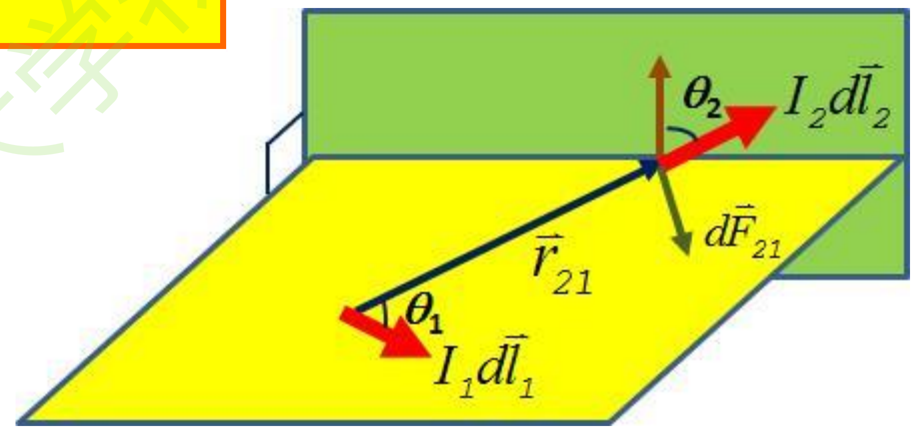
第4章 磁力与磁场

安培定律

两个电流元之间的相互作用力：

$$d\vec{F}_{21} = k\frac{I_2 d\vec{l}_2 \times \left(I_1 d\vec{l}_1 \times \vec{e}_r\right)}{r_{21}^2}$$



$$k = \frac{\mu_0}{4\pi} = 10^{-7}\ NA^{-2}$$

$$d\vec{F}_{21} = \frac{\mu_0}{4\pi}\frac{I_2 d\vec{l}_2 \times \left(I_1 d\vec{l}_1 \times \vec{r}\right)}{r_{21}^3}$$

毕奥-萨伐尔定律 (Biot-Savart Law)

关于电流产生的磁感应强度的定律

磁感应强度的定义：

$$d\vec{F}_0 = I_0 d\vec{l}_0 \times d\vec{B}$$

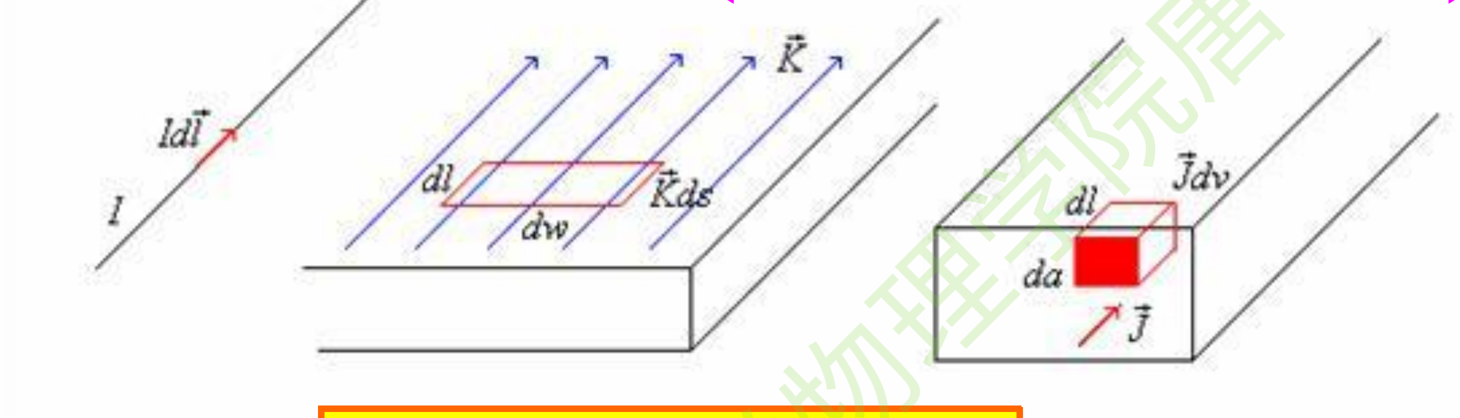
安培定律：

$$d\vec{F}_{21} = \frac{\mu_0}{4\pi}\frac{I_0 d\vec{l}_0 \times (I d\vec{l} \times \vec{r})}{r^3}$$

比较可得电流元产生的磁感应强度：

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \vec{r}}{r^3}$$

毕奥-萨伐尔定律 (Biot-Savart Law)



线电流元:

$$d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$$

面电流元:

$$d\vec{B}=\frac{\mu_0}{4\pi}\frac{\vec{i}\,dS\times\vec{r}}{r^3}$$

体电流元:

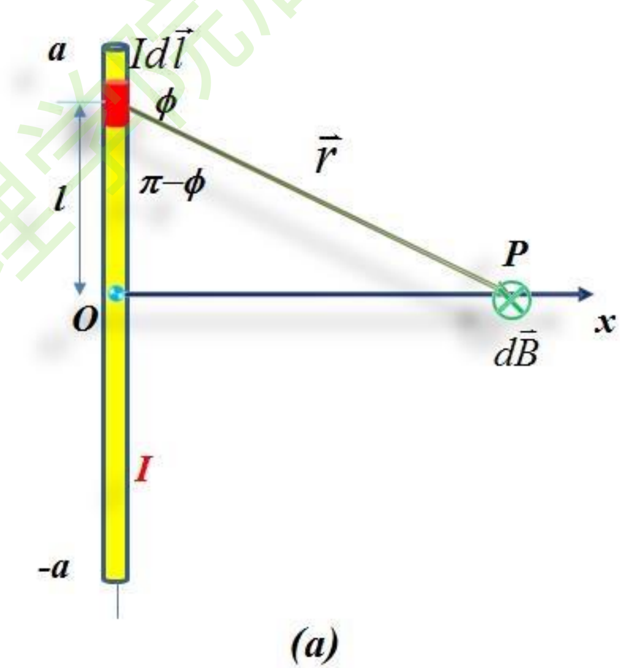
$$d\vec{B}=\frac{\mu_0}{4\pi}\frac{\vec{j}\,dV\times\vec{r}}{r^3}$$

叠加原理:

$$\vec{B}=\int d\vec{B}$$

【例4.1】长直线电流I，求在距离为r_0处一点P的磁场。

【解】沿电流方向取一电流元$Id\vec{l}$



代入毕奥-萨伐尔定律：

$$d\vec{B}=\frac{\mu_0}{4\pi}\frac{(Id\vec{l}\times\vec{r})}{r^3}$$

$$dB=\frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$$

$$r=\frac{r_0}{\sin\phi}$$

$$l=-r_0\cot\phi$$

$$dl=\frac{r_0 d\phi}{\sin^2\phi}$$

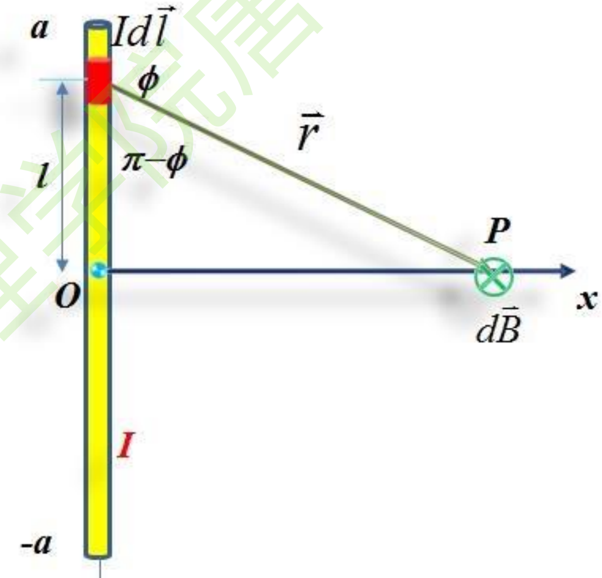
$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{I\left(\frac{r_0 d\phi}{\sin^2\phi}\right)\sin\phi}{\left(\frac{r_0}{\sin\phi}\right)^2} = \frac{\mu_0 I}{4\pi r_0}\sin\phi\, d\phi$$

$$B = \int dB = \frac{\mu_0 I}{4\pi r_0}\int_{\phi_1}^{\phi_2}\sin\phi\, d\phi = \frac{\mu_0 I}{4\pi r_0}(\cos\phi_1 - \cos\phi_2)$$

对无限长直导线： $\phi_1 = 0$， $\phi_2 = \pi$

$$B = \frac{\mu_0 I}{2\pi r_0}$$

【例4.1】长直线电流I，求在距离为r_0处一点P的磁场。



【解】沿电流方向取一电流元$Id\vec{l}$

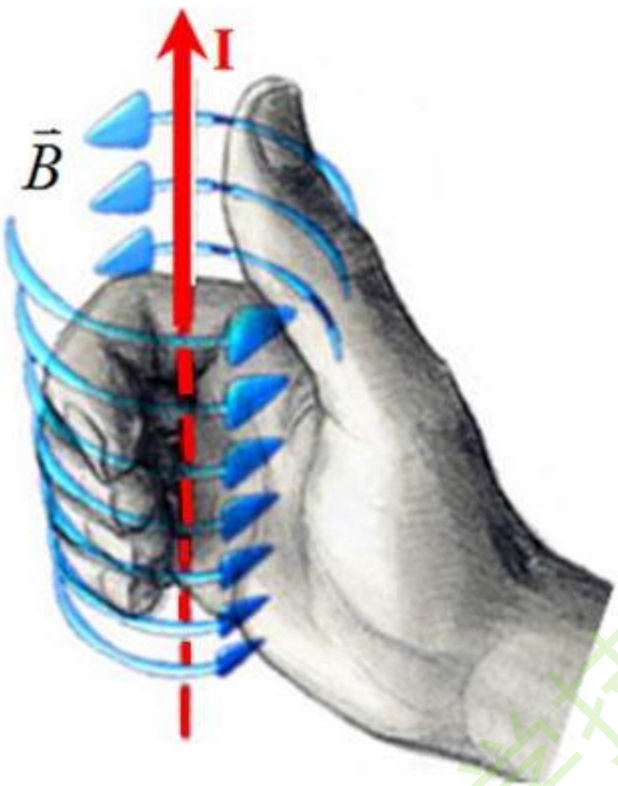
代入毕奥-萨伐尔定律：

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{\left(Id\vec{l} \times \vec{r}\right)}{r^3}$$

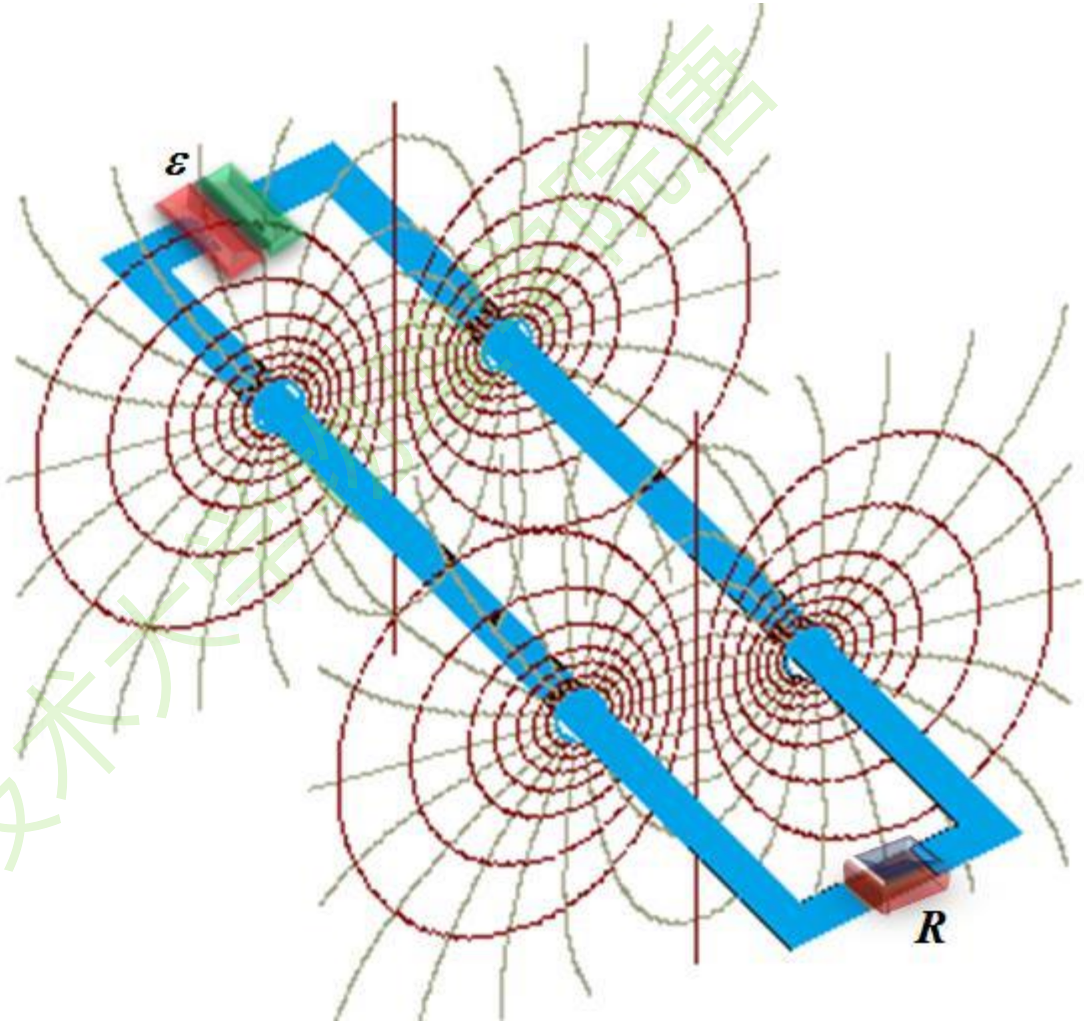
$$dB = \frac{\mu_0}{4\pi}\frac{Idl \cdot |l|}{r^3} = \frac{\mu_0}{4\pi} Idl \cdot |l| \cdot (r_0^2 + l^2)^{-\frac{3}{2}}$$

$$B = \int dB = 2\int_0^{\infty} \frac{\mu_0 I}{4\pi} l dl (r_0^2 + l^2)^{-\frac{3}{2}}$$

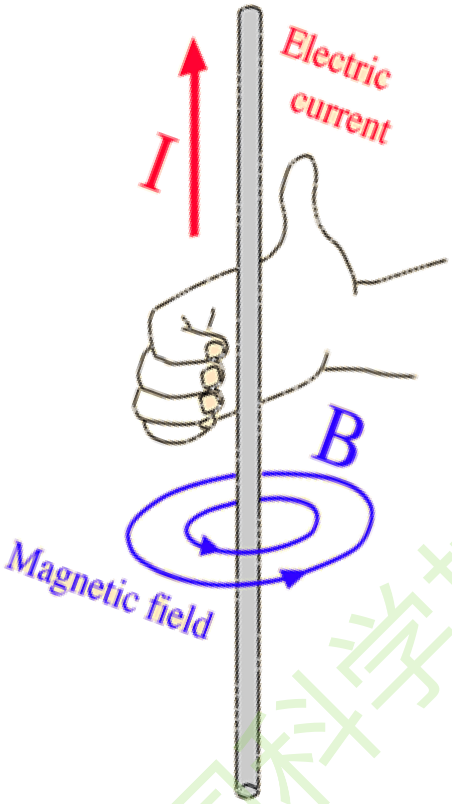
$$= -\frac{\mu_0 I}{2\pi}(r_0^2 + l^2)^{-\frac{1}{2}}\Big|_{l=0}^{l=\infty} = -\frac{\mu_0 I}{2\pi r_0}$$



(b)

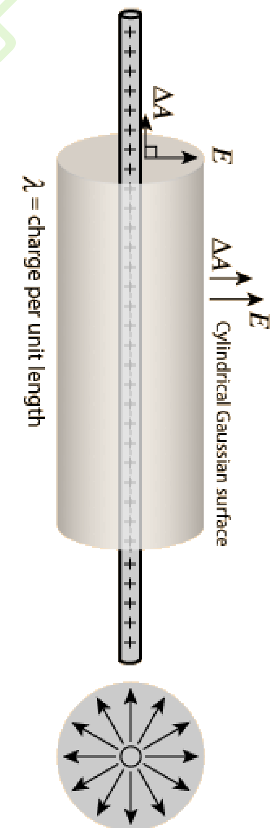


$$B = \frac{\mu_0 I}{2\pi r}$$



稳恒电流

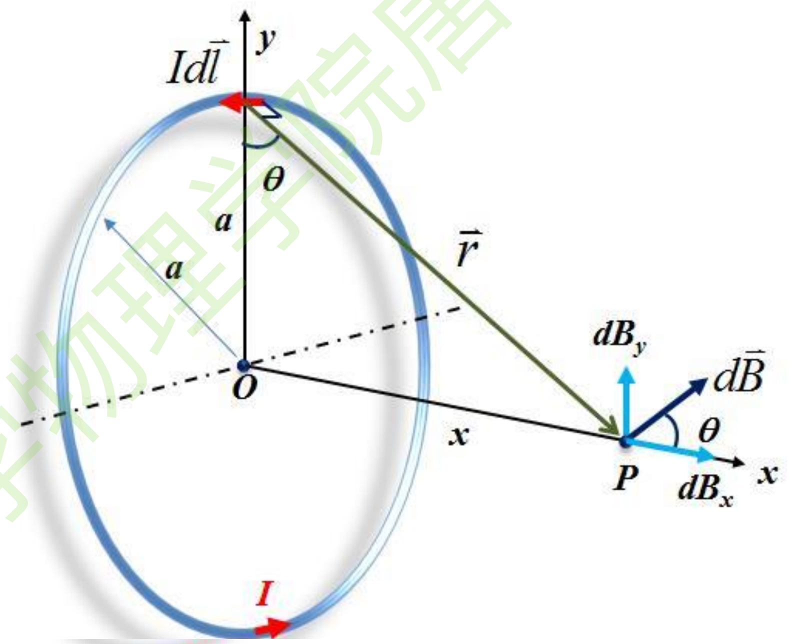
$$E = \frac{\lambda}{2\pi \varepsilon_0 r}$$



静止电荷

【例4.2】半径为a的圆电流I，求轴线上距离圆心为x的P点处的磁场。

【解】由对称性可知，只有x分量



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{(Id\vec{l}\times\vec{r})}{r^3}$$

$$dB_x = \frac{\mu_0}{4\pi}\frac{Idl\cos\theta}{r^2}$$

$$B_x = \oint_L dB_x = \oint_L \frac{\mu_0}{4\pi}\frac{Idl\cos\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\cos\theta}{r^2}\oint_L dl$$

$$B_x = \frac{\mu_0}{4\pi}\frac{I\cos\theta}{r^2}2\pi a = \frac{\mu_0}{2}\frac{Ia\cos\theta}{r^2} = \frac{\mu_0}{2}\frac{Ia^2}{r^3}$$

圆形电流的磁矩

$$\vec{\mu} = I\vec{S} = I\pi a^2 \vec{e}_x$$

则轴线上

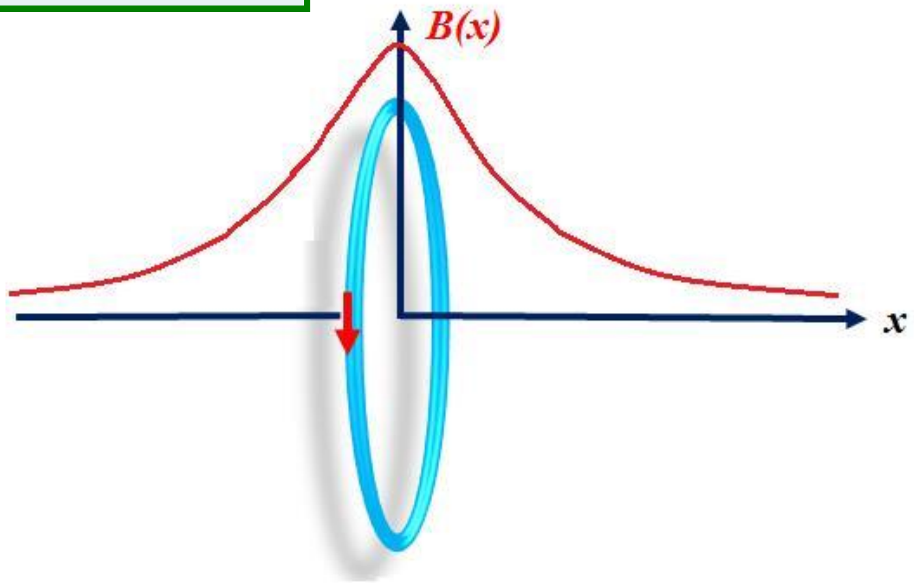
$$\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{r^3}$$

圆心处

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{a^3} = \frac{\mu_0}{2}\frac{I}{a}\vec{e}_x$$

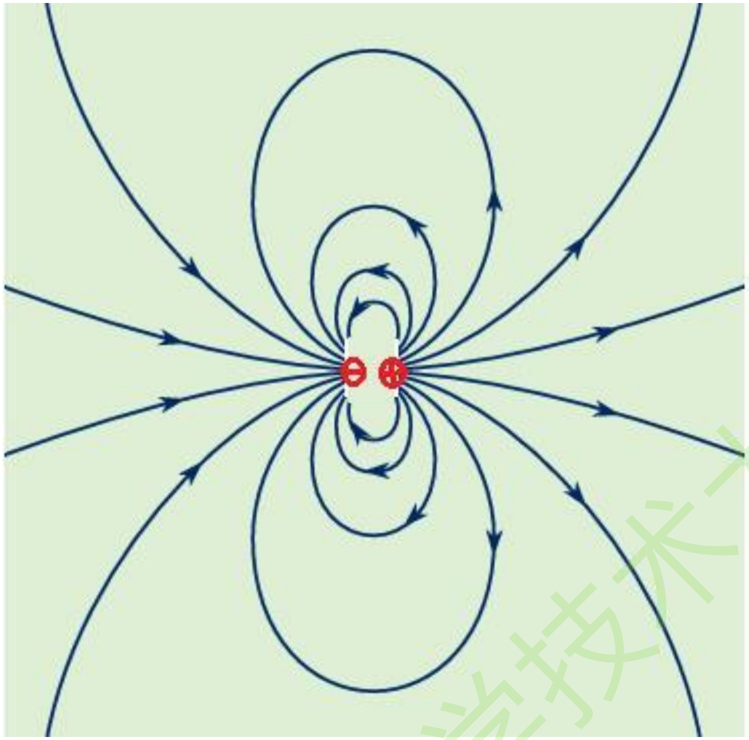
无限远处

$$\vec{B} \approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{x^3}$$



电偶极子

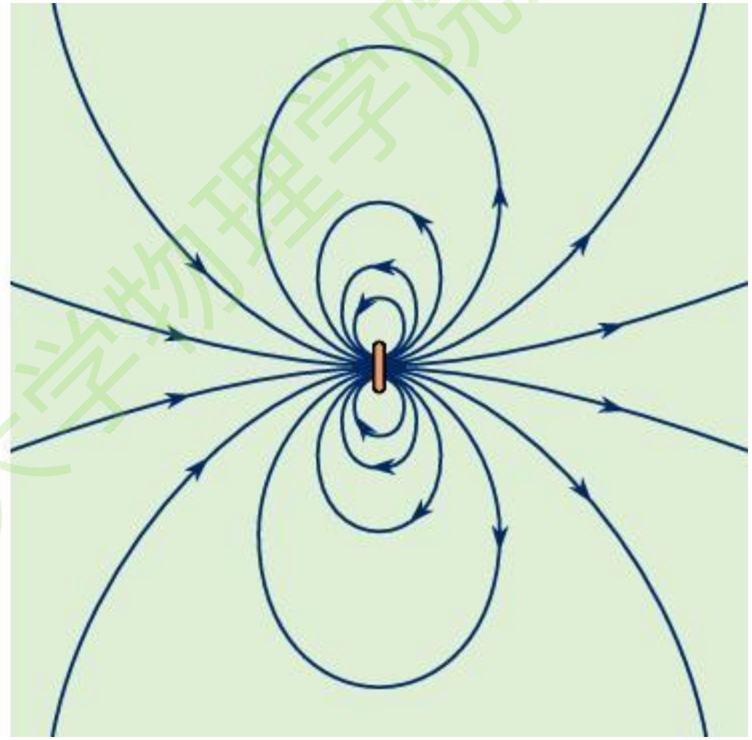
Electric dipole



$$\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}[3(\vec{p}\cdot\vec{e}_r)\vec{e}_r - \vec{p}]$$

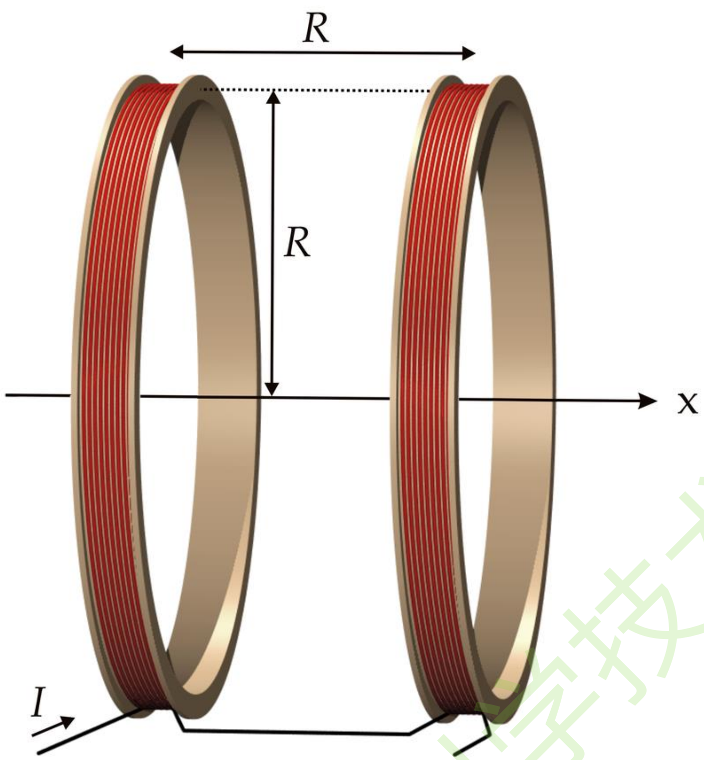
磁偶极子

Magnetic dipole



$$\vec{B} = \frac{\mu_0}{4\pi r^3}[3(\vec{\mu}\cdot\vec{e}_r)\vec{e}_r - \vec{\mu}]$$

亥姆霍兹线圈 (Helmholtz coil)



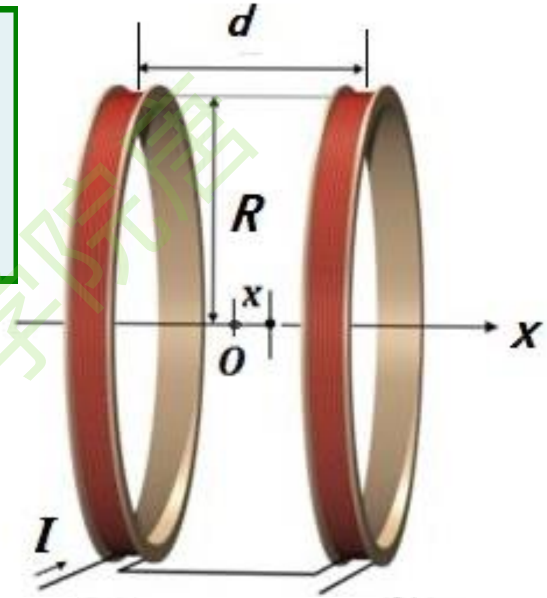
两个圆形线圈共轴、大小一致；

电流方向一致；

线圈距离正好等于线圈半径。

线圈之间的轴线上磁场很均匀。

$$B_x = \frac{\mu_0}{2}\frac{Ia^2}{r^3} = \frac{\mu_0}{2}\frac{Ia^2}{(a^2 + x^2)^{3/2}}$$

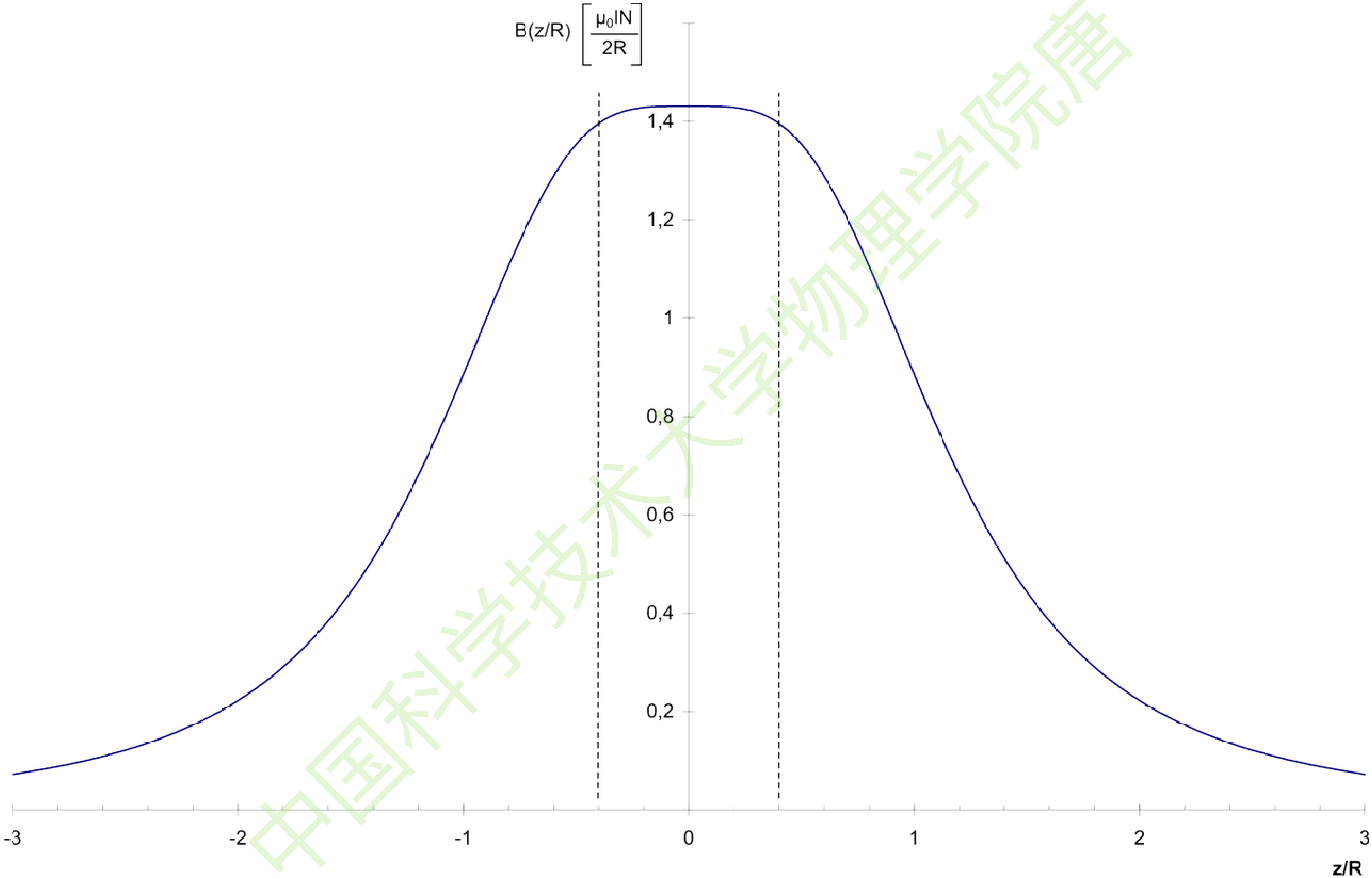


$$B_x = \frac{\mu_0 I R^2}{2}\left[\frac{1}{\left(R^2 + \left(\frac{d}{2} + x\right)^2\right)^{3/2}} + \frac{1}{\left(R^2 + \left(\frac{d}{2} - x\right)^2\right)^{3/2}}\right]$$

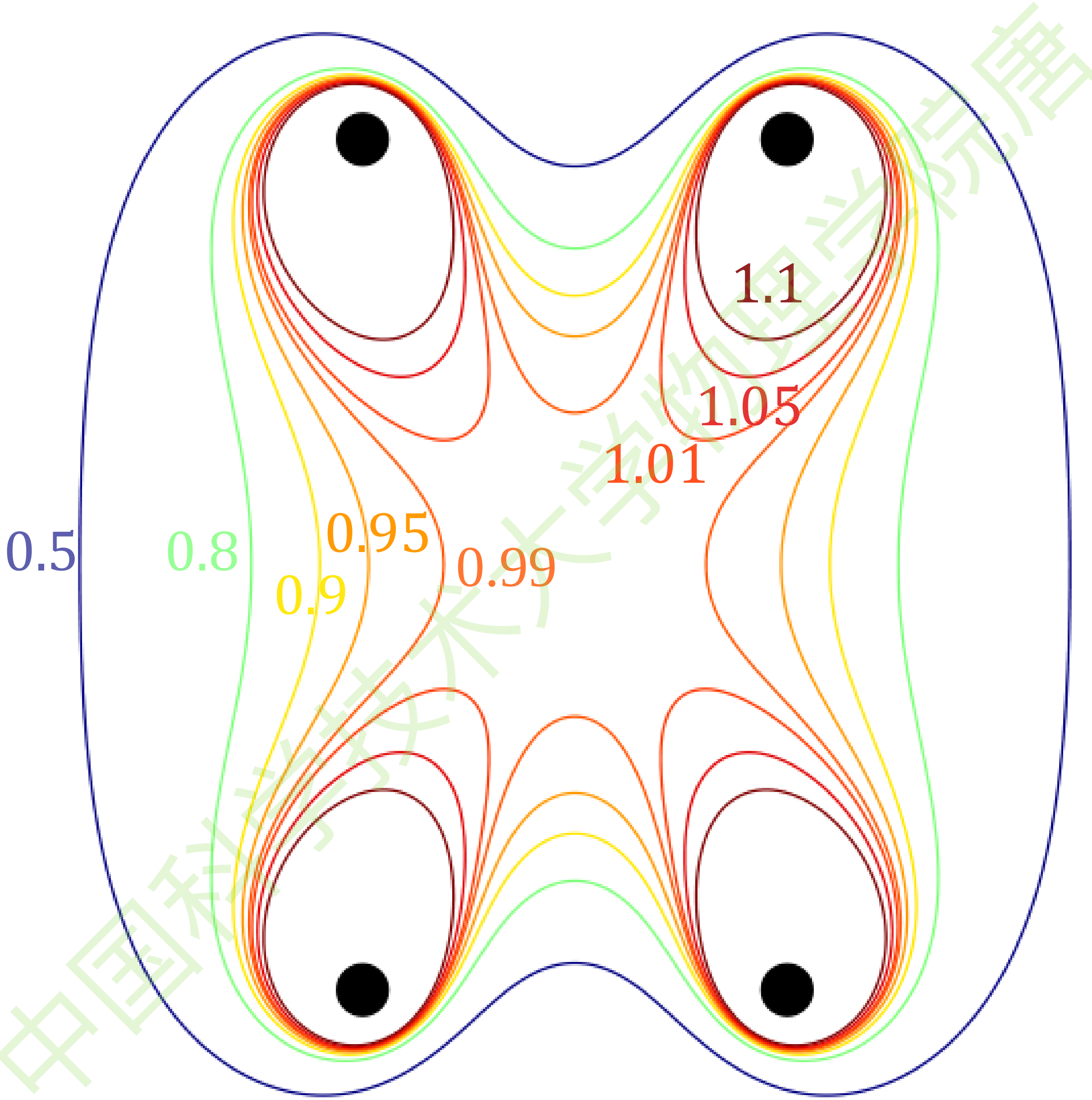
$$\frac{dB_x(x)}{dx} = 0 \Rightarrow x = 0$$

$$\frac{d^2B_x(x)}{dx^2}\bigg|_{x=0} = 0 \Rightarrow d = R$$

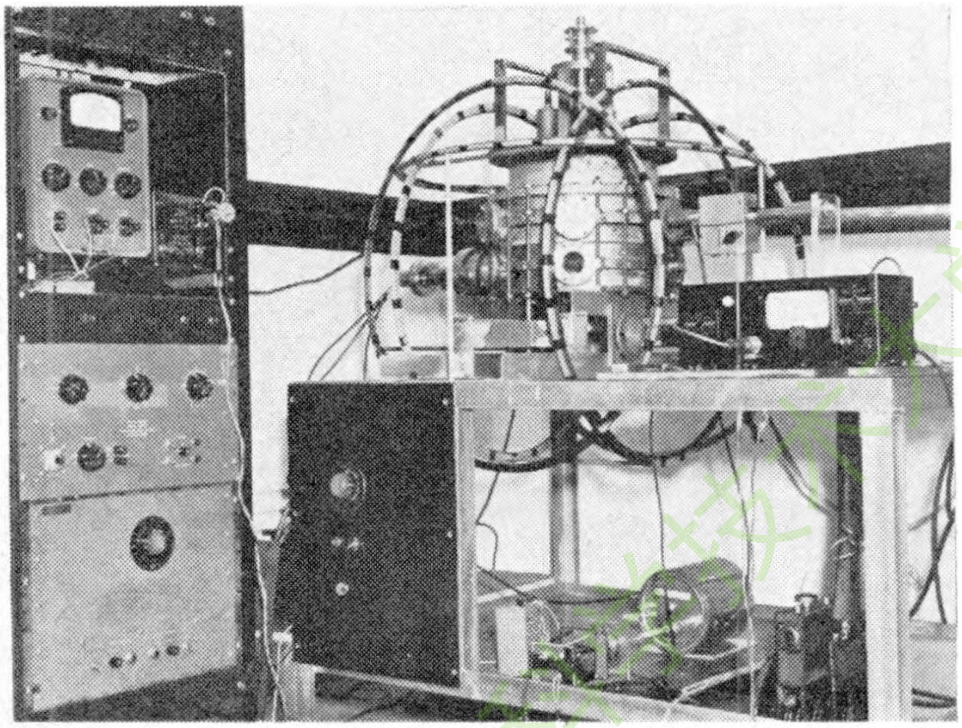
$$B_{0x} = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 I N}{R}$$

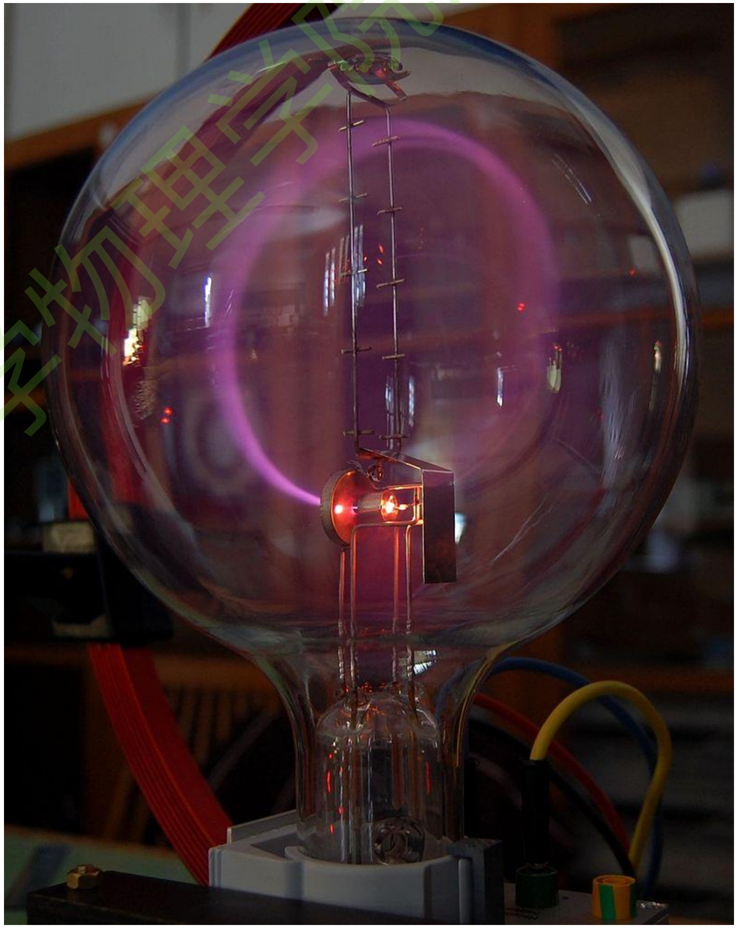


B(z/R) $\left[\frac{\mu_0 IN}{2R}\right]$
1,4
1,2
1
0,8
0,6
0,4
0,2
-3
-2
-1
0
1
2
3
z/R


0.5
0.8
0.9
0.95
0.99
1.01
1.05
1.1

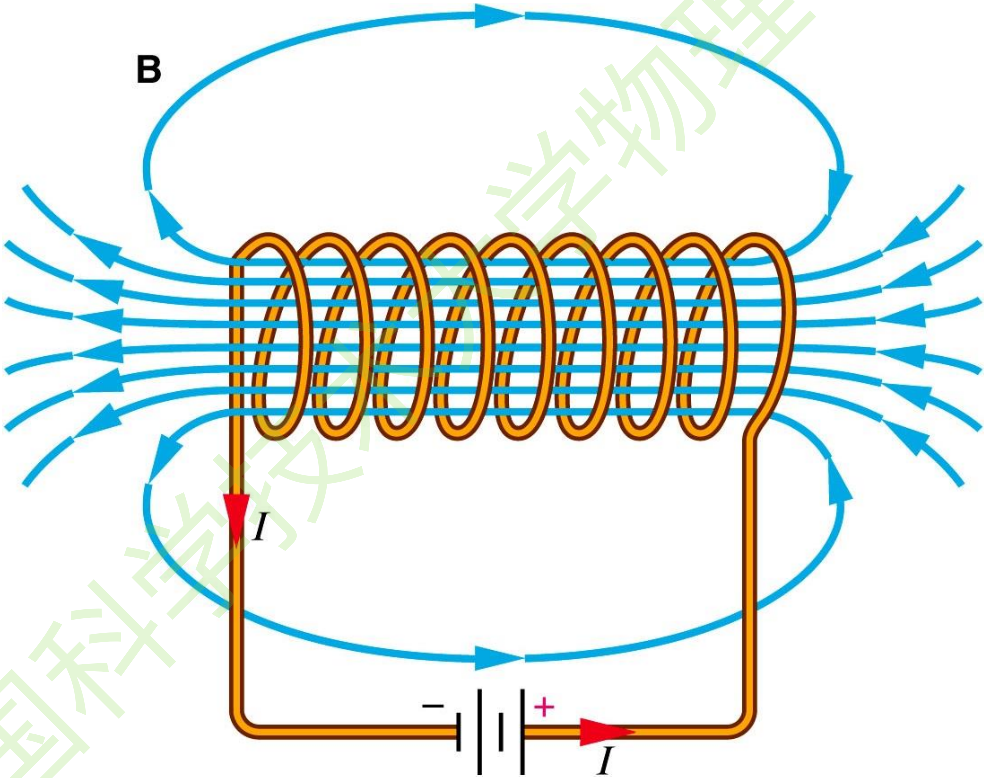
亥姆霍兹线圈

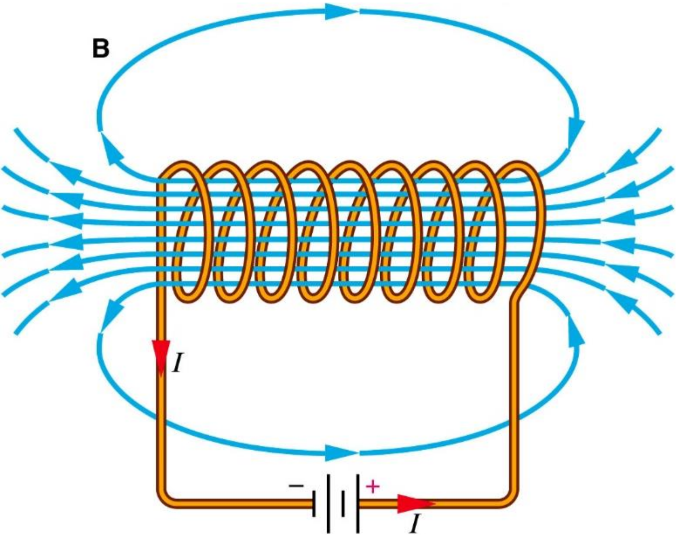


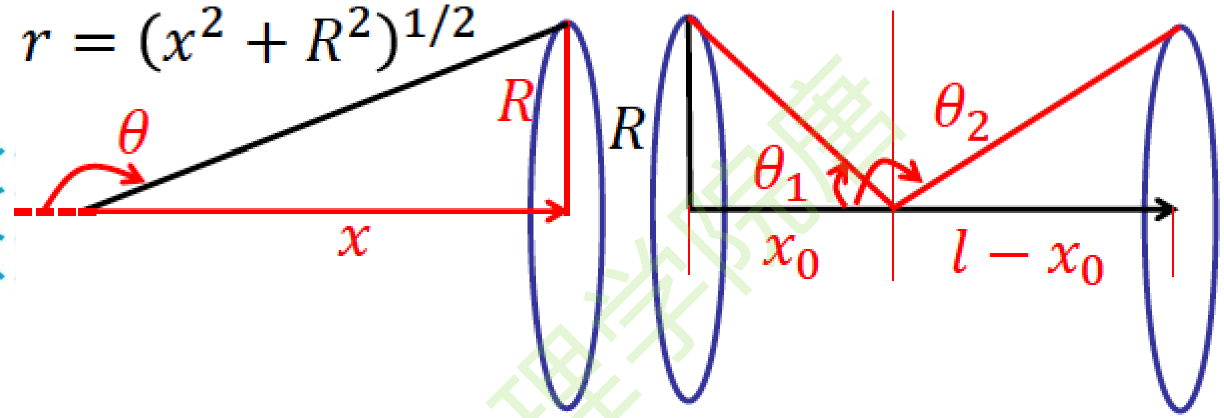


螺线管（Solenoid Coils）

【例】绕在圆柱面上的螺旋形线圈叫螺线管。设它的长度为l，半径为R，单位长度的匝数为n，电流强度为I，求螺线管轴线上的磁感应强度分布。







$$B_x = \frac{\mu_0}{2}\frac{Ia^2}{r^3}$$

$$dB_x = \frac{\mu_0}{2}\frac{R^2}{r^3}(I \cdot ndx)$$

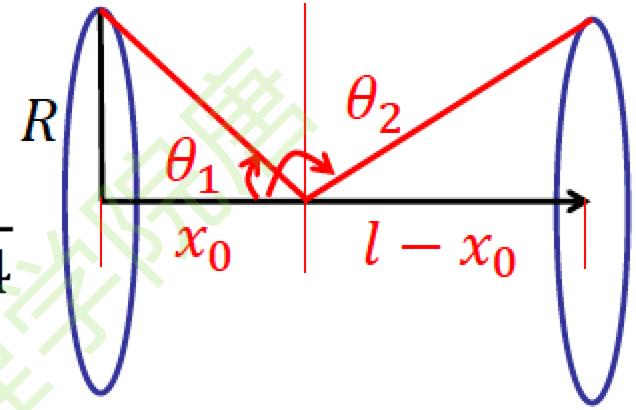
$$B_x = \int dB_x = \int_{-x_0}^{l-x_0} \frac{\mu_0 R^2 nI}{2} r^{-3} \quad dx$$

$$x = -R\cot\theta \, , r = R/\sin\theta$$

$$dx = \frac{R}{sin^2\theta} d\theta$$

$$B_x = \frac{\mu_0 nI}{2}\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = \frac{\mu_0 nI}{2}(\cos\theta_1 - \cos\theta_2)$$

$$B_x = \frac{\mu_0 n I}{2}(\cos\theta_1 - \cos\theta_2)$$



中心处： $x_0 = \frac{l}{2}$ $\quad \cos\theta_1 = \frac{l}{2}/\sqrt{R^2 + l^2/4}$

$\cos\theta_2 = -\cos\theta_1$

$$B_{cx} = \frac{\mu_0 n I}{2} 2\cos\theta_1 = \frac{\mu_0 n I l}{2\sqrt{R^2 + l^2/4}} = \frac{\mu_0 n I l}{\sqrt{4R^2 + l^2}}$$

左侧端点： $x_0 = 0$ $\quad \cos\theta_1 = 0$ $\quad \cos\theta_2 = -l/\sqrt{R^2 + l^2}$

$$B_{0x} = -\frac{\mu_0 n I}{2}\cos\theta_2 = \frac{\mu_0 n I l}{2\sqrt{R^2 + l^2}}$$

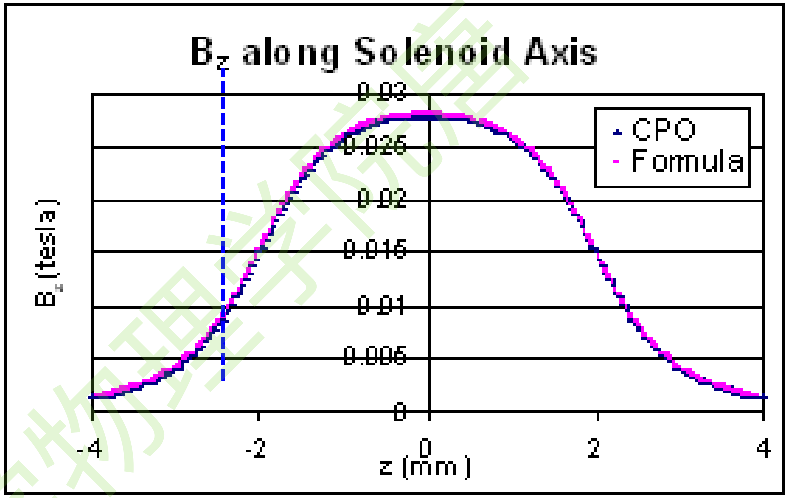
右侧端点： $x_0 = l$ $\quad \cos\theta_1 = l/\sqrt{R^2 + l^2}$ $\quad \cos\theta_2 = 0$

$$B_{lx} = -\frac{\mu_0 n I}{2}\cos\theta_2 = \frac{\mu_0 n I l}{2\sqrt{R^2 + l^2}}$$

$$B_x = \frac{\mu_0 n I}{2}(\cos\theta_1 - \cos\theta_2)$$

$$B_{cx} = 0.028T, B_{0x} = 0.015T$$

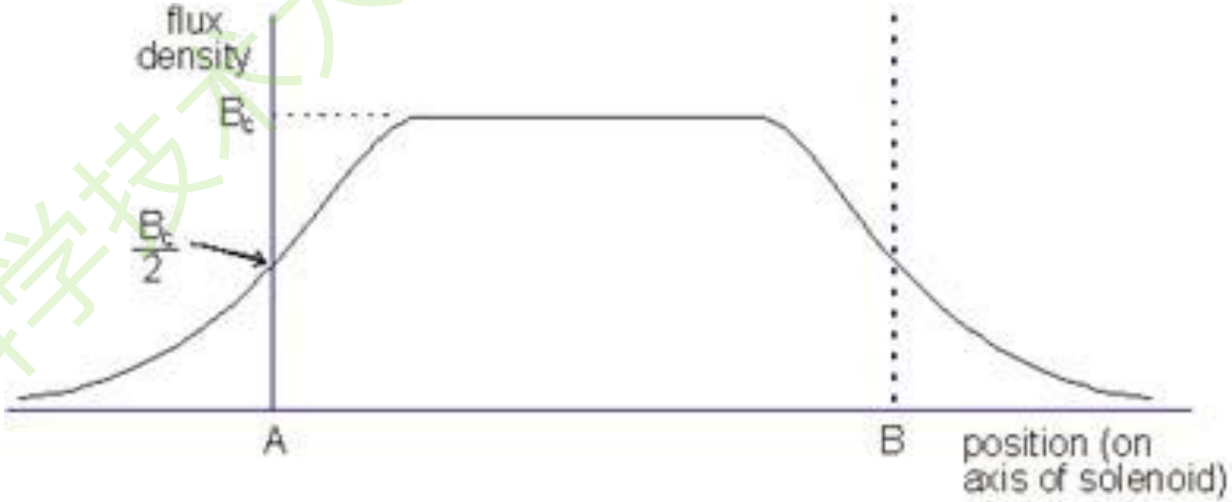
R=1cm, l=4cm,I=1A, n=25000



当$l \gg R$：

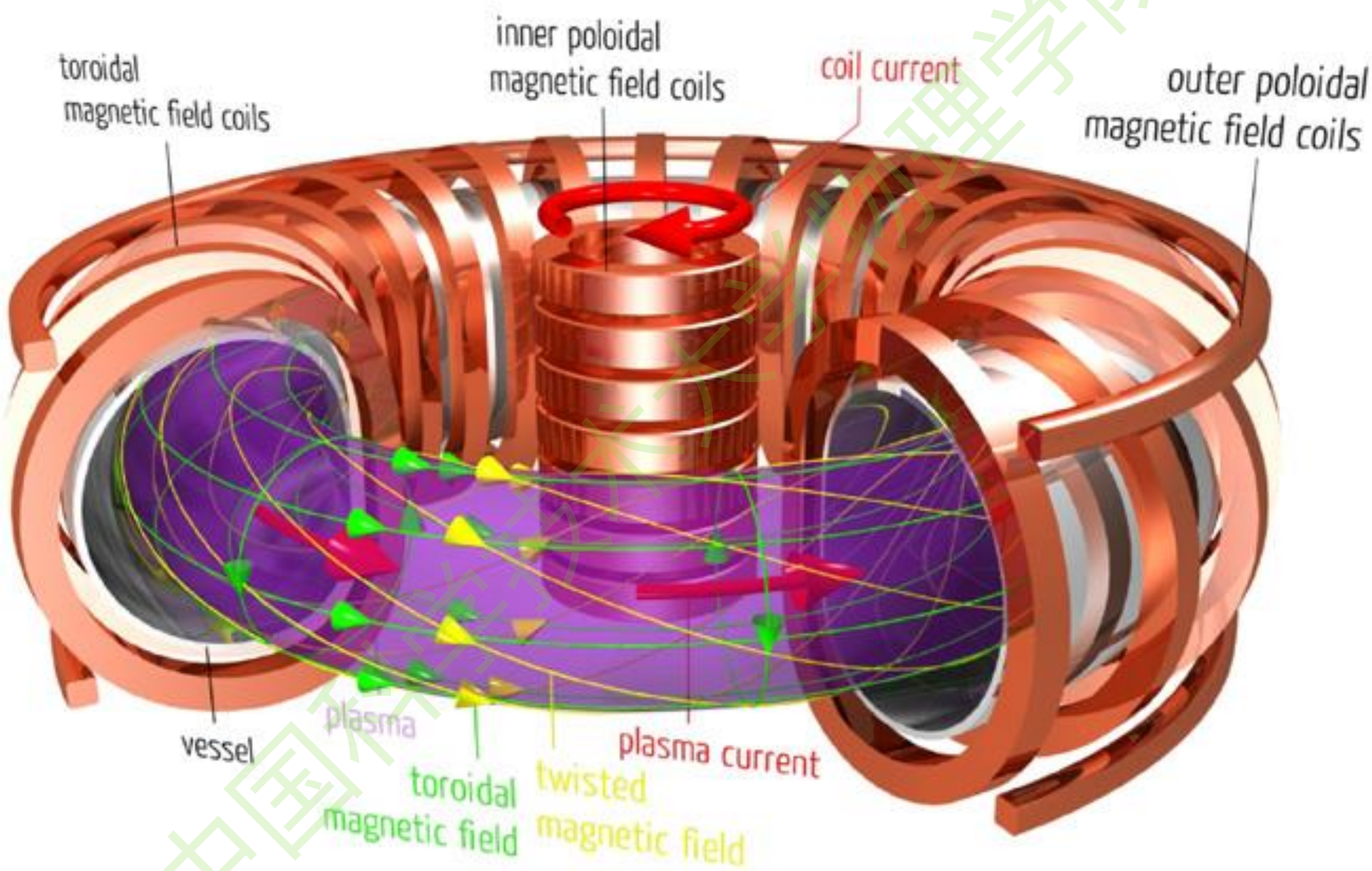
$$B_{cx} = \mu_0 n I$$

$$B_{0x} = \frac{\mu_0 n I}{2}$$



托克马克中的磁场

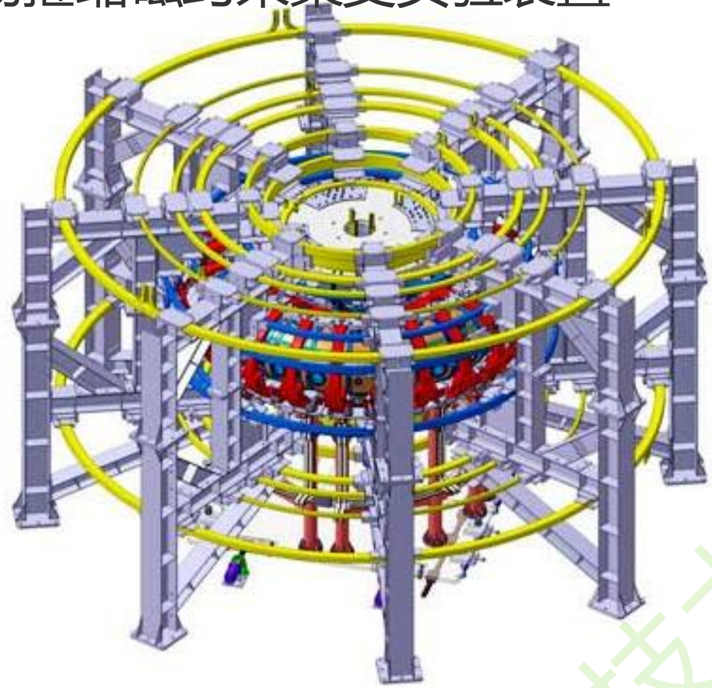
Tokamak

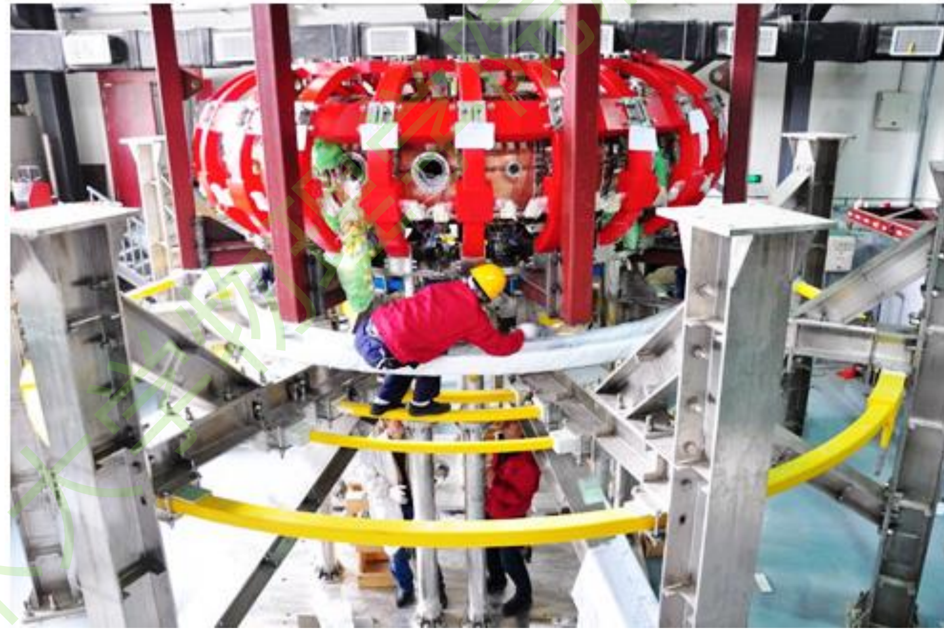


科大一环 (KTX)

反场箍缩磁约束聚变实验装置

Keda Torus eXperiment





科大一环者，非道路之名，非珍珠钻石之实，我国首台反场箍缩磁约束聚变装置之称也。昨日中国科学报头版推出，人民日报、人民网、安徽财经、科学网、环球网、合肥晚报诸媒体，纷纷跟进，俨然网络熟词也。余嗟曰：名成一日，事成十年。

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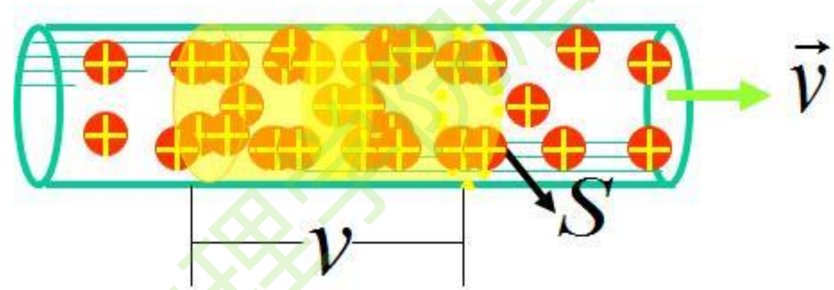
科大一环者，聚变之一道，其价不输珍珠钻石也。

物理学院 刘万东 教授

运动电荷产生的磁场

一束载流子定向运动



$$\vec{j} = nq\vec{v}$$

取一电流元 $\vec{j}dV$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{j}dV \times \vec{r}}{r^3}$$

电流元中的电荷数

$$dN = ndV$$

单个运动电荷产生的磁场

$$\vec{B} = \frac{d\vec{B}}{dN} = \frac{\mu_0}{4\pi}\frac{\vec{j}\,dV \times \vec{r}}{ndVr^3} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \vec{r}}{r^3}$$

$$\vec{B}=\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$$

$$\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q\vec{r}}{r^3}$$

$$\vec{B}=\vec{v}\times\frac{\mu_0}{4\pi}\frac{q\vec{r}}{r^3}=\mu_0\varepsilon_0\vec{v}\times\frac{1}{4\pi\varepsilon_0}\frac{q\vec{r}}{r^3}=\mu_0\varepsilon_0\,\vec{v}\times\vec{E}$$

$$\vec{B}=\frac{1}{c^2}\vec{v}\times\vec{E}$$

← 相对论下依然成立

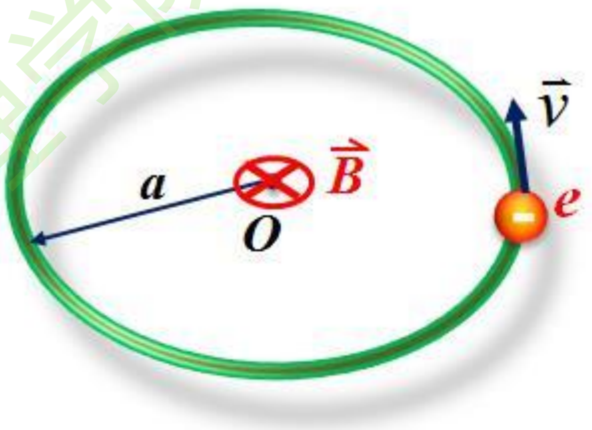
相对论下：

$$\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{q\vec{e}_r}{r^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}$$

运动电荷产生的磁场

一个电子做匀速圆周运动产生的磁场

圆心处：
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{ev}{a^2}\vec{e}_x$$



轴线其他地方：对时间求平均，垂直于轴线部分抵消

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{ev}{r^2}\frac{a}{r}\vec{e}_x = \frac{\mu_0}{4\pi}\frac{eva}{r^3}\vec{e}_x$$

§ 4.2.2 磁场对电流的力与力矩

均匀磁场中的力

线电流：$d\vec{F} = Id\vec{l} \times \vec{B}$

$$\vec{F} = \int_L Id\vec{l} \times \vec{B}$$

面电流：$d\vec{F} = \vec{i}\, dS \times \vec{B}$

$$\vec{F} = \iint_S \vec{i}\, dS \times \vec{B}$$

体电流：$d\vec{F} = \vec{j}\, dV \times \vec{B}$

$$\vec{F} = \iiint_V \vec{j}\, dV \times \vec{B}$$

运动电荷：$\vec{F} = q\vec{v} \times \vec{B}$

均匀磁场中的力矩

线电流：

$$\vec{M}=\int_L \vec{r}\times\left(Id\vec{l}\times\vec{B}\right)$$

面电流：

$$\vec{M}=\iint_S \vec{r}\times\left(\vec{i}\times\vec{B}\right)dS$$

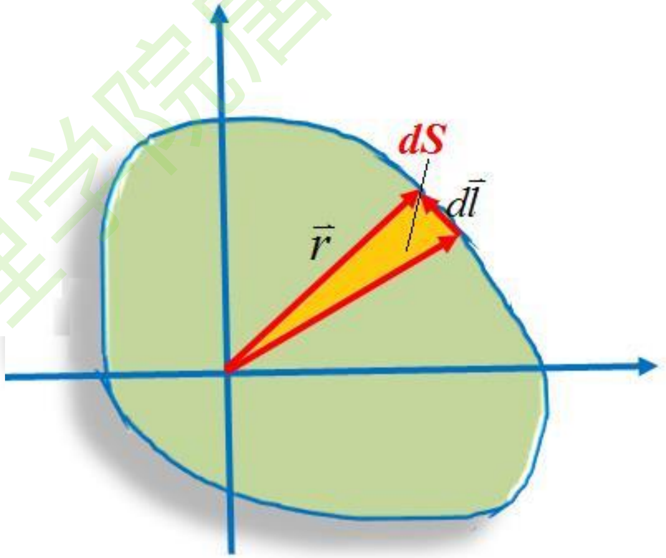
体电流：

$$\vec{M}=\iiint_V \vec{r}\times\left(\vec{j}\times\vec{B}\right)dV$$

运动电荷：

$$\vec{M}=q\vec{r}\times\left(\vec{v}\times\vec{B}\right)$$

【例】求一电流强度为I的载流线圈在均匀磁场B中所受的力与力矩。



【解】由于 $d\vec{l} = d\vec{r}$

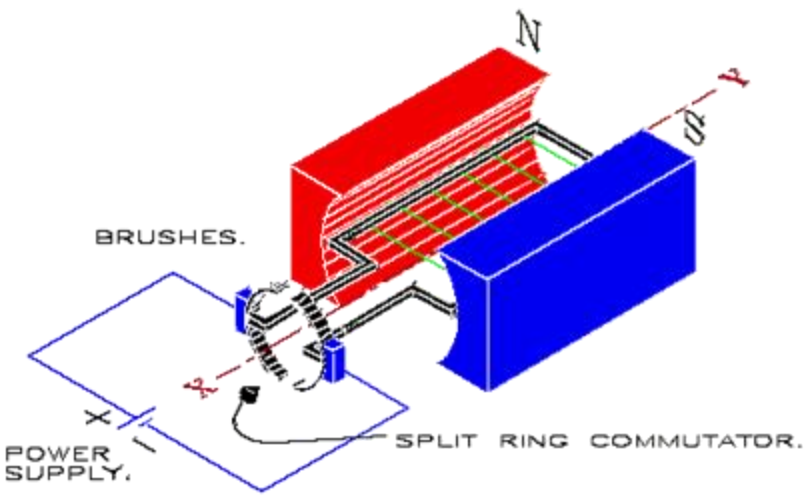
$$\vec{F} = \oint_L I\,d\vec{r} \times \vec{B} = I\left(\oint_L d\vec{r}\right) \times \vec{B} = 0$$

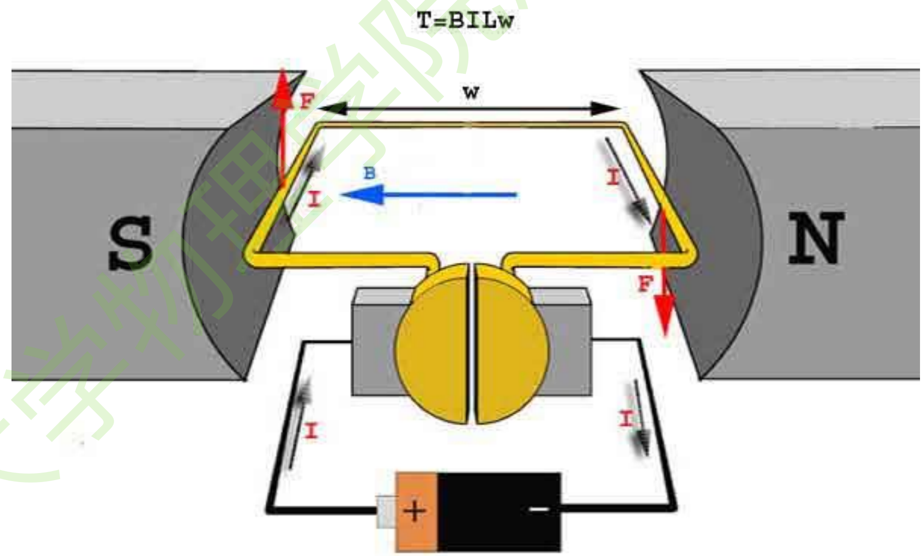
$$\vec{M} = \oint_L \vec{r} \times (I d\vec{r} \times \vec{B}) = I\left(\frac{1}{2}\oint_L \vec{r} \times d\vec{r}\right) \times \vec{B} = \vec{\mu} \times \vec{B}$$

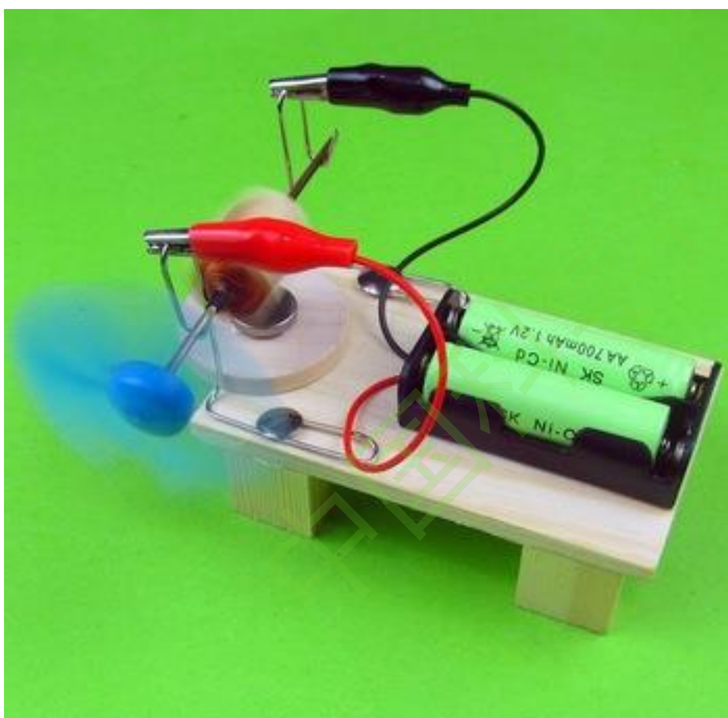
磁矩：$\vec{\mu} = I\vec{S}$

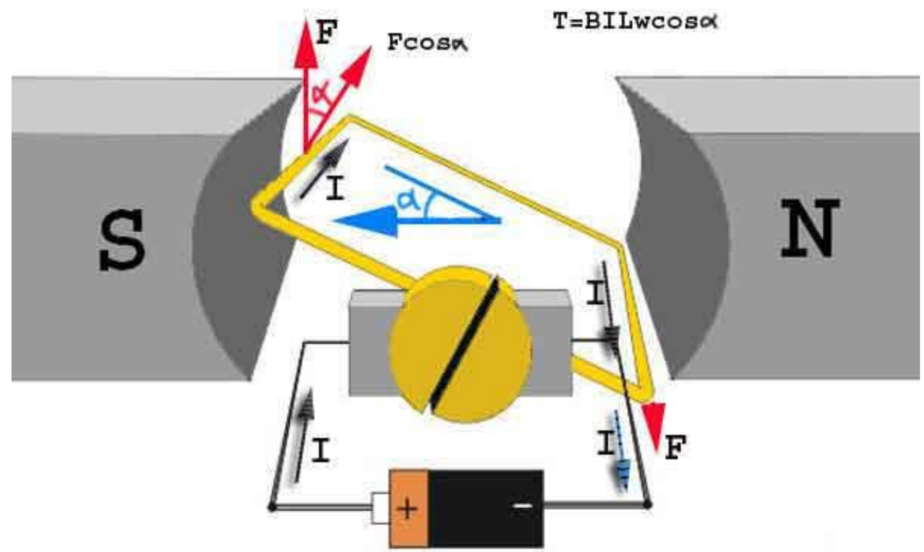
闭合线圈在均匀磁场中受力为零，力矩不为零

直流电动机


N
S
Y
X
BRUSHES.
SPLIT RING COMMUTATOR.
POWER SUPPLY.


T=BILw
w
F
I
B
S
N
+
−




F
Fcosα
T=BILwcosα
α
I
S
N
+
−

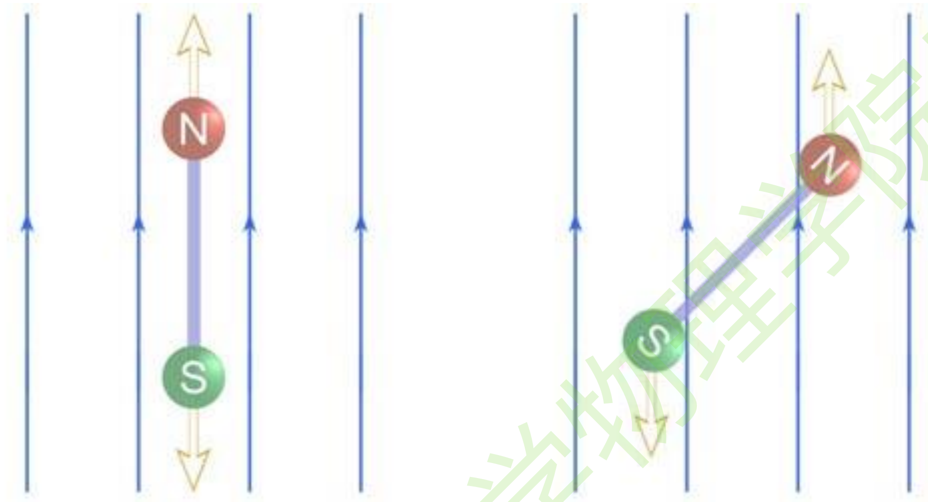
闭合线圈在非均匀磁场中的力与力矩

梯度力：

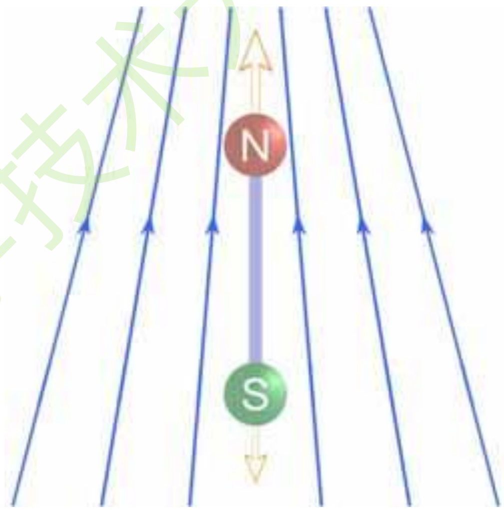
$$\vec{F} = (\vec{\mu} \cdot \nabla)\vec{B} = \left(\mu_x \frac{\partial}{\partial x} + \mu_y \frac{\partial}{\partial y} + \mu_z \frac{\partial}{\partial z}\right)\vec{B}$$

$$\vec{M} \approx \vec{\mu} \times \vec{B}$$

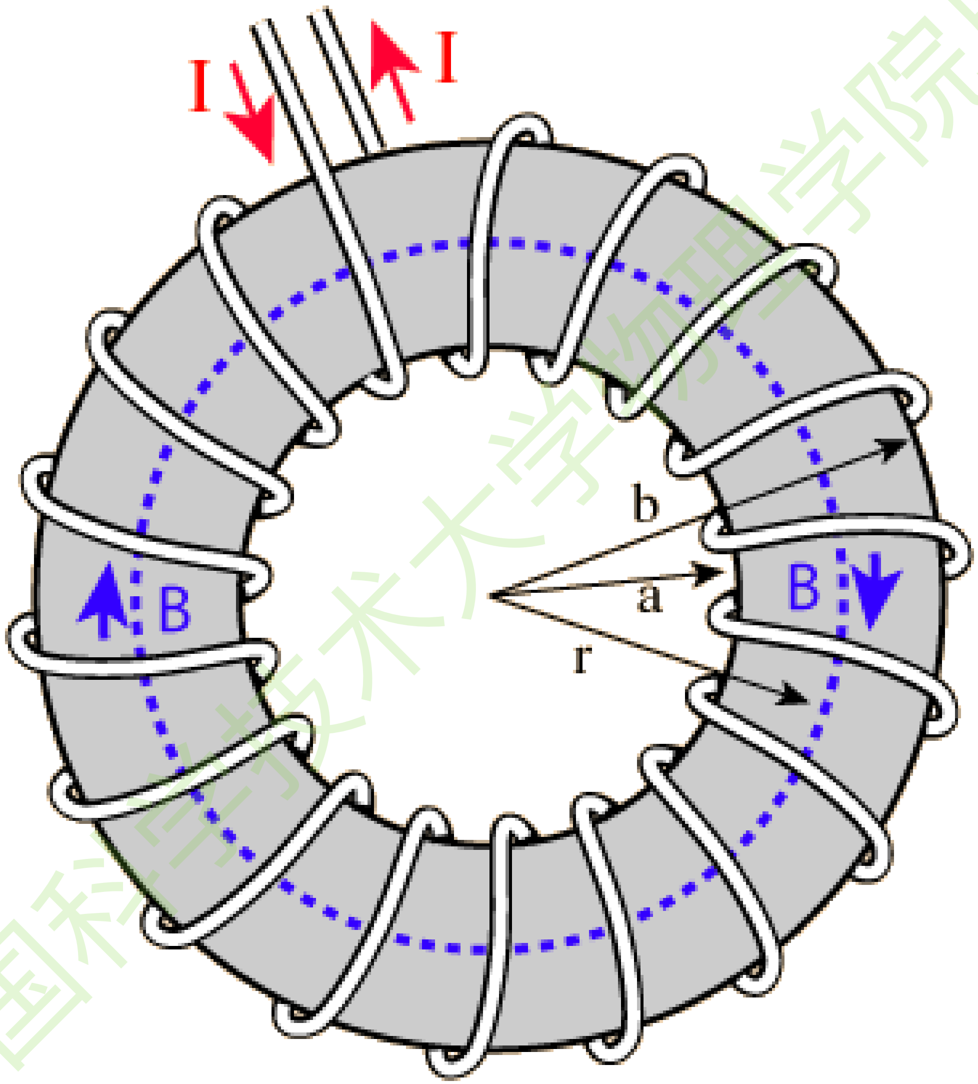
均匀磁场



非均匀磁场



§ 4.3 静磁场的基本定理

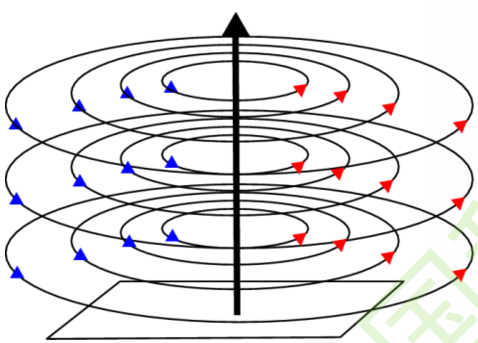


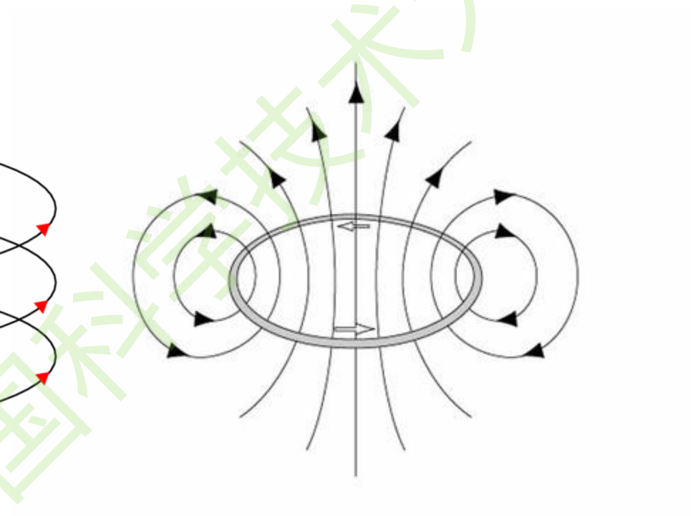
§ 4.3.2 磁场高斯定理

高斯定理： 通过任意**闭合**曲面的磁通量等于零

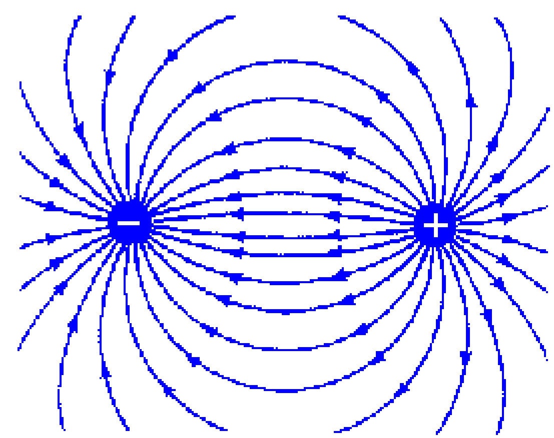
$$\Phi = \oiint_S \vec{B} \cdot d\vec{S} = 0$$

物理意义： 反映了磁场的“无源性”，即孤立磁荷不存在。





磁场线没有源头



电场线有源头