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The experiments of Biot and Savart concerning the force exerted by a current on a magnetic needle

Herman Erlichson
Department of Applied Sciences, The College of Staten Island, The City University of New York,
Staten Island, New York 10314

Francois Arago brought the news of Oersted's discovery of the effect of a current-carrying wire on a magnet to Paris on September 4, 1820. André-Marie Ampère and the team of Jean-Baptiste Biot and Félix Savart quickly set to work to establish a quantitative law for this effect. Their approaches were quite different. This paper describes the experiments of Biot and Savart and their results. It also briefly discusses the approach of André-Marie Ampère, who coined the name ''electrodynamics'' and whose fundamental formula gave the force between two infinitesimal current elements. Ampere's formula fell into disuse after the advent of Maxwell's field approach. Biot and Savart's experimental law, in the modern form of the differential magnetic field due to a current element, became the standard starting point for calculating the magnetic field due to steady currents. © *1998 American Association of Physics Teachers.*

I. AMPERE'S FORMULA AND AMPERE'S CIRCUITAL LAW

The scene is Paris in the fall of 1820. François Arago, returned from travel abroad, brought the news of Oersted's discovery of the magnetic effect of an electric current on a magnet. This was on September 4, 1820. On September 11, 1820 Arago reproduced Oersted's experiments.[1] Sometime between September 25 and October 2, Ampere discovered the attraction and repulsion between two long, parallel current-carrying wires.[2] In the months that followed Ampère gave careful consideration to the nature of the new electrodynamic force and decided to model it after the Newtonian gravitational force.

Like Newton, who stressed the ''mathematical principles'' of natural philosophy, Ampère's great work on electrodynamics was called by him ''On the mathematical theory of electrodynamic phenomena, experimentally deduced.''[3] Like Newton who sought to avoid ''hypotheses,'' Ampère did likewise, and proclaimed

> ''I have relied solely on experimentation to establish the laws of the phenomena and from them I have derived the formula which alone can represent the forces which are produced; I have not investigated the possible cause of these forces, convinced that all research of this nature must proceed from pure experimental knowledge of the laws and from the value, determined solely by deduction from these laws, of the individual forces in the direction which is, of necessity, that

of a straight line drawn through the material points between which the forces act.'' [4]

The experimental basis for Ampère's formula was a set of four ''equilibrium'' experiments. Ampere described the principle of his experiments as follows:

> ''it consists in establishing by experiment that a moving conductor remains exactly in equilibrium between equal forces, or between equal rotational moments, these forces and these moments being produced by portions of fixed conductors of arbitrary shape and dimension without equilibrium being disturbed in the conditions of the experiment, and in determining directly therefrom by calculation what the value of the mutual action of the two infinitesimal portions must be for equilibrium to be, in fact, independent of all variations of shape and dimension compatible with the conditions.'' [5]

From these equilibrium experiments Ampere deduced his formula. This formula for the force between two infinitesimal current elements was

$$dF \alpha \frac{ii' \, ds \, ds'}{r^2} \{\sin \theta \sin \theta' \cos \omega - (1/2)\cos \theta \cos \theta'\},$$

where i and i' are the currents in the two elements, ds and ds' are the lengths of the elements, r is the distance between the elements, θ and θ' are the angles between the elements and the line r, and ω is the angle between the plane defined by ds and r and the plane defined by ds' and r.

Maxwell lavished praise on Ampère's work and hailed it as follows:

> ''The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the 'Newton of electricity.' It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electro-dynamics.'' [6] However, the development of Maxwell's field theory of electromagnetism focused attention on the field vector **B**, and Ampère's formula fell into essentially complete disuse. Sometime around the 1930s Ampère's name came to be associated with ''the first law of circulation'' (a designation common in England). This law is

$$\oint \mathbf{B} \cdot \mathbf{ds} = \mu_0 i,$$

where the line integral of B is around a closed path and i is the net current traversing the area bounded by the closed path. This first law of circulation is commonly ascribed to Ampère in many textbooks, where it is called ''Ampère's Circuital Law.'' However, it is not due to Ampere.[7]

II. THE MEASUREMENT OF THE MAGNETIC FORCE ON A COMPASS NEEDLE BY THE METHOD OF OSCILLATIONS

We now turn to our main concern, the experiments of Biot and Savart. These experiments were clever, painstaking, and highly original. On September 30, 1820, Biot and Savart announced their results for the distance dependence of the magnetic force exerted by a long, straight current-carrying wire on a magnetic needle to the French Academy. These

were the first quantitative measurements of Oersted's effect. They involved the timing of damped oscillations of a magnetic needle by an accurate Bréguet chronometer. We describe in some detail the theory of these measurements.

Figure 1 shows a magnetic needle suspended from a vertical axis through a point A. It is situated in a plane which is perpendicular to a long, straight current-carrying wire. When the current is started in the wire, magnetic forces act on the two poles of the needle in directions which are perpendicular to the line from the wire to each of the two poles. For simplicity we have shown only the components of forces which produce a torque about A. These components produce a torque which is proportional to the magnetic force F and to $\sin \theta$. We write the rotational analog of Newton's second law:

$$-lF \sin \theta = I \frac{d^2 \theta}{dt^2}, \tag{1}$$

where l is the length of the needle and I is its moment of inertia about the rotation axis A. The system is arranged so that the angular departure from the equilibrium position is relatively small. This permits the use of the approximation

$$\sin \theta \approx \theta,$$

so that the equation of motion is

$$-lF \theta = I \frac{d^2 \theta}{dt^2}. \tag{2}$$

This equation of motion is the well-known equation of simple harmonic motion whose prototype is the spring-mass system of force constant k and mass m:

$$-kx = m \frac{d^2 x}{dt^2}. \tag{3}$$

The solution of Eq. (3) is an undamped sinusoidal oscillation of angular frequency

$$\omega = \sqrt{\frac{k}{m}}.$$

By analogy, the solution of (2) is an undamped sinusoidal oscillation of angular frequency

$$\omega = \sqrt{\frac{lF}{I}}. \tag{4}$$

The period of this oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{lF}}.$$

Thus

$$F \alpha \frac{1}{T^2}. \tag{5}$$

The magnetic force F exerted on each pole of the needle is inversely proportional to the square of the period of oscillation. In the actual experiment of Biot and Savart, damping is present and what is observed is a damped oscillation of period very closely equal to T. The inverse square of the period is therefore a measure of the magnetic force F, and by changing the distance D from the long straight wire to the magnetic needle the variation of the force with distance can be observed by noting the change in the period of oscillation.
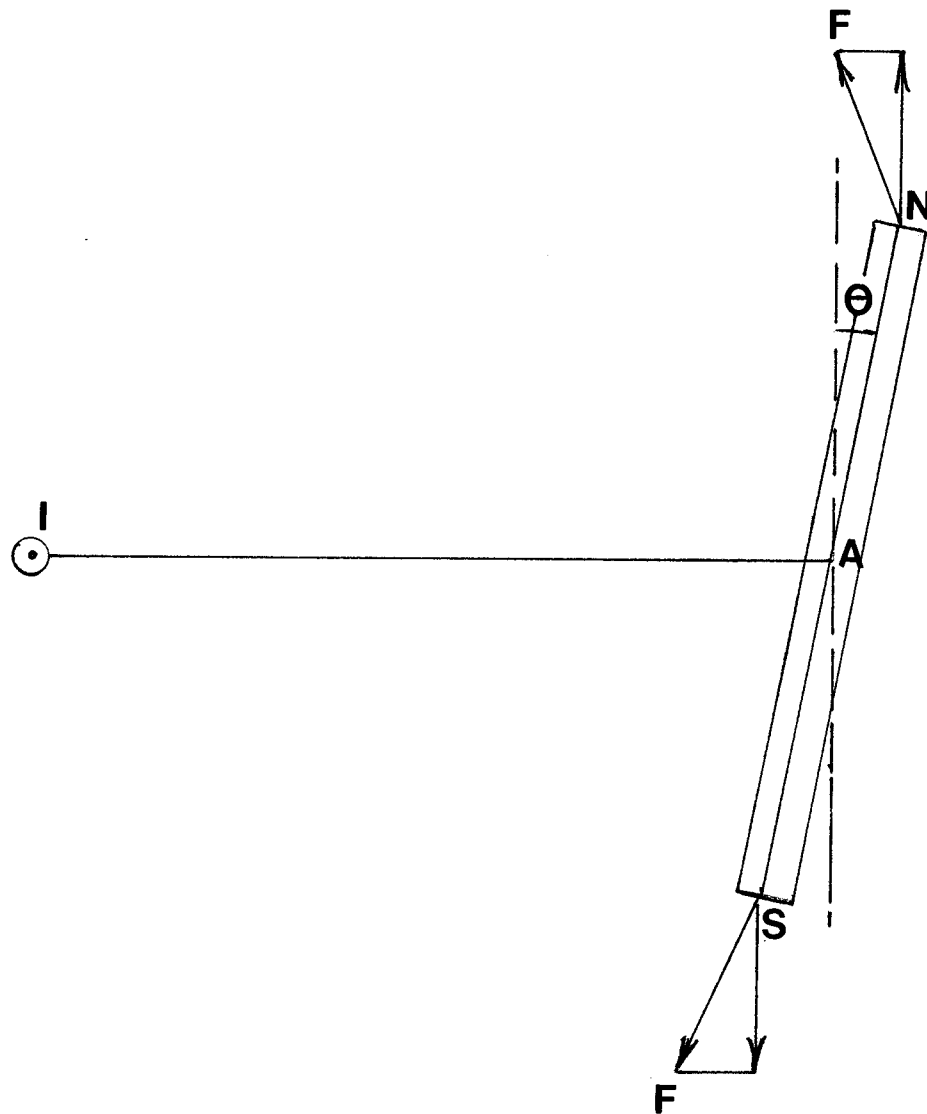
Herman Erlichson

Fig. 1. A magnetic needle is acted upon by two forces due to the magnetic field of a long straight wire.

This is the theory of the first Biot–Savart experiment. Their result was that the force varied inversely as the distance from the wire to the needle. This result was presented by them to the Académie des Sciences on October 30, 1820 and reported in the *Annales de Chimie et Physique* as follows:

> "By these procedures MM. Biot and Savart arrived at the following result which rigorously represents the action experienced by a molecule of austral or boreal magnetism when placed at some distance from a fine and indefinite ('indefinitely long'-Au.) cylindrical wire which is made magnetic by voltaic current. Drawing a perpendicular to the axis of the wire from the point where the magnetic molecule resides, the force influencing the molecule is perpendicular to this line and to the axis of the wire. Its intensity is inversely proportional to the distance." [8]

III. DETAILS OF THE FIRST BIOT–SAVART EXPERIMENT

The best source for the details of both the first and the second Biot–Savart experiments is the third edition (1824) of Biot's textbook, *Précis Élémentaire de Physique*. Here, we utilize the English translation of O. M. Blunn as contained in Tricker's book.

Figure 2 shows the experimental arrangement for the first Biot–Savart experiment. *CMZ* is the long straight wire through which current is passed to produce the magnetic field which exerts forces on the magnetic needle *AB*. *A'B'* is a bar magnet which compensates for the magnetic field of the Earth. The magnetic needle is suspended horizontally on a silkworm thread inside a glass cage as shown. The compensating magnet and the magnetic needle are aligned along the local magnetic meridian, and the distance from the compensating magnet to the needle is adjusted until the compensating field is equal and opposite to the Earth's field.
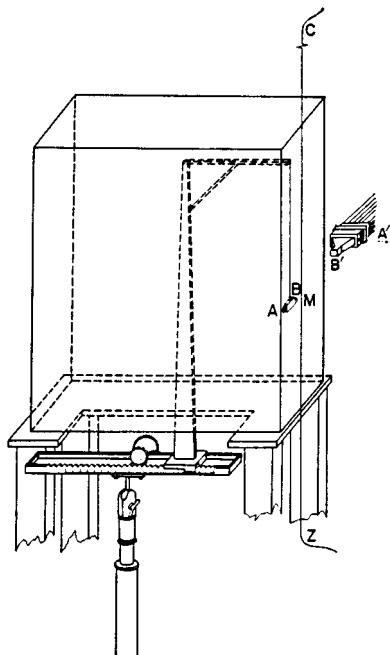
Fig. 2. The experimental arrangement for the first Biot–Savart experiment (taken from Tricker, Ref. 2, p. 120).

Order of the observations	Distances from the connecting wire to the center of the needle (mm)	Duration of ten oscillations (sec)	Ratio of the observed forces, compared with the force at 30 mm
1	30	42·25	
2	40	48·85	3/4 $(1-0·008508)$
3	30	42·00	
4	20	33·50	3/2 $(1-0·023090)$
5	30	41·00	
6	50	54·75	3/5 $(1-0·036673)$
7	30	42·25	
8	60	56·75	2/6 $(1-0·095460)$
9	30	41·75	
10	120	89·00	3/12 $(1-0·103892)$
11	30	42·50	
12	15	30·00	2/1 $(1-0·067010)$
13	30	43·15	

The description of the experiment by Biot leaves no doubt that the experimenters were skillful and meticulous in their conduct of the experiment. For example, in discussing the length of the long straight wire (supposed to be of infinite length) Biot remarked ''It was long enough for its extremities to be bent back and connected to the poles of the battery and still only exert such a feeble effect on the needle that it could confidently be ignored.'' [9]

The experimenters determined that the force on each pole was in the plane perpendicular to the wire, and also perpendicular to the line from the wire to the pole. To see how they concluded the latter, consider Fig. 3, which is taken from Biot's text as reproduced by Tricker. The figure shows the needle AB in the static equilibrium position under the action of the symmetric magnetic forces AD' and BD, plus a third force due to the outbound tension component in the silk thread. From the symmetry of the problem the angle FAD' equals the angle FBD. In addition, if the pole at A is imagined at B, the force on it would make the same angle with FB as does the force on the actual pole at B, except in the opposite direction. Thus this force would be in the BD' direction. So, if angle FBD' = angle FBD, the sum of these two angles is 180 deg, each of these angles must be equal to 90 deg. Biot concludes:

> ''when an indefinite ('indefinitely long'—Au.) connecting wire, animated by voltaic current, acts on an element of austral or boreal magnetism situated at a certain distance FA or FB from its center, the resultant of the actions which it exerts is perpendicular to the shortest distance between the molecule and the wire.'' [10]

Among the experimental difficulties confronted by Biot and Savart was the problem of identifying the precise moment of the start of an oscillation. The magnetic needle was instantaneously stationary at the end points of the oscillations and therefore hard to pinpoint in time. To solve this problem Biot and Savart used vertical threads to mark the equilibrium position of the oscillations and counted the oscillations from these equilibrium positions. They also compensated for progressive weakening of the battery which supplied the current to the long wire by using an averaging technique. This technique involved taking a first and third run at their standard distance (which was 30 mm) and comparing this average with the data taken in between the two runs at the standard distance. Using this averaging technique, they presented their results in Table I.

Biot and Savart then computed a theoretical time for ten oscillations at their standard distance of 30 mm based on assuming an exact inverse distance dependence of the force with distance. They presented their results in a second table, here shown (Table II). Noting that the differences between the calculated and observed times for ten oscillations were alternately positive and negative, as they should be, and declaring that ''a more probably physical interpretation can be placed on the proportionality which this law discloses if it is
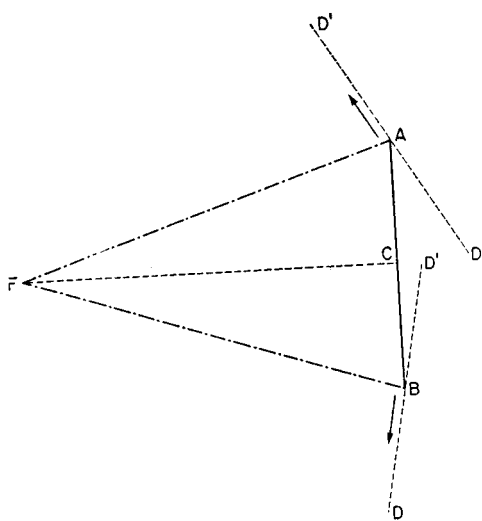

Fig. 3. Biot's drawing to show that the magnetic force is transverse to the line from the wire to each pole of the needle (taken from Tricker, Ref. 2, p. 124).

Order of the observations	Distances to the wire	Duration of ten oscillations		Difference (sec)
		Calculated (sec)	Observed (sec)	
2	40	48·62	48·85	−0·23
4	20	33·88	33·50	+0·38
6	50	53·74	54·75	−1·01
8	60	59·40	56·75	+2·65
10	120	84·25	89·00	−4·75
12	15	30·99	30·00	+0·99

assumed to apply, not to the distance from the centre of the needle to the wire, but to the distance between the wire and the two poles of the needle,'' [11] they took this as their interpretation.

IV. THE SECOND BIOT−SAVART EXPERIMENT— FROM THE OVERALL FORCE TO THE DIFFERENTIAL FORCE OF A CURRENT ELEMENT

According to Biot, Laplace had calculated what the differential action of a differential section of the wire should be in order that the action of the total wire should vary as $(1/d)$, where d is the perpendicular distance from the wire to either pole of the magnetic needle.[10] Biot did not reproduce Laplace's calculation, but it can be easily done. Figure 4 shows an infinite wire and a magnetic pole at point P. We take it to be a North pole (in the language of Biot, an Austral pole). The action of the wire, in which current is flowing up, on the North pole is out of the plane of the paper. For simplicity we also assume this to be the direction of the infinitesimal action of an infinitesimal current element $i\,ds$ shown in the figure. We have

$$dF = \frac{ki\,ds}{r^2} f(\theta),\qquad(6)$$

where it has been assumed that the magnetic force is inverse square, k is a constant of proportionality, r is the distance from the current element to the Austral pole, and $f(\theta)$ is
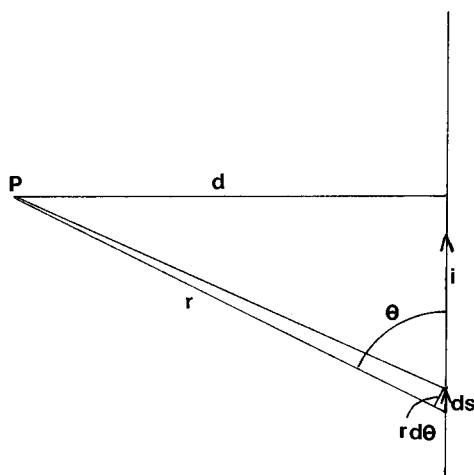

Fig. 4. Diagram for calculating the total force exerted by an infinite current-carrying wire on a magnetic pole.
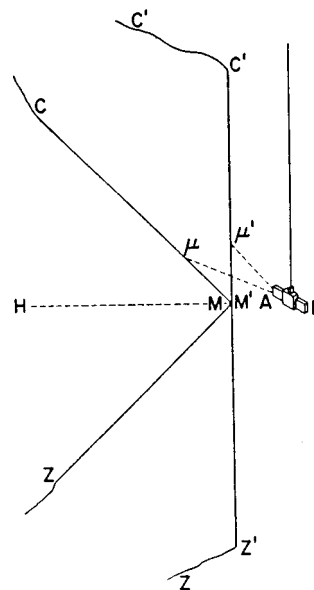
Fig. 5. The second Biot−Savart experiment (taken from Tricker, Ref. 2, p. 135).

some unknown function of the angle θ between the vector **r** and the current element.

The action of the entire wire (θ going from 0° to 180°) is inversely proportional to d, as was known from Biot and Savart's first experiment. To obtain this, we integrate, but we first make the substitution

$$ds(\sin\theta) = r\,d\theta.$$

We have

$$dF = \frac{k\,i\,d\theta}{r\sin\theta} f(\theta).$$

Now we integrate on θ from 0 to $\pi/2$ and double the result:

$$F = 2ki \int_0^{\pi/2} \frac{f(\theta)\,d\theta}{r\sin\theta}.$$

We note that $\sin\theta = d/r$. Hence,

$$F = \frac{2ki}{d} \int_0^{\pi/2} f(\theta)\,d\theta.\qquad(7)$$

We see from this result that any $f(\theta)$ is compatible with the inverse d dependence of the total force F. This is why Biot said ''the distance ratio may be modified by a coefficient which depends on the inclination of each distance to the general direction of the wire; that is to say, such a coefficient, whatever its composition, does not prevent the total action of a straight and indefinite wire being reciprocal to the shortest distance to the magnetic element, in accordance with our observations.'' [12] Then he continued, ''It was therefore necessary to carry out fresh experiments to find out whether the coefficient did in fact exist, and to discover how it was composed; the simplest and most direct way of doing this was obviously to compare the actions exerted on the same magnetic element by two equal portions of indefinite wires in different directions.'' [13]

The second Biot−Savart experiment was designed to answer these questions. The experimental arrangement is

Table III. The ratio of the action of the inclined wire to the straight wire.

i	$\tan(i/2)$	$i/90°$
10°	0.087	0.111
20°	0.176	0.222
30°	0.268	0.333
40°	0.364	0.444
45°	0.414 214	0.500
50°	0.466	0.556
60°	0.577	0.667
70°	0.700	0.778
80°	0.839	0.889
90°	1.000	1.000

Fig. 6. Figure for the derivation that the total action goes as the tangent of $\alpha/2$.

shown in Fig. 5, taken from Biot's text. The two long wires ZMC and $Z'M'C'$ are identical in material and cross section, and stretched in a vertical plane. $Z'M'C'$ is straight and ZMC is bent at the point M, with the two segments ZM and MC making the same angle with the horizontal. The two points M and M' were separated only by a strip of very fine paper. The magnetic needle AB was suspended with its longitudinal axis AB at the same height as M and M'. The vertical plane of the two wires was arranged to be perpendicular to the local magnetic meridian.

The basic idea of the experiment was to compare the force exerted by the inclined wire with the force exerted by the straight wire, by comparing the inverse of the squares of the periods of the harmonic oscillation produced by each. The experimenters were once again careful to have the equilibrium position of the needle coincide with the magnetic meridian of the Earth. They partially neutralized the magnetic field of the Earth, but partly retained it as a stabilizing effect for the neutral position of the needle. The early results of this comparison seemed to indicate that the action of the long inclined wire was proportional to θ but Biot observed that

> ''other laws were equally admissible and, for example, the inclination i to the horizontal could have been replaced by the tangent to half of the inclination, that is to say, $\tan(i/2)$; then, by putting F for the observed action on the needle of the vertical wire at a certain distance, $F\tan(i/2)$ would be the action of the oblique wire animated by the same current; formerly it would have been represented by $Fi/90°$, a value which can only ever differ from the foregoing in hundredths.'' [14]

What Biot was saying was that the ratio of the actions of the wire inclined at i to the vertical wire was $i/90$ on the one supposition and $\tan(i/2)$ on the other. Consider Table III, which compares these two possibilities. It is clear from this table that the two ratios differ in the hundredths place, as remarked upon by Biot.

To determine which was indeed the correct value of the action, Biot and Savart studied the particular case $i=45°$ with great care. They doubled the oblique wire so that this doubled the action. This gave them the theoretical value of $2(0.414\,214)=0.828\,427$ for the ratio of the actions on the assumption that $\tan(i/2)$ is the correct action. After their corrections Biot and Savart found the experimental value of $0.827\,545$, which they took as confirmation that the total action was correctly represented by $\tan(i/2)$.
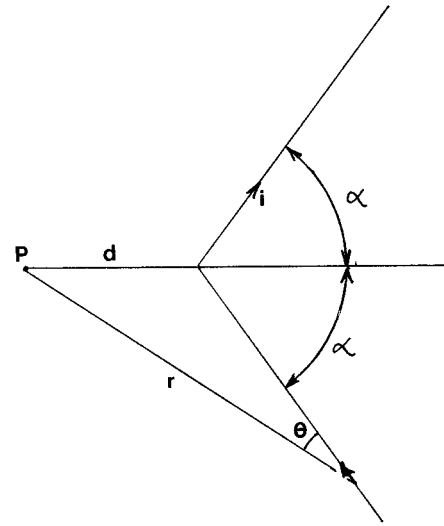
To conclude their paper it only remained for the investi-

gators to determine what function $f(\theta)$ in Eq. (1), when integrated for the case of the inclined wire, would yield a result which went as the tangent of 1/2 the angle of inclination. Biot stated that the function $\sin\theta$ would satisfy the condition. We verify this by considering Fig. 6 and doing the integration. Figure 6 shows the two semi-infinite wires, each inclined at an angle α to the horizontal. (Note that our angle α in Fig. 6 corresponds to Biot's inclination angle i).

The current element $i\,ds$ is a distance r from the magnetic pole at P. The angle between the directed current element and the r direction is θ. The differential force exerted by the current element on the pole is

$$dF = \frac{ki\,ds}{r^2}\sin(\theta),$$

$$F = 2ki\int_0^\alpha \frac{ds\,\sin(\theta)}{r^2}. \tag{8}$$

As before,

$$r\,d\theta = ds\,\sin\theta,$$

$$F = 2ki\int_0^\alpha \frac{ds\,\sin(\theta)}{r^2} = 2ki\int_0^\alpha \frac{d\theta}{r}.$$

From the law of sines,

$$\frac{\sin\theta}{\sin\alpha} = \frac{d}{r}.$$

Hence,

$$F = \frac{2ki}{d\sin\alpha}\int_0^\alpha \sin\theta\,d\theta = \frac{2ki(1-\cos\alpha)}{d\sin\alpha}$$

$$F = \frac{2ki}{d}\tan\frac{\alpha}{2}. \tag{9}$$

This verifies Biot's statement that Eq. (8), when integrated, yields a total action which goes as the tangent of half the inclination angle.

V. THE SIGNIFICANCE OF BIOT AND SAVART'S WORK

The rise of the field approach to electricity and magnetism soon converted the Biot–Savart force law on a magnetic pole to the modern form for the field vector **dB**:

$$\mathbf{dB} = \frac{\mu_0}{4\pi} i \frac{\mathbf{ds} \times \hat{\mathbf{r}}}{r^2}. \tag{10}$$

Since the direction of the force on a North magnetic pole is the same as that of the magnetic flux density **B**, we see that the historic Biot–Savart law, Eq. (8), and the modern form, Eq. (10), are essentially the same. This important law permits the calculation of **B** for any arrangement of steady currents. Biot foresaw this importance when he wrote

> "this makes the elementary action of a section proportional to $R^{-2} \sin \omega$; using this experimentally determined expression, knowing the absolute direction of the force to be perpendicular to a plane through each distance in the direction of each longitudinal element of wire, the total resultant of the action exerted by a wire, or some straight or curved, limited or unlimited portion of wire, can be assigned by calculation." [15]

The development of field theory as the dominant theory in electricity and magnetism has accentuated the importance of the Biot–Savart law and virtually relegated the attempt of Ampère to base electrodynamics on a Newtonian-type force law to the role of historical curiosity piece. Without taking away from the importance of Ampère, it would seem appropriate to also stress the importance of the painstaking laboratory work of Biot and Savart. Ampère and the team of Biot and Savart were the giants of the early history of electrodynamics.

[1]L. Pearce Williams, "What were Ampere's Earliest Discoveries in Electrodynamics?" Isis **74**, 492–508 (1983).

[2]L. Pearce Williams, "What were Ampere's Earliest Discoveries in Electrodynamics?" Isis **74**, 508 (1983). In his book entitled *Early Electrodynamics, The First Law of Circulation* (Pergamon, Oxford, 1965) R. A. R. Tricker said that Ampère had established this within a week of September 11 (p. 23).

[3]This work will be referred to simply as Ampère's *1827 Mémoire*. The full title is *Mémoire sur la théorie mathématique des phénomènes électrodynamiques uniquement déduite de l'expérience, dans lequel se trouvent réunis les Mémoires que M. Ampère a communiqués à l'Académie royale des Sciences, dans les séances de 4 et 26 décembre 1820, 10 juin 1822, 22 décembre 1823, 12 septembre et 21 novembre 1823, 12 septembre et 21 novembre 1825*. The *1827 Mémoire* was republished with a foreword by Edmond Bauer (Paris, 1958). Portions of the *1827 Mémoire* were translated into English by O. M. Blunn and were included in Tricker's book (Ref. 2).

[4]Ampère, *1827 Mémoire*, in Ref. 2, p. 156.

[5]Ampère, *1827 Mémoire*, in Ref. 2, p. 160. Ampere's description of these equilibrium experiments with accompanying diagrams can be found on pp. 161–172 of Tricker (Ref. 2).

[6]J. C. Maxwell, *A Treatise on Electricity and Magnetism* (Clarendon, Oxford, 1891, republished by Dover, 1954), 3rd ed., Vol. Two, p. 175.

[7]See J.-P. Mathieu, "Sur le théorème d'Ampère," Revue d'Histoire des Sciences et de leurs Applications **43**, 333–338 (1990). Also see James R. Hoffman, *André-Marie Ampère* (Cambridge U.P., Cambridge, 1996, first published in 1995 by Blackwell Publishers, Oxford), p. 349. For additional bibliographic information on Ampère, Biot, and Savart see the articles on them in *The Dictionary of Scientific Biography*, edited by C. C. Gillispie (Scribner's, New York, 1970). Additional interesting references are: L. Pearce Williams, "Why Ampère did not discover electromagnetic induction," Am. J. Phys. **54**, 306–311 (1986); "André-Marie Ampère," Sci. Am. **260**, 90–97 (January, 1989); M. O. Peach and L. K. Shirely, "What Ampère could have done: A simple demonstration of the Biot-Savart and magnetic force laws," Am. J. Phys. **50**, 410–414 (1982).

[8]Ann. Chim. Phys. **15**, as translated by O. M. Blunn in Tricker, Ref. 2, pp. 118–119.

[9]*Précis Elémentaire de Physique*, Vol. II, 3rd ed., 1824, as translated by O. M. Blunn in Tricker, Ref. 2, p. 122.

[10]In Ref. 9, p. 126.

[11]In Ref. 9, p. 131.

[12]In Ref. 9, p. 134.

[13]In Ref. 9, p. 134.

[14]In Ref. 9, pp. 136–137. In the second edition of Biot's *Précis Elémentaire de Physique*, Biot had stated that the force found in his early runs of the second experiment was proportional to the angle of inclination i, and that this overall result for the inclined wire could be analyzed and "appears to indicate" that the force exerted by a differential element was proportional to sine θ, where θ is the angle between the direction of the current element and the line from the current element to the magnetic needle. Ampere noted that this was an error and that it was an overall force proportional to the tangent of $i/2$ which would be the correct result if the force of the differential element was proportional to sine θ. Ampere further noted that Savary had found this out and that Biot had not acknowledged this in his corrected version in the third edition of the *Précis*. We quote the relevant statement by Ampère in the Blunn translation as given on the bottom of pp. 198–199 in Tricker: "It is remarkable that this law, which is a corollary of the formula by which I have represented the interaction of two elements of conducting wires when, according to my theory, each magnetic element is replaced by a very small electrodynamic solenoid, was first found through a mathematical error; indeed, for the law to be valid, the *absolute intensity* ought to have been proportional, not to the angle ZMH, but to the tangent of half this angle, as demonstrated later by M. Savary in his dissertation at the Académie, 3 February 1823, and which has meanwhile been published in the Journal de Physique, vol. XCVI, pp. 1–25 con't. It appears that M. Biot later discovered the error himself, for in the third edition which has just appeared, he describes, without reference to the Mémoire where it had first been corrected, new experiments where the intensity of the total force is, in accordance with the calculation of M. Savary, proportional to the tangent of half the angle ZMH, and he concludes therefrom with more reason than he had with his first experiments, that the force which he calls elementary, is proportional for equal distance to the sine of the angle between the direction of the element of conducting wire and the direction of the straight line joining its mid-point to the magnetic molecule."

[15]In Ref. 9, p. 139.

EMPTY NUMBERS

Scientists make observations to perhaps three or four digits, and process the results on an eight-digit calculator. Careful scientists then discard meaningless digits. They also use numbers and numerical words with care. But some writers use *words* with a precision corresponding to only one significant digit!

Vernon Booth, *Communicating in Science—Writing a Scientific Paper and Speaking at Scientific Meetings* (Cambridge University Press, 1993), 2nd ed., p. 41.