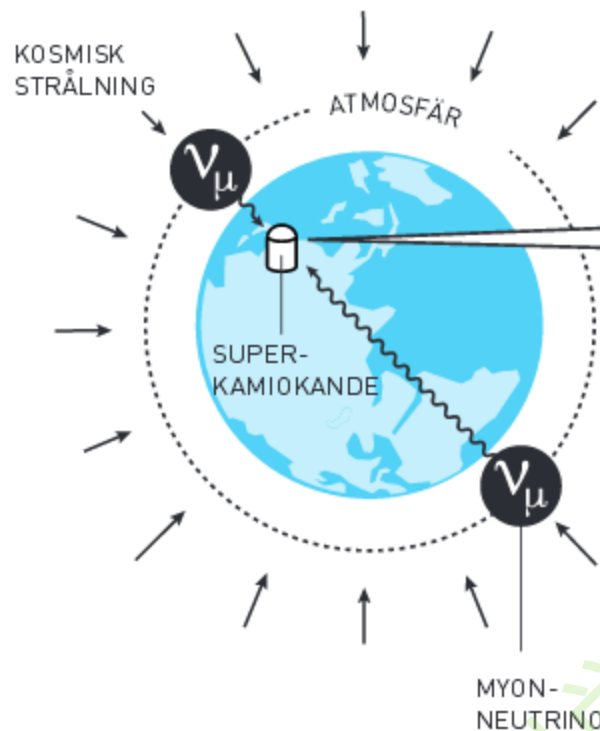
International Center for Elementary Particle Physics, University of Tokyo, Japan



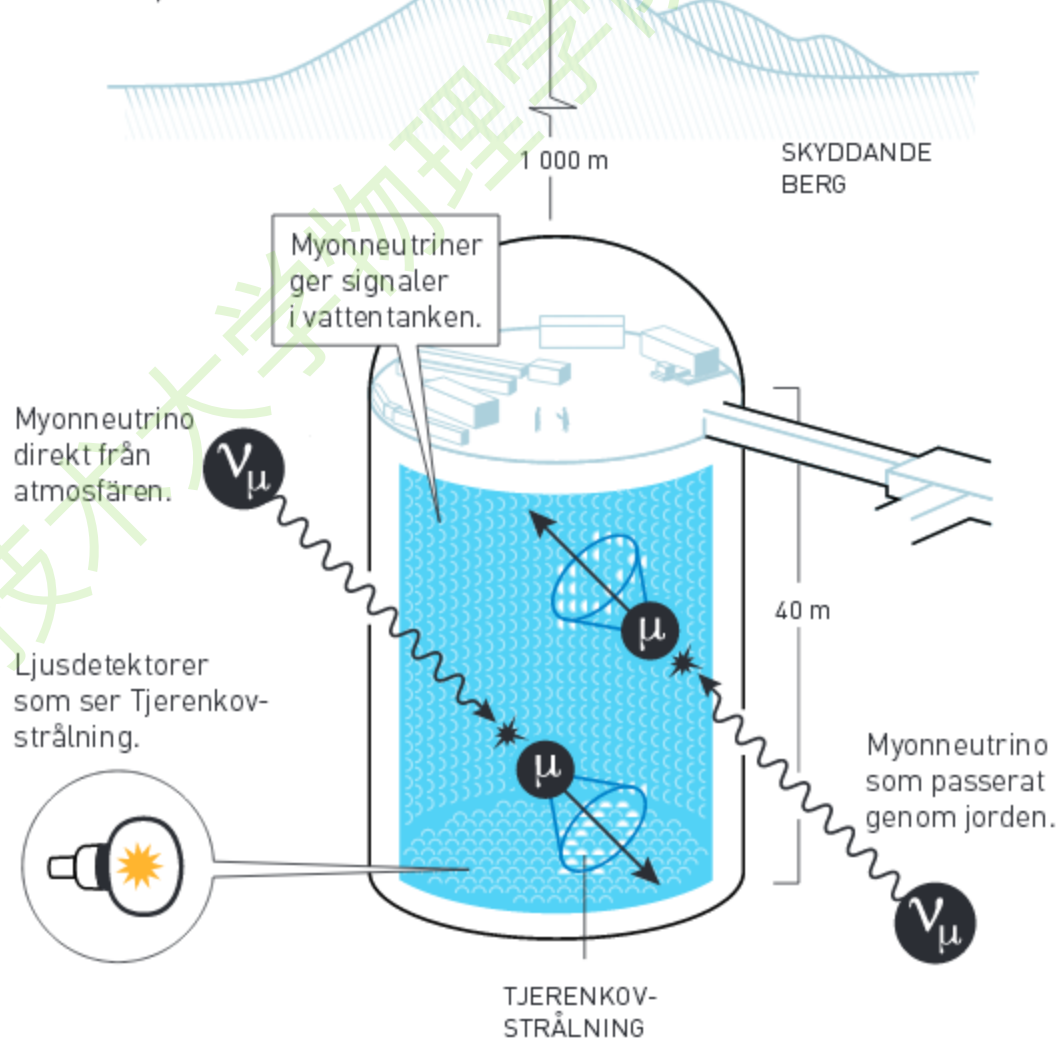
Raymond Davis Jr

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, USA,

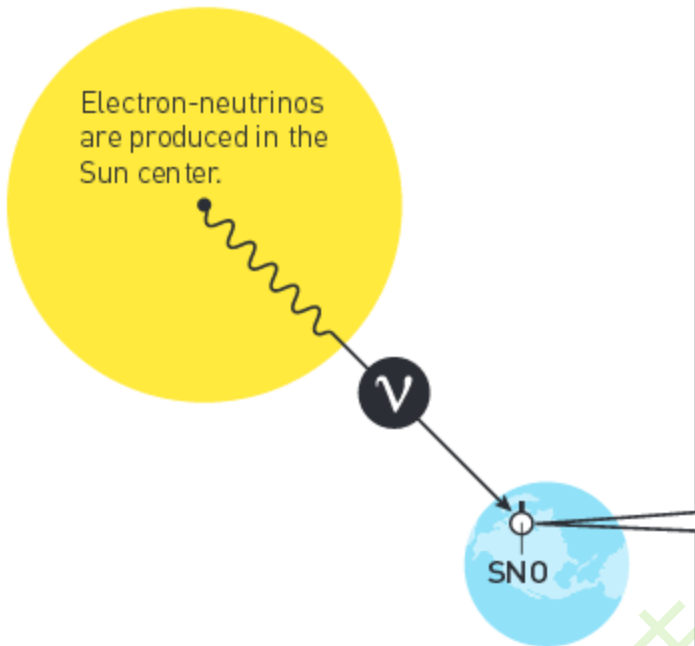
NEUTRINER FRÅN KOSMISK STRÅLNING



SUPER-KAMIOKANDE KAMIOKA, JAPAN

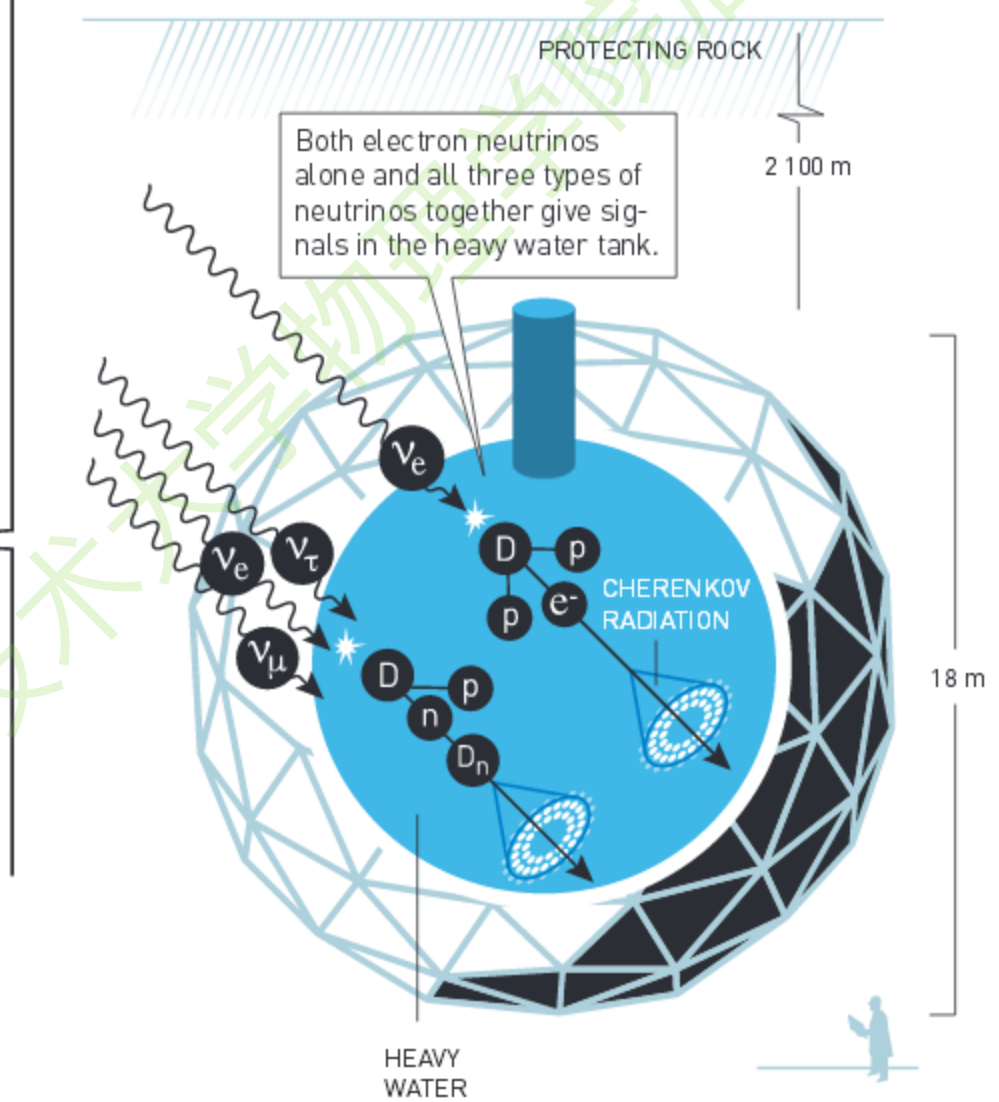


NEUTRINOS FROM THE SUN

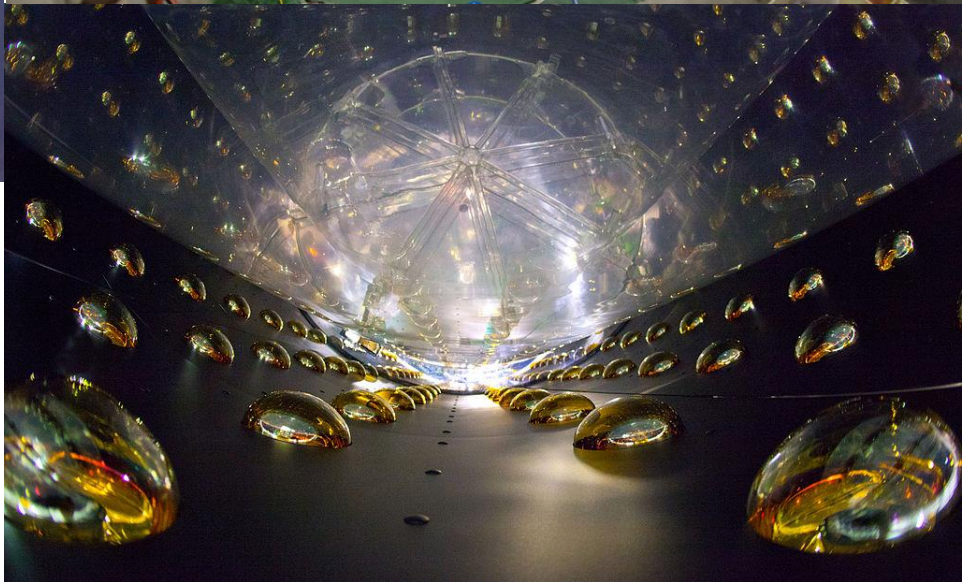
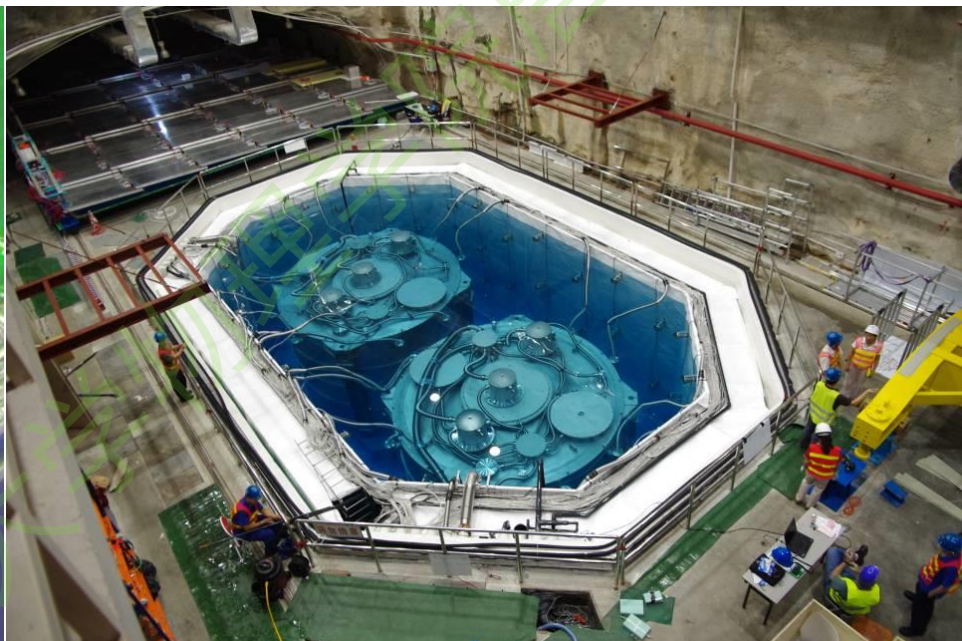


SUDBURY NEUTRINO OBSERVATORY (SNO)

ONTARIO, CANADA



大亚湾中微子实验

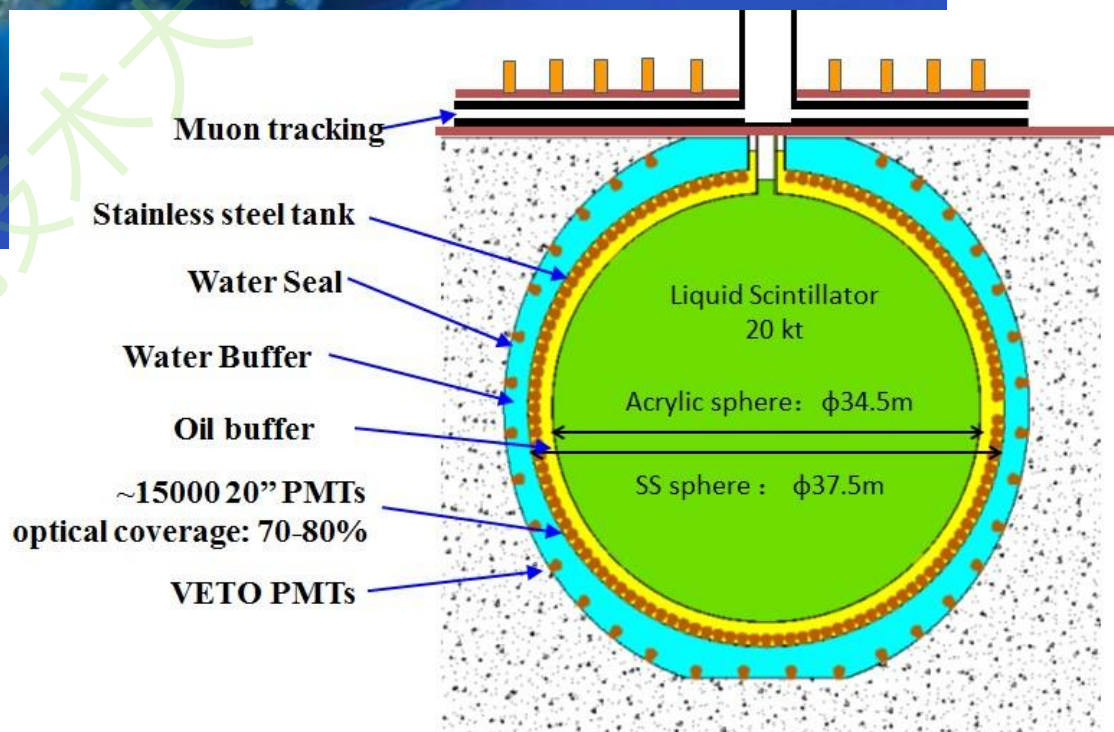
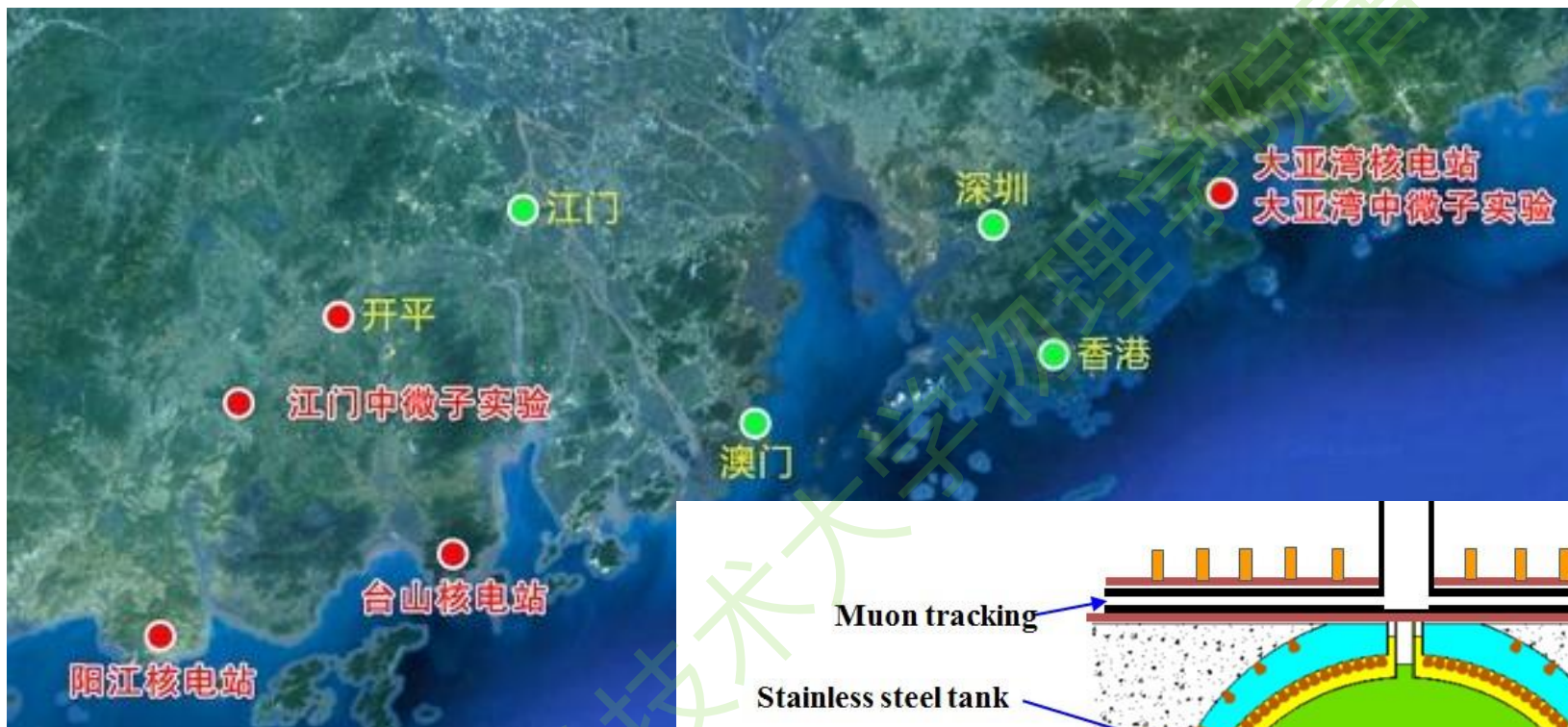


测量中微子振荡混合角 θ_{13}

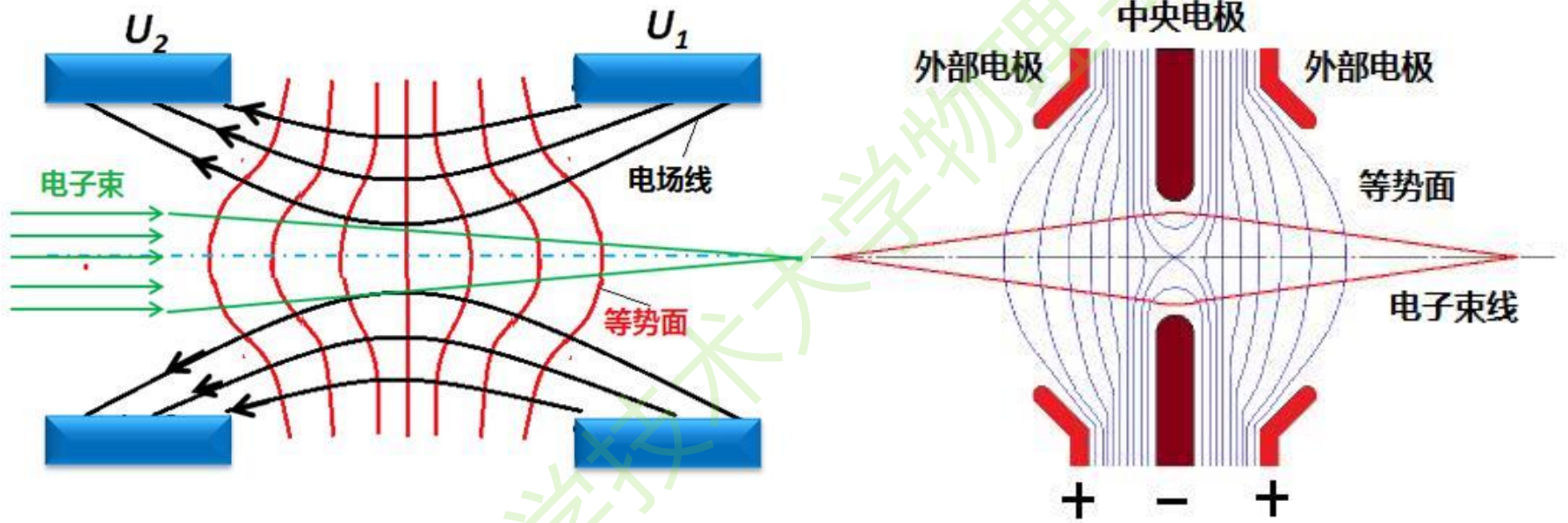
获得2016基础物理突破奖

5个中微子实验分享300万美元

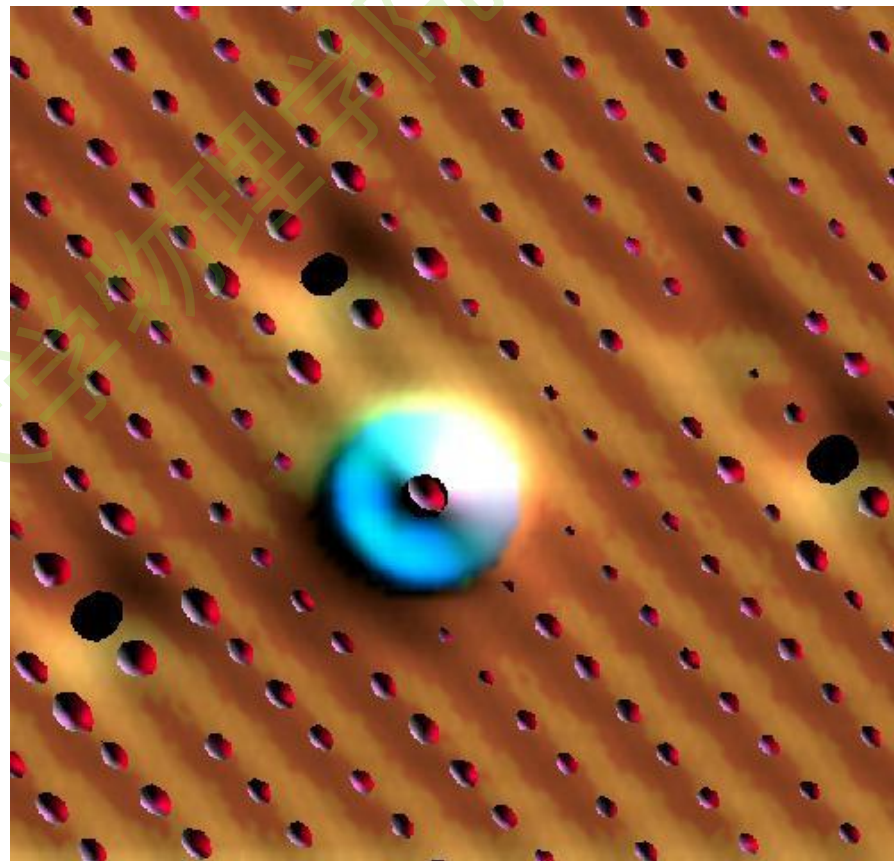
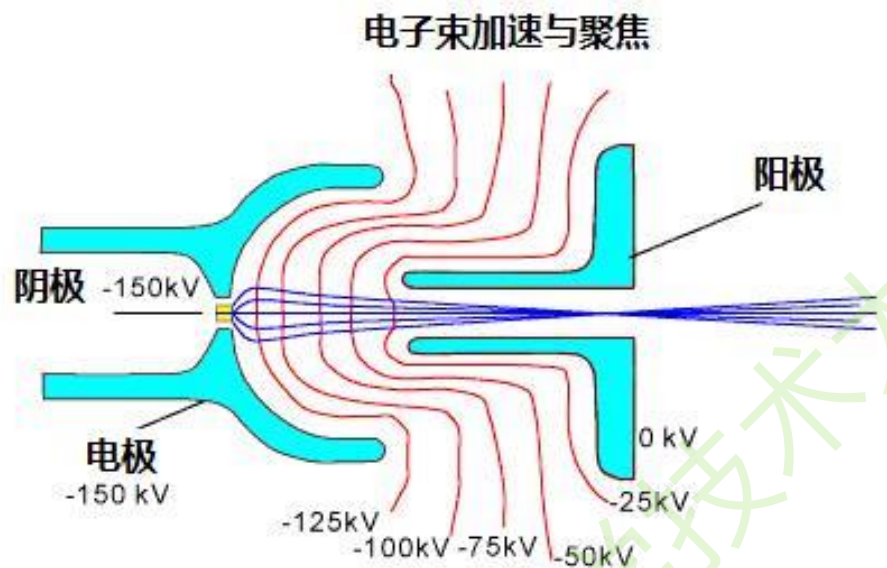
江门中微子实验JUNO



静电透镜



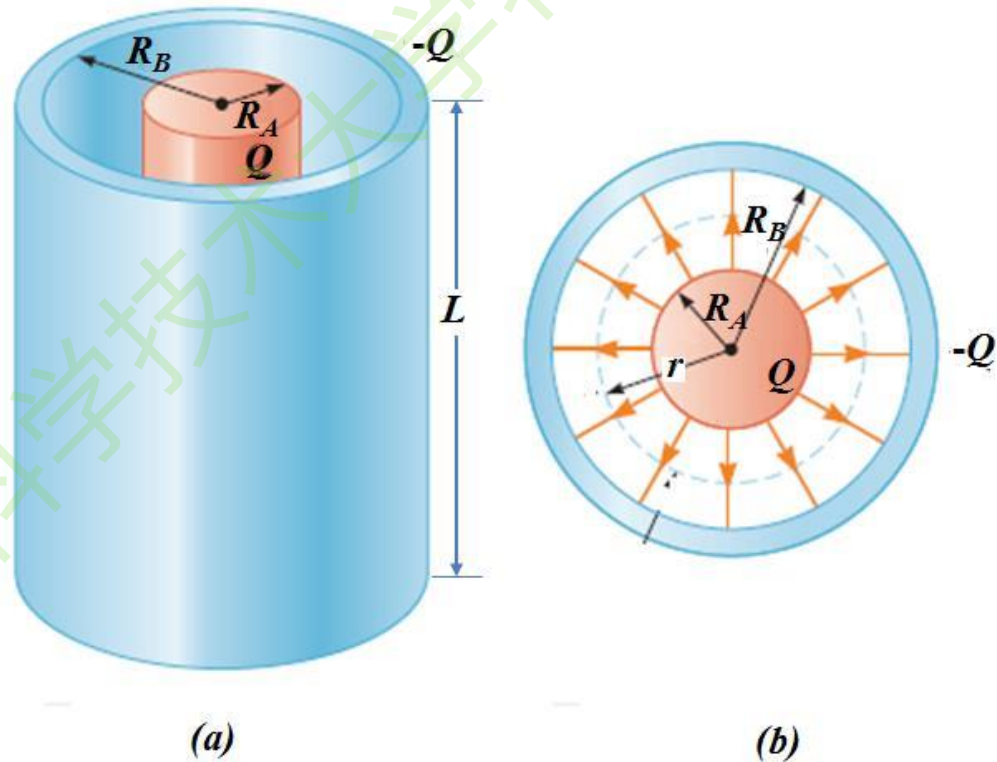
电子显微镜



中国科学技术大学

练习

1. 同轴电缆由同心导体圆柱体和圆柱面构成，半径分别为 R_A 和 R_B ，圆柱体所带电荷全分布在外表面上、电量为 $+Q$ ，圆柱面所带电荷为 $-Q$ ，假定电缆长度 L 远大于 R_B ，求全空间中的电势分布。



作业

- 1.9

中国科学技术大学物理学院唐