Segmented Entanglement Establishment With All-Optical Switching in Quantum Networks

Gongming Zhao, *Member, IEEE*, Jingzhou Wang, *Graduate Student Member, IEEE*, Yangming Zhao, *Member, IEEE*, Hongli Xu, *Member, IEEE*, Liusheng Huang, *Senior Member, IEEE, Member, ACM*, and Chunming Qiao, *Fellow, IEEE*

***Abstract*— There are two conventional methods to establish an entanglement connection in a Quantum Data Networks (QDN). One is to create single-hop entanglement links first and then connect them with quantum swapping, and the other is forwarding one of the entangled photons from one end to the other via all-optical switching at intermediate nodes to directly establish an entanglement connection. The two methods both have pros and cons. Respectively, the former method has a higher success probability of constructing entanglement link, but it would consume more quantum resources. The latter method, however, has a lower success probability to deliver a photon across multiple quantum links with fewer quantum resources. Accordingly, we are expecting to establish significantly more entanglement connections with limited quantum resources by first creating entanglement segments, each spanning multiple quantum link, using all-optical switching, and then connecting them with quantum swapping. In this paper, we design SEE, a Segmented Entanglement Establishment approach that seamlessly integrates quantum swapping and all-optical switching to maximize quantum network throughput. SEE first creates entanglement segments over one or multiple quantum links with all-optical switching, and then connect them with quantum swapping. Accordingly, SEE can theoretically outperform conventional entanglement link-based approaches. Large scale simulations show that SEE can achieve up to 100.00% larger throughput compared with the state-of-the-art entanglement link-based approaches, *e.g.*, Redundant Entanglement Provisioning and Selection (REPS).**

***Index Terms*— Quantum networks, quantum swapping, all-optical switching, throughput.**

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Gongming Zhao, Jingzhou Wang, Yangming Zhao, Hongli Xu, and Liusheng Huang are with the School of Computer Science and Technology, University of Science and Technology of China, Hefei, Anhui 230027, China, and also with the Suzhou Institute for Advanced Research, University of Science and Technology of China, Suzhou, Jiangsu 215123, China (e-mail: gmzhao@ustc.edu.cn; wjzwgh@mail.ustc.edu.cn; zhaoym.ustc@gmail.com; xuhongli@ustc.edu.cn; lshuang@ustc.edu.cn).

Chunming Qiao is with the Department of Computer Science and Engineering, University at Buffalo, Buffalo, NY 14260 USA (e-mail: qiao@computer.org).

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I. INTRODUCTION

QUANTUM networks have been proposed for many decades in order to support highly secure communications [2], [3], [4], [5], [6], [7]. The main function of conventional quantum networks is to support *quantum key distribution* (QKD), which is used to establish a shared encryption key between two (classical) computers. In a QKD network, the information is still carried by classic bits. However, with the development of quantum computing, we need to network multiple quantum computers and build a large quantum computing system. To this end, we have to transmit the quantum states without measuring and transferring them into classic data bits. The QKD networks are not adequate for this purpose. To transmit the data quantum bits (called *qubits*), which carry the quantum state information to be delivered, we need to build *Quantum Data Networks* (QDNs) [8], [9].

In a QDN, a number of *quantum nodes*, each serving as a source (Alice), destination (Bob) or *quantum repeaters*, are interconnected with *quantum links*, which are fibers or free-space optical links. Each quantum node has some *quantum memory* to store qubits, and each quantum link carries *quantum channels* (*e.g.*, wavelengths) that can be used to deliver qubits from one of its end to the other. Since a *data qubit* is likely to be lost if it were to be transmitted over one or more channels, and moreover, the qubit cannot be simply copied by Alice for retransmission once it is lost due to the no-cloning theorem [10], the prevailing approach used in a quantum network is to establish an *entanglement connection* between Alice and Bob, and then use an approach unique to quantum communication known as *teleportation* to transfer the quantum state information carried by the data qubit from Alice to Bob. Since an entanglement connection can be used to teleport one and only one data qubit, to achieve a high-throughput QDN, we should maximize the number of entanglement connections that can be established with the limited quantum resources.

To establish an entanglement connection between Alice and Bob who are not directly connected with each other, the conventional way is to connect multiple *entanglement links*. More specifically, we will first figure out a physical path from Alice to Bob. Then, over each link between every two physically adjacent quantum nodes along this path, a *Bell pair of photons* (*i.e.*, a pair of entangled photons) are generated and distributed to the two end nodes to create an entanglement link. As a result, there will be an *entanglement path* consisting of entanglement links from Alice to Bob. Along this entanglement path, Alice holds one qubit of a Bell pair and Bob holds a qubit of another Bell pair, while each repeater along the path holds two qubits, belonging to two different Bell pairs. At last, intermediate quantum nodes (*i.e.*, repeaters) along the entanglement path can perform quantum swapping to connect all these entanglement links and establish an entanglement connection. During above procedure to establish an entanglement connection between Alice and Bob, one quantum channel over each quantum link along the path will be consumed to distribute the Bell pair photons. In addition, to store Bell pair photons, one unit of quantum memory will be reserved at Alice and Bob, respectively, while two units of quantum memory will be consumed at each and every of the repeater along the path.

To conquer the challenge of limited quantum resources, with the help of all-optical switching, we argue that though it is difficult to establish a long entanglement connection by sending a photon from one end to the other, the success probability is not that low to create an *entanglement segment*, which is a partial entanglement connection over several quantum links (referred to as a *physical segment*), by directly distributing a Bell pair of photons to the two ends of a segment using all-optical switching, especially in a room size QDN. By connecting these entanglement segments with quantum swapping, we can also establish entanglement connections. Since entanglement segment is a more general concept (and an entanglement link is a special case of an entanglement segment), such an entanglement segment based method brings a significant potential to improve the network throughput. More specifically, it can save quantum memory, the most precious resource in QDNs [11], at the intermediate repeaters to create entanglement segments across several quantum links via all-optical switching, and leave more quantum resources to establish entanglement connections.

In this paper, we propose a Segmented Entanglement Establishment (SEE) approach to maximize the throughput of QDNs. As in previous works [8], [9], we assume a QDN works in a time slot fashion, and hence SEE maximizes the number of entanglement connections that can be established in each time slot. Given the topology, network resources, the success probabilities associated with creating entanglement segments through different physical segments, the success probability to perform swapping at each repeater, as well as a set of source-destination (SD) pairs, SEE will determine i). which entanglement segments (over which physical segments) will be created; ii). how to perform quantum swapping to connect the entanglement segments successfully created and establish entanglement connections. Since there are exponential combinations on how to create entanglement segments; and for each entanglement segment, there are multiple choices of its physical segments, our problem is more challenging than existing entanglement link based works [8], [9].

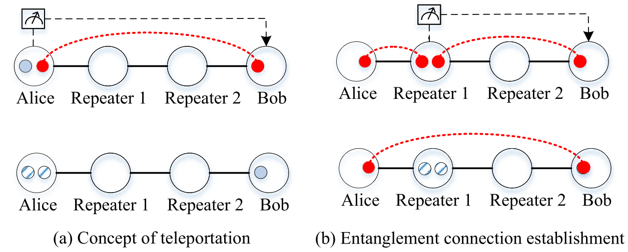


Fig. 1. Teleportation and entanglement connection establishment.

To solve the above throughput maximization problem, SEE first calculates which entanglement connections we should try to establish for each SD pair. Then SEE figures out the entanglement segments to be created in order to establish the desired entanglement connections. Based on the entanglement segments that are created successfully, another efficient algorithm is proposed to determine the way to perform quantum swapping such that the network throughput can be maximized. Extensive simulations show that SEE can increase the throughput over the state-of-the-art technique by up to 100%.

As far as we know, SEE is the first work that integrates all-optical switching and quantum swapping to maximize the throughput of QDNs. The main technical contributions of this paper can be summarized as follows:

1) Propose a novel approach named SEE to integrate the all-optical switching capability and quantum swapping to achieve segmented entanglement establishment in order to maximize the throughput of QDNs;
2) Design several effective algorithms for SEE, and analyze the performance of the proposed algorithms to show that we can acquire a feasible solution and a near-optimal performance can be achieved with high probability;
3) Conduct extensive simulations to demonstrate the superior performance of the proposed SEE approach, which can achieve up to 100.00% larger throughput compared with the state-of-the-art approaches.

The remainder of the paper is organized as follows. In Section II, we first present related background and give a motivating example. We show our motivation and give an overview in Section III. Then, in Section IV, the algorithm details of SEE are discussed. Extensive simulations are conducted in Section V to show the superior performance of SEE. We conclude this paper in Section VI.

II. Background and Previous Works

In this section, we first present some preliminary background of our work, including how a data qubit is teleported to its destination, how to establish an entanglement connection, and how to create entanglement segments.

A. Teleportation and Entanglement Connection

To teleport a data qubit from Alice to Bob, an entanglement connection (*i.e.*, Alice and Bob each host one qubit from a

Bell pair) has to be established between them as shown in the upper plot of Fig. 1(a). Then, Alice measures her two qubits (*i.e.*, the data qubit and the Bell pair photon), and sends the measurement results to Bob through a classic network. Based on the measurement results, Bob will perform some unitary operation on his Bell pair photon. Such an operation transfers the state of his photon to be the same as the data qubit. After above operations, as shown in the lower plot of Fig. 1(a), the entanglement between the two Bell pair photons will be destroyed by the measurement. In addition, the state of Alice's data qubit will also collapse due to the measurement. Accordingly, we can observe that i). the data qubit is not physically sent to Bob; Alice only teleports the state of the data qubit to Bob. ii). an entanglement connection can be used to teleport one and only one data qubit.

To establish an entanglement connection between Alice and Bob, Alice can generate a pair of entangled photons (called Bell pair) using *e.g.*, an Entangled Photon Source (EPS, meaning the location where the pair of entangled photons are generated), keep one of the photons to herself and send the other one to Bob. This method is impractical when Alice and Bob are far away from each other, since the photon will be lost on its way to Bob with high probability. To solve this problem, we can first generate multiple entanglement segments (similar to entanglement connection but not directly connecting Alice and Bob) to form an *entanglement path*, and then connect these entanglement segments through *swapping*. As shown in the upper plot of Fig. 1(b), there is an entanglement segment between Alice and Repeater 1, and another entanglement segment between Repeater 1 and Bob. To establish an entanglement connection between Alice and Bob, Repeater 1 will perform quantum swapping to connect these two entanglement segments. Through an entanglement path with more than 2 hops, all intermediate repeaters can perform quantum swapping simultaneously and we can connect all those entanglement segments together to establish an entanglement connection. It should be noted that the entanglement links in all previous works [8], [9], [12] are in fact special entanglement segments. The former should be created over single hop quantum links, while the later can be created over a multi-hop physical segment.

In this paper, we only need the very basic repeaters that can perform quantum swapping as in [8], [9], [12], and [13]. Physically, such repeaters only need to measure the two photons on their hand and send the results to corresponding destination. Such a swapping operation has been experimentally demonstrated by many different research groups, in many different scenarios (through a free-space link over 143 kilometers [14], across the Danube [15], and over a ground-to-satellite uplink [16]).

B. Failure of Entanglement Segment Creation and Swapping

An entanglement segment cannot always be created successfully. This may be due to following reasons: i). an EPS may fail to generate a pair of entangled photons; ii). when Alice sends a photon to Bob, due to the signal attenuation, the photon may be lost during the transmission; and iii). when the photon arrives at Bob's side, Bob may fail to detect its arrival.

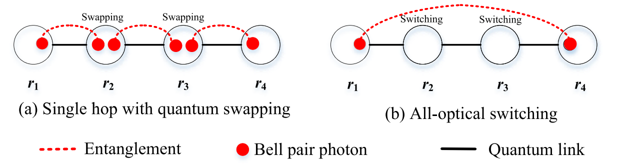


Fig. 2. Two ways to generate entanglement segments.

Actually, the success probability to generate an entanglement segment over a single-model fiber for one attempt is about 2.18×10^{-4} [17]. Though we can have many tries to create such an entanglement segment, the success probability of creating an entanglement segment within a time slot is still low according to the state of current technology [8], [9], [12].

Mathematically, the success probability of creating an entanglement segment over a physical segment can be expressed as $p = e^{-\alpha l} + \delta$, where l denotes the length of physical segment and α denotes a parameter determining the success probability to create an entanglement link. These two parameters are representing the physical attenuation. Moreover, to simulate that switching is not error-free and a repeater may force some loss on the establishment of the segment, we also set δ in the formula, which is a random variable uniformly distributed on $[-0.05, 0]$.

The quantum swapping operation may also fail. To perform quantum swapping, Repeater 1 (in the upper plot of Fig. 1(b)) has to read out its two photons from the quantum memory, and measure their states. Regardless of reading or measuring the photons, Repeater 1 may encounter an error which will result in a failure of establishing the entanglement connection. However, the success probability of swapping would be much higher than creating an entanglement segment. Usually, such success probability will be larger than 0.9 [8], [18].

C. Two Existing Ways to Generate Entanglement Segments

To create an entanglement segment, there are two alternate ways. The most intuitive way is to stitch single hop entanglement links created over each physical link using quantum swapping. This method is widely adopted in most previous works [8], [9], [19]. For instance, as shown in Fig. 2(a), we first create an entanglement link over each of the physical links along a physical segment between two nodes. Then, every intermediate repeater will perform quantum swapping to stitch these entanglement links. By using p_{uv} to denote the probability to create the entanglement link over physical link $r_u \rightarrow r_v$, and p_u to denote the probability to execute quantum swapping at node r_u, for a physical segment: $r_1 \rightarrow r_2 \rightarrow \ldots \rightarrow r_k$, the success probability to create such an entanglement segment can be described as

$$\prod_{k-1}^{2} q_i \prod_{k}^{2} p_{(i-1,i)} \tag{1}$$

The other method is leveraging all-optical switching to create an entanglement segment as creating an entanglement link. As shown in Fig. 2(b), Repeaters 1 and 2 set up all-optical circuits to forward one of the entangled photons from one node to the other, without detecting or storing it,

TABLE I

SUCCESS PROBABILITY TO CREATE AN ENTANGLEMENT LINK IS ASSUMED TO BE 0.9. WHEN A PHYSICAL SEGMENT IS LONG ENOUGH (*e.g.*, EXCEEDING THREE HOPS), THE PROBABILITY WOULD BE VERY SMALL

Physical Segments	Prob.
A Single Physical Link	0.9
$S_1 \to R_1 \to R_3$	0.85
$R_3 \to R_2 \to D_1$	0.85
$R_1 \to R_2 \to D_2$	0.8
$S_2 \to R_1 \to R_2 \to D_2$	0.55
$S_1 \to R_1 \to R_3 \to R_2 \to D_1$	0.1
Other Physical Segments	<0.1

let alone performing quantum swapping. By doing so, the quantum memory in Repeaters 1 and 2 can be saved. The success probability p to create entanglement segment is only determined by the physical distance between the two ends of an entanglement segment, but rather the quantum swapping performed by intermediate repeaters.

Compared with the former alternative, the latter can benefit the QDN throughput by leveraging all-optical switching in two folds: i). all intermediate repeaters do not need to reserve quantum memory to create and host entanglements, which saves the precious quantum memory; ii). the intermediate nodes along a physical segment do not need to perform quantum swapping, which promotes the success probability to establish an entanglement segment. However, we should note that, when the physical distance is too long, the success probability of creating an entanglement segment with all-optical switching will dramatically decrease. For instance, when we deliver a photon over a long-distance path via all-optical switching, the receiver may not be able to detect the photon due to its attenuation during transmission.

By using all-optical switching to create an entanglement segment, one unit of quantum memory is required at each end of it to store one entangled photon. In addition, since we need many attempts in order to create an entanglement segment, one dedicated quantum channel, *e.g.*, a wavelength, will be reserved over all the quantum links along the physical segment to create such an entanglement segment. Compared with conventional entanglement link based methods, we do not need to reserve quantum memory at the intermediate nodes when creating entanglement segments. It helps us save quantum memory and establish more entanglement segments, and entanglement connections consequentially.

D. Previous Works

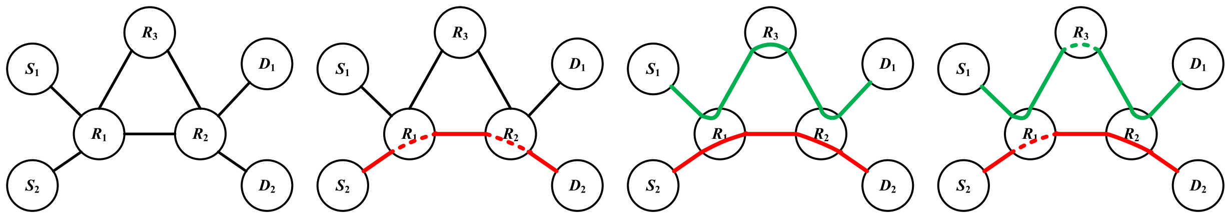
For many decades, quantum networks have been proposed for Quantum Key Distribution (QKD) systems [2], [3], [4], [6], and several real QKD systems have been built around the world, including the US, Europe, Japan, and China [4], [5], [6], [7]. QKD network is fundamentally different from QDNs since it is used only to establish a shared encryption key between two (classical) computers, and the data in a QKD network is still sent as classic bits. However, a QDN is used to deliver the accurate state of qbits. Due to the no-clone theory [10], we cannot keep a copy of any qubit for retransmission purpose in case that the data qubit is lost during transmission. Once data loss happens, we will not be able to recovery the data to be transmitted. Accordingly, reliability is a critical issue in QDN.

Teleportation can significantly improve the qubit transmission reliability and is widely adopted by QDNs. To improve the throughput of a QDN based on teleportation, we have to maximize the number of entanglement connections that can be established. Early works in this area discussed how to fully utilize the quantum resources to maximize the number of established entanglement connections on some specific types of topology, such as diamond topology [20], ring or sphere topologies [21], star topology [22], and chain topology [23]. After that, [24] and [19] were proposed to establish entanglement connections on a general topology. However, both of them assume the entanglement links have been successfully created and only focus on how to connect the existing entanglement links to form entanglement connections.

Works [8] and [9] considered how to create the entanglement links with limited quantum resources and how to perform quantum swapping to establish entanglement connections. They also took into consideration the success probability to create entanglement links and perform swapping. However, neither of them considered the alternative based on entanglement segments. Reference [12] is another representative to establish entanglement connections. This work mainly focused on how to physically create the entanglement links and perform swapping, such that the probability to establish an entanglement connection can be maximized.

Fidelity and purification issues in quantum networks are discussed in works [25] and [26]. Reference [26] discussed several purification schemes but when it comes to entanglement routing, the main idea is to assign each link a cost which is inversely proportional to the total number of sacrificial pairs supported by the link, and then use the Dijkstra's algorithm to find a shortest path. Reference [25] elaborated entanglement routing solution based on the idea of first purifying the links was proposed, so that only the links whose fidelity can be purified above a given threshold will be used in the routing. However, the work [13] argues that these two methods still cannot guarantee the fidelity. Thus, [13] presents the first algorithm to maximize the number of entanglement connections, whose fidelity can be guaranteed to be above a given threshold.

The most recent works [27] and [28] focus on some interesting aspects in quantum networks. Reference [27] noted that existing solutions all have to wait until the entanglement links along an entanglement path are created successfully before the intermediate repeaters can perform quantum swapping, which leads to a waste of quantum resources. They design an opportunistic way to manage entanglement resources, allowing requests move forward along its path as soon as possible as long as part of entanglement links on its entanglement path are created. Consequently, average total waiting time is significantly reduced. Reference [28] considers quantum entanglement routing problem to simultaneously maximize the number of quantum-user pairs and their expected throughput, which outperforms existing solutions in both



(a) Topology. (b) REPS solution. (c) E2E solution. (d) SEE solution.

Fig. 3. A motivation example. (Solid black lines are the quantum links; green and red lines are the entanglement connections created for SD pair (S_1, D_1) and (S_2, D_2), respectively. Dotted lines are the internal swapping operations to connect multiple entanglement segments.)

TABLE II

METHODS COMPARISON. REPS WILL CREATE 0.59 CONNECTIONS IN TOTAL, AND CONSUME 6 MEMORIES AND 3 CHANNELS. ON AVERAGE, REPS CREATES ONE CONNECTION CONSUMING 10.17 MEMORIES AND 5.08 CHANNELS. E2E WILL CREATE 0.65 CONNECTIONS IN TOTAL, AND CONSUME 4 MEMORIES AND 7 CHANNELS. ON AVERAGE, E2E CREATES ONE CONNECTION USING 6.15 MEMORIES AND 10.77 CHANNELS. SEE WILL CREATE 1.3 CONNECTIONS IN TOTAL, AND CONSUME 8 MEMORIES AND 7 CHANNELS. ON AVERAGE, SEE CREATES ONE CONNECTION WITH 6.15 MEMORIES AND 5.38 CHANNELS

Methods	Expected Number of Entanglement Connections			Network Resources		Resources per Connection	
	(S_1,D_1)	(S_2,D_2)	Total	Mem.	Chan.	Mem.	Chan.
REPS	0	0.59	0.59	6	3	10.17	5.08
E2E	0.1	0.55	0.65	4	7	6.15	10.77
SEE	0.65	0.65	1.3	8	7	6.15	5.38

served quantum-user pairs numbers and the network expected throughput.

III. MOTIVATION AND OVERVIEW

A. A Motivation Example

As discussed in last subsection, we have multiple alternatives to create entanglement segments. Under different environments, *e.g.*, length of physical segments and interference from the environment, *etc.*, these alternatives can achieve different success probabilities by adopting different physical routing and switching schemes. By carefully choosing these alternatives, we will not only maximize the probability to establish an entanglement segment, but also optimally utilize the quantum resources, especially the quantum memory, which benefits establishing more entanglement connections with limited quantum resources, *i.e.*, increasing the network throughput. Fig. 3 shows an example to motivate our work on segmented entanglement establishment via integrating quantum swapping and all-optical switching to maximize the QDN throughput. Table I illustrates the specific probabilities of creating entanglement segments over different physical segments. The comparison results are shown in Table II.

Fig. 3(a) shows the network topology of the motivation example. Each of R_1, R_2 and R_3 has 2 units of quantum memory, while each of the remaining 4 nodes (working as sources and destinations) has only 1 unit of quantum memory; every link carries only 1 quantum channel. In each time slot, the success probability to create an entanglement link over any physical link is assumed to be 0.9, and the swapping success probability at any node is also 0.9. Table I shows the success probabilities to create different entanglement segments.

In this motivation example, we would like to establish entanglement connections for two SD pairs, *i.e.*, (S_1, D_1) and (S_2, D_2). With REPS method [9], namely, using quantum swapping to connect single hop entanglement links, we can establish at most one entanglement connection due to the limitation of quantum memory. By taking into consideration the success probability, the optimal solution is shown in Fig. 3(b) which is able to establish an entanglement connection through the path $S_2 \to R_1 \to R_2 \to D_2$, with the success probability $0.9^5 = 0.59$ (three entanglement links and two quantum swapping). That means the expected number of entanglement connections that can be established is 0.59. To establish such an entanglement connection, REPS has to reserve 6 units of quantum memory (1 unit of quantum memory in S_2 and D_2, respectively; 2 units of quantum memory in R_1 and R_2, respectively) and 3 quantum channels ($S_2 \to R_1$, $R_1 \to R_2$, $R_2 \to D_2$). We calculate resources per connection by dividing consumed network resources by total expected number of connections. Accordingly, the REPS needs $6/0.59 = 10.17$ units of memory and $3/0.59 = 5.08$ channels to create one entanglement connection on average.

We can also only use all-optical switching to establish entanglement connections, just as create end-to-end (E2E) entanglement segments (so we refer to this as E2E methods). The solution is shown in Fig. 3(c) and the entanglement connections are $S_1 \to R_1 \to R_3 \to R_2 \to D_1$ and $S_2 \to R_1 \to R_2 \to D_2$. We should note that, if the transmission distance is too long (*e.g.*, in this example, exceeding three hops is regarded as a long distance.), the probability of creating an entanglement segment is very small. According to Table I, the success probabilities are 0.1 and 0.55, respectively. Thus, the total expected number of entanglement connections is 0.65. Though E2E method needs only 4 units of quantum memory (1 unit of memory in S_1, S_2, D_1, D_2), and 7 quantum channels (all the physical links). On average, it needs 6.15 units of memory and 10.77 channels to establish one entanglement connection, which is not as resource efficient as REPS.

By integrating quantum swapping and all-optical switching, we can propose segmented entanglement establishment (SEE) approach, which will derive a solution as shown in Fig. 3(d). By creating entanglement segments over the segments $S_1 \rightarrow R_1 \rightarrow R_3$, $R_3 \rightarrow R_2 \rightarrow D_1$ and $R_1 \rightarrow R_2 \rightarrow D_2$. We can save the quantum memory at node R_1, and such saved quantum memory can be used to create entanglement links (S_1, R_1) and (R_1, R_2) for the SD pair (S_2, D_2). Then, R_1 will perform quantum swapping to establish an entanglement connection between S_2 and D_2.

With the solution shown in Fig. 3(d), the expected number of entanglement connections that can be established is $0.85 \times 0.9 \times 0.85 + 0.9 \times 0.9 \times 0.8 = 1.3$, which outperforms REPS and E2E methods by 2.2x and 2x, respectively. SEE can create one entanglement connection with 6.15 units of memory and 5.38 channels, which is more resource-efficient compared with REPS and E2E.

In this example, it can be observed that with SEE approach that integrates all-optical switching and quantum swapping, we can not only increase the probability to establish entanglement connections, but also save quantum resources (*e.g.*, quantum memory and quantum channels), which enables us to establish more entanglement connections.

B. SEE in a Nutshell

Motivated by the work of REPS [9], we assume SEE works in a time-synchronous network operating in time slots, and it also provisions redundant entanglement to deal with the entanglement link failure. Since an entanglement segment would cross multiple quantum links, different from entanglement links in [9], entanglement segments connecting the same two ends may be created over different physical segments. For example, on the topology shown in Fig. 3(a), if we want to create two entanglement segments connecting S_1 and D_1, one of them may go through $S_1 \rightarrow R_1 \rightarrow R_3$, while the other one may go through $R_3 \rightarrow R_2 \rightarrow D_1$.

In SEE, a central controller maintains all the basic network information, such as the network topology, quantum resources at each node and link, the success probability of swapping on each node, and especially the success probabilities of creating entanglement segments over different physical segments.

With above information, SEE will teleport a batch of data qubits in a time slot through following five steps, which is different from the traditional method, *i.e.* single-hop with quantum swapping in [9]:

i). The central controller collects the information of whole quantum data network, including topology, quantum resources, and SD pairs. Then, we derive a set of entanglement paths to establish the entanglement connections for SD pairs. The details will be described in specific in Alg. 1.

ii). With the output of the first step, we determine the optimal set of entanglement segments that are to be created. For each entanglement segment, in addition to its two ends, the central controller should also figure out the physical segment to create it. Some of these entanglement segments will be used as backups. This step will be introduced in Alg. 2.

iii). The central controller notifies the corresponding nodes to reserve quantum memory, generate Bell pairs, set up all-optical switching circuits, and send out photons, in order to create entanglement segments, though not all the entanglement segments can be created successfully.

iv). Every node reports back the successfully created entanglement segments. Based on this information, the central controller tries to figure out how to perform the swapping operation to establish entanglement connections. This step will be described in detail in Alg. 3.

v). Corresponding nodes (*i.e.*, the destination node of every entanglement connection) report the swapping result to the source node. If all related swapping operations associated with an entanglement path succeeds, the source node can teleport data qubits to their destination.

Apparently, the key steps in SEE are the first, second and fourth steps where the central controller has to determine how to identify entanglement paths, create entanglement segments and how to connect the successfully created entanglement segments to establish entanglement connections. In the next section, we will describe these three algorithms in detail.

IV. SEE Design

In this section, we first review some important concepts in this paper in Section IV-A, and then we introduce our quantum data network model in Section IV-B. We formulate the problem to be solved in Section IV-C, and then design efficient algorithms to solve the formulation in Section IV-D. Theoretical analysis on the proposed algorithms will be presented in Section IV-E.

A. Key Concepts in SEE

Before we move on, we need to review some similar concepts and strengthen the difference between them to avoid ambiguous statement.

Physical Link/Quantum Link: A physical edge in the topology, which connects two adjacent quantum nodes using optical links.

Physical Segment: A physical path consists of one or more physical links. Specially, A single physical link can be regarded as a physical segment.

Entanglement Link: For two physically adjacent quantum nodes, if each of them host one photon from the same Bell pair (*i.e.*, entangled photon pair), we say there is an entanglement link between these two nodes.

Entanglement Segment: This is a generalization of the entanglement link. For two arbitrary quantum nodes, if each of them hosts one photon from the same Bell pair, we say there is an entanglement segment. Compared with entanglement link, the two ends of an entanglement segment are not necessarily physically adjacent.

Entanglement Path: For a given SD pair, an entanglement path is a sequence of entanglement segments from the source to the destination.

TABLE III
NOTATION LIST

Parameters	Description
$(\boldsymbol{V}, \boldsymbol{E})$	Network topology. $\boldsymbol{V}$ is the set of quantum nodes, while $\boldsymbol{E}$ is the set of quantum links.
s_i	Source node of i^{th} SD pair.
d_i	Destination node of i^{th} SD pair.
c_{uv}	Number of quantum channels over link $(u, v) \in \boldsymbol{E}$
m_u	Quantum memory size at node u.
q_u	Success probability of a quantum swapping operation at node u.
N_i	The number of entanglement connections we are trying to establish for SD pair i.
n_i	$0 \le n_i \le N_i$. The index of an entanglement connection we are trying to establish for SD pair i. We also use n_i to refer to the n_i^{th} entanglement connection established for SD pair i. Without ambiguity, we may ignore the subscript.
k_{uv}	The number of physical segments over which we can create entanglement segments between u and v.
C_{uv}^k	The k^{th} physical segment to create entanglement segments between u and v.
p_{uv}^k	Success probability of creating an entanglement segment between u and v over the k^{th} physical segment between u and v.
Variables	**Description**
$f_i^n(u, v)$	Binary variable indicating if or not an entanglement segment between u and v is used to establish the n^{th} entanglement connection for SD pair i.
t_i^n	Binary variable indicating if or not the n^{th} entanglement connection for SD pair i will be established.
x_{uv}^k	Number of entanglement segments between u and v that will be created through the k^{th} physical segment between u and v.

Entanglement Connection: This is a special case of the entanglement segment. When the two ends of an entanglement segment are the source and the destination of an SD pair, respectively, such an entanglement segment is referred to as an entanglement connection.

B. Quantum Data Network Model

To transmit the data quantum bits (called qubits), which carry the quantum state information to be delivered, we need to build Quantum Data Networks (QDNs). In a typical QDN, there are mainly two kinds of components: the quantum node set and the quantum link set. We use $\boldsymbol{V}$ to denote the set of quantum nodes and use $\boldsymbol{E}$ to denote the quantum links. The network topology is denoted as $(\boldsymbol{V}, \boldsymbol{E})$.

Each quantum node u may be a repeater in the network, or the source/destination node of a SD pair. The quantum memory of node u is denoted as m_u. The success probability of quantum swapping executed in node u is denoted as q_u. We assume the number of quantum channel over physical link (u, v) is c_{uv}. In the topology, we have a set of SD pairs. The source/destination node of i^{th} SD pair is denoted as s_i/d_i.

With the help of our motivating example in Fig. 3, we introduce our parameters and constants in network model. The network topology in Fig. 3(a) can be expressed as $(\boldsymbol{V}, \boldsymbol{E})$, where $\boldsymbol{V}$ is the set of quantum nodes and $\boldsymbol{E}$ is the set of quantum links. In our example, each of R_1, R_2, R_3 has 2 units of quantum memory and each of other nodes only has 1 unit of quantum memory, i.e., $m_{R_1}, m_{R_2}, m_{R_3} = 2$ and $m_{S_1}, m_{D_1}, m_{S_2}, m_{D_2} = 1$. Since c_{uv} represents the number of quantum channels over link $(u, v) \in \boldsymbol{E}$, in this example, each link carries only 1 quantum channel, i.e., $c_{uv} = 1, \forall (u, v) \in \boldsymbol{E}$. Moreover, in our motivating example, the success probability of quantum swapping is set to 0.9, *i.e.*, we have $q_u = 0.9, \forall u \in \boldsymbol{V}$. Since we use s_i/d_i to denote the source/destination node of the i^{th} SD pair. In this example, we have two SD pairs (S_1, D_1) and (S_2, D_2).

C. Problem Formulation

In this section, we formulate the problem to maximize the network throughput in terms of the number of entanglement connections that can be established in each time slot. For i^{th} SD pair, we try to establish N_i entanglement connections. Between arbitrary two nodes u and v, we find k_{uv} physical segments between them. We use C_{uv}^k to represent the specific path of the k^{th} physical segments. The success probability of creating an entanglement segment over k^{th} physical segments between u and v is p_{uv}^k.

We use variable $f_i^n(u, v)$ to denote whether an entanglement segment between u and v is used to establish the n^{th} entanglement connection for SD pair i ($f_i^n(u, v) = 1$) or not ($f_i^n(u, v) = 0$). The variable t_i^n means whether the n^{th} entanglement connection for SD pair i will be established ($t_i^n = 1$) or not ($t_i^n = 0$). Let x_{uv}^k represent the number of entanglement segment between u and v that will be created through the k^{th} physical segment between u and v. For clear presentation, the notations used in this section are summarized in Table III.

The formulation is shown in (2). Compared with the formulation in [9], we directly determine if each entanglement connection should be established or not without estimating the number of entanglement connections that should be established for each SD pair. Accordingly, the variables to formulate the entanglement path $f_i^n(u, v)$ are binary variable, rather than integer variables, which will benefit our algorithm design (see details in Section IV-D).

$$\max \quad \sum_i \sum_n t_i^n \tag{2}$$

Subject to:

$$\sum_v f_i^n(u, v) - \sum_v f_i^n(v, u) = t_i^n, \ \forall u = s_i, \quad n \le N_i \tag{2a}$$

$$\sum_v f_i^n(u, v) - \sum_v f_i^n(v, u) = -t_i^n, \ \forall u = d_i, \quad n \le N_i \tag{2b}$$

$$\sum_{v} f_i^n(u,v) - \sum_{v} f_i^n(v,u) = 0,\ \forall u \neq s_i, d_i,\quad n \leq N_i \tag{2c}$$

$$\sum_{i,n} [f_i^n(u,v) + f_i^n(v,u)] \leq \sum_{k} p_{uv}^k x_{uv}^k \sqrt{q_u q_v},\quad \forall u, v \tag{2d}$$

$$\sum_{u,v,k:(i,j)\in C_{uv}^k} x_{uv}^k \leq c_{ij},\ \forall (i,j) \in E \tag{2e}$$

$$\sum_{v,k} x_{uv}^k \leq m_u,\ \forall u \tag{2f}$$

$$t_i^n \geq t_i^{n+1},\ \forall i, n < n_i \tag{2g}$$

$$f_i^n(u,v),\ t_i^n \in \{0,1\},\ x_{uv}^k \in \boldsymbol{N} \tag{2h}$$

The objective of (2) is to maximize the number of entanglement connections that will be established. The first three constraints, *i.e.*, (2a)–(2c), are flow conservation constraints which should be held in all the routing related problems. Constraint (2d) states that the number of entanglement segments from node u to node v used by all the SD pairs to establish entanglement connections cannot exceed the expected number of entanglement segments that can be created. It should be noted that in this constraint, we apportion the success probability of performing quantum swapping at each intermediate repeater onto the success probability of creating its adjacent entanglement segments as in [9]. Constraint (2e) says that the number of entanglement segments that we are trying to create going through quantum link (u,v) must be less than or equal to the number of quantum channels carried by (u,v). To create an entanglement segment that incidents upon node u, one quantum memory is required. Constraint (2f) states that the number of entanglement segments incident on node u cannot exceed its quantum memory size. Constraint (2g) is an auxiliary constraint to limit the solution space and reduce the problem complexity. Given an entanglement path, we can assign it an arbitrary index, which significantly increases the solution space without bringing any benefit to improve the objective. This constraint can limit an entanglement path to be built unless all the entanglement paths labeled by a smaller index have been built. Constraint (2h) states that the number of entanglements on every edge to be integer.

The problem (2) is difficult to solve due to the integer natural of the variables. In fact, we have following theorem.

Theorem 1: The problem formulated in (2) is NP-hard.

Proof: By setting the success probability to create an entanglement segment over multi-hop physical segments to be 0, the success probability to create an entanglement segment over single-hop quantum links to be 1, and the success probability of quantum swapping operation to be 1, the problem in (2) will be reduced to a classic integer multi-commodity flow problem, which is a well-know NP-hard problem [29]. □

Due to the complexity of the proposed problem, we will design efficient algorithms to solve it in the next subsection.

D. Algorithm Design

In this subsection, we will propose a series of algorithms to solve the problem formulated in last subsection. At first, we will derive a set of entanglement paths to establish the entanglement connections with Entanglement Path Identification (EPI) algorithm. To establish as many entanglement connections as possible according to the entanglement paths identified by Algorithm EPI, Entanglement Segment Creation (ESC) algorithm is leveraged to determine how many entanglement segments will be created over each physical segment. Since some of the entanglement segments cannot be successfully created, Entanglement Connection Establishment (ECE) algorithm is proposed to determine how to establish entanglement connections by connecting the entanglement segments successfully created.

1) Entanglement Path Identification (EPI): The entanglement paths identified by solving (2) will maximize the network throughput. However, (2) is difficult to solve. Accordingly, we propose an Entanglement Path Identification (EPI) algorithm based on randomized rounding to derive a near-optimal solution. The basic idea of Algorithm EPI can be summarized as follows: we first relax the integral constraint of (2h), and solve the derived linear programming (LP) model. Then, we derive a solution to (2) based on the solution of this LP via randomized rounding. The details of this algorithm are shown in Algorithm 1.

This algorithm contains two steps. In the first step (Lines 1–3), we relax the Problem 2 and solve it. The solution of the relaxed model is usually infeasible to Problem 2 due to two reasons: i). an entanglement path may not be fully satisfied, *i.e.*, $0 < t_i^n < 1$; and ii). an entanglement path will be split onto multiple paths, *i.e.*, $0 < f_i^n(u,v) < 1$. Algorithm EPI solves these two problems in Step 2. At first, it figures out which entanglement paths will be built up (Line 5), *i.e.*, rounds t_i^n to be a binary value, and then indicates the corresponding concrete path (Lines 6–11), *i.e.*, round $f_i^n(u,v)$ to be a binary value. The first rounding is based on how much fraction of each corresponding entanglement path is satisfied according to the LP solution, while the second rounding is based on the fraction of the entanglement path carried by different paths.

We will show the effectiveness of Algorithm EPI in terms of its ability to achieve optimal throughput and produce feasible entanglement paths using theorems in Section IV-E.

2) Entanglement Segment Creation (ESC): With randomized rounding, we cannot ensure the entanglement paths derived by Algorithm EPI are feasible solutions to (2). In addition, Algorithm EPI only specifies the entanglement segments that will be created to establish each entanglement connection, but not how many entanglement segments should be created over different physical segments and how to connect the successfully created entanglement segments to establish entanglement connections. We will propose Entanglement Segment Creation (ESC) algorithm and Entanglement Connection Establishment (ECE) algorithm to address these two issues, respectively. The goals of Algorithm ESC are that i). establish as many entanglement connections identified by Algorithm EPI (Say the set of entanglement paths identified by Algorithm EPI is $\boldsymbol{T}$) as possible; and ii). pursue the fairness among all the SD pairs. To pursue the first goal, we first reserve quantum resources to the entanglement paths with fewer hops, and use the physical segments that have higher probability to

Algorithm 1 Entanglement Path Identification (EPI) Based on Randomized Rounding

Input: The formulation of (2)

1: **Step 1: Solving the Relaxed Formulation**
2: Construct a linear program by relaxing the integral constraints (2h) as $f_i^n(u,v) \in [0,1]$, $t_i^n \in [0,1]$, and $x_{uv}^k \geq 0$
3: Solve the LP and obtain the optimal solutions $\{\widetilde{f}_i^n(u,v)\}$ and $\{\widetilde{t}_i^n\}$
4: **Step 2: Identify entanglement paths via randomized rounding**
5: Set $t_i^n = 1$ with the probability $\widetilde{t}_i^n$
6: **for** All n_i such that $t_i^n = 1$ **do**
7: Calculate the set of paths traversed by the entanglement connection n_i according to $\{\widetilde{f}_i^n(u,v)\}$
8: Say the path set is $\{P_{ni}^{(r)}\}$, $P_{ni}^{(r)}$ also denotes the fraction of flow going through the corresponding path
9: Select one path $P_{ni}^{(r)}$ with probability $P_{ni}^{(r)}/\widetilde{t}_i^n$
10: Set $f_i^n(u,v) \leftarrow 1$ for all $(u,v) \in P_{ni}^{(r)}$ and $f_i^n(u,v) \leftarrow 0$ for all $(u,v) \notin P_{ni}^{(r)}$
11: **end for**
12: **return** $f_i^n(u,v)$ and t_i^n

Algorithm 2 Entanglement Segment Creation (ESC) Algorithm

Input: The set of entanglement paths identified by Algorithm ESC $\boldsymbol{T}$

1: Reorder all the entanglement paths
2: Initialize the number of entanglement segments created over each physical segment $x_{uv}^k \leftarrow 0$, and the set of all the entanglement paths for which we have allocated quantum resources $\boldsymbol{D} \leftarrow \Phi$
3: **for** Any path $p \in \boldsymbol{T}$ **do**
4: $\boldsymbol{D} \leftarrow \boldsymbol{D} \cup p$
5: **for** Any entanglement segment $(u,v) \in p$ **do**
6: Assign minimum quantum resources on segment $\langle u,v \rangle$ such that $\sum_{p\in \boldsymbol{D}} I_{\langle u,v\rangle \in p} \leq \sum_k p_{uv}^k x_{uv}^k$
7: Update the quantum resource assignment x_{uv}^k
8: **if** qunatum resources are not enough **then**
9: Release all the quantum resources assigned for p, $\boldsymbol{D} \leftarrow \boldsymbol{D}/p$
10: **break**;
11: **end if**
12: **end for**
13: **end for**
14: **return** $\{x_{uv}^k\}$ and $\boldsymbol{D}$

successfully create entanglement segments directly with all-optical switching of a Bell pair photon (rather than using quantum swapping). For the second goal, we reserve quantum resources to each SD pairs following round robin principle. Following the line of these thoughts, we propose the Algorithm ESC shown in Algorithm 2.

In Line 1, Algorithm ESC first sorts all the entanglement paths in the increasing order of path length (entanglement

Algorithm 3 Entanglement Connection Establishment (ECE) Algorithm

Input: The number of entanglement segments successfully created over each segment $\{e_{uv}\}$, the set of entanglement paths for which we have reserved enough quantum resources $\boldsymbol{D}$, and the set of entanglement paths identified by Algorithm ESC $\boldsymbol{T}$

1: Initialize $\boldsymbol{O} \leftarrow \Phi$
2: **for** Any entanglement path $p \in \boldsymbol{D}$ **do**
3: **if** $e_{uv} \geq 1$ for all $\langle u,v \rangle \in p$ **then**
4: $e_{uv} \leftarrow e_{uv} - 1$ for all $\langle u,v \rangle \in p$, $\boldsymbol{O} \leftarrow \boldsymbol{O} \cup p$
5: **end if**
6: **end for**
7: Initialize an auxiliary graph $G = \langle \boldsymbol{V}, \boldsymbol{S} \rangle$, where $\boldsymbol{S}$ is the set of all entanglement segments successfully created
8: Set the weight of each node $u \in \boldsymbol{V}$ as $-\ln q_u$
9: **while** More entanglement connections can be established **do**
10: **for** Any SD pair i with fewer than N_i entanglement connections in $\boldsymbol{O}$ **do**
11: Set the weight of an edge $\langle u,v \rangle$ to be 10^{-5} if $e_{uv} \geq 1$, and 10^9 if $e_{uv} = 0$
12: Find the shortest path from s_i to d_i, say the path is p
13: $e_{uv} \leftarrow e_{uv} - 1$ for all $\langle u,v \rangle \in p$, $\boldsymbol{O} \leftarrow \boldsymbol{O} \cup p$
14: **end for**
15: **end while**
16: **return** $\boldsymbol{O}$

segment number first and then physical hop number). This is to increase the network throughput since the entanglement paths with fewer hops will require less quantum resources. Then, with the equal path length, all the entanglement paths will be ordered based on round robin with respect to SD pairs in order to pursue the fairness among all SD pairs. In Lines 3–13, we reserve quantum resources along each path $p \in \boldsymbol{T}$ to ensure that the expected number of entanglement segments that can be created will be enough to build all the entanglement paths for which we have already reserved resources (Line 6). Here I represents the total number of entangle paths for which we have allocated quantum resources. To save the quantum resource, *i.e.*, minimize the number of entanglement segments that we are trying to create, the physical segments with higher probability to create an entanglement segment will be used first. It should be noted that if we cannot assign enough resources to an entanglement path, all the quantum resources reserved for this entanglement path should be released (Line 9). When all the entanglement paths have been traversed, Algorithm ESC returns the number of entanglement segments we will try to create over each physical segment and the set of entanglement paths for which we have reversed enough quantum resources.

3) Entanglement Connection Establishment (ECE): Algorithm ESC tells us how many entanglement segments will be created over different physical segments. However, we should note that only part of these entanglement segments will be created successfully, and we still have to determine how to perform quantum swapping to establish the entanglement connections. To this end, we propose Entanglement Connection Establishment (ECE) algorithm as shown in Algorithm 3.

The input of Algorithm ECE is the entanglement segments that are successfully created in the second step of each time slot, e_{uv}. It should be noted that when an entanglement segment is created, we do not care about the physical segment over which it was created. Based e_{uv}, Algorithm ECE first assigns the created entanglement segments to the entanglement paths in $\boldsymbol{D}$, *i.e.*, the entanglement paths for which we have reserved enough quantum resources (Lines 2–6). On the one hand, since some of the entanglement segments may fail to be created, we may not be able to build all the entanglement paths in $\boldsymbol{D}$. As a result, there may leave some entanglement segments. On the other hand, we may create more entanglement segments than we expect since we have created some redundant entanglement segments in case some of the entanglement segments may fail to be created. It will also leave some entanglement segments that are successfully created but cannot be used by the entanglement paths in $\boldsymbol{D}$. Accordingly, Algorithm ECE leverages these entanglement segments to establish more connections and improve the network throughput (Lines 9–15). To this end, Algorithm ECE first constructs an auxiliary graph on which the vertexes represent the repeaters while each edge stands for an entanglement segment (Line 7). Then, the weight of each vertex u is set to be $-\ln q_u$ (Line 8), and the weight of each edge is set to be a small number (10^{-5} in Algorithm ECE) if there are still remaining corresponding entanglement segments, while a large number (10^9 in Algorithm ECE) if all corresponding entanglement segments are assigned to some entanglement paths (Line 11). In this way, maximizing the probability to establish an entanglement connection is equivalent to minimize the length of the corresponding entanglement path from Alice to Bob (Line 12). Algorithm ECE will end when it cannot find out more entanglement paths based on the remaining entanglement segments in the network.

E. Algorithm Analysis

This section analyzes the efficiency of Algorithm EPI.

Theorem 2: Suppose O_{LP} is the optimal objective value to the relaxed version of formulation (2), while O_{ALG} is the objective value achieved by Algorithm EPI, then we have $\mathbf{Pr}[O_{ALG} \leq (1-\epsilon)O_{LP}] \leq e^{-\epsilon O_{LP}/2}$.

Proof: According to Algorithm EPI, we have $E[t_i^n] = 1\times\widetilde{t}_i^n + 0\times(1-\widetilde{t}_i^n) = \widetilde{t}_i^n$. Then, $E[O_{ALG}] = \sum_i\sum_n E[t_i^n] = \sum_i\sum_n \widetilde{t}_i^n = O_{LP}$. Based on Chernoff Bound, we know $\mathbf{Pr}[O_{ALG} \leq (1-\epsilon)E[O_{ALG}]] \leq e^{-\epsilon E[O_{ALG}]/2}$. Combining above discussions, we conclude $\mathbf{Pr}[O_{ALG} \leq (1-\epsilon)O_{LP}] \leq e^{-\epsilon O_{LP}/2}$. □

This concludes the near-optimal performance of the EPI algorithm.

Before we move on, we first let y_{uv} be the number of entanglement segments (u,v) we are trying to create and $C_{uv} = \cup_k C_{uv}^k$. In addition, we assume there is only one segment between (u,v). Correspondingly, the superscript k of x and C_{uv} can be removed. We assume there to be only one segment between every pair of nodes because this is a realistic assumption. When one segment is found for a pair of nodes, it usually has the highest success probability to create the corresponding entanglement segment. Thus, other segments are not likely to be built. Constraint (2d), (2e), (2f) can be replaced with the following inequalities, respectively.

$$\sum_{i,n}[f_i^n(u,v) + f_i^n(v,u)] \leq p_{uv}x_{uv}\sqrt{q_u q_v},\ \forall u,v \tag{2d'}$$

$$\sum_{u,v:(i,j)\in C_{uv}} x_{uv} \leq c_{ij},\ \forall(i,j)\in E \tag{2e'}$$

$$\sum_{v} x_{uv} \leq m_u,\ \forall u \tag{2f'}$$

Then, we have the following theorems:

Theorem 3: $\mathbf{Pr}[\sum_{u,v:(i,j)\in C_{uv}} y_{uv} \geq (1+\epsilon)c_{ij}]] \leq e^{-\frac{\epsilon^2}{2+\epsilon}c_{ij}/2}$ for all quantum link $(i,j) \in \boldsymbol{E}$

Proof: According to Algorithm EPI, we know $f_i^n(u,v)$ would be set to 1 with the probability $\widetilde{t}_i^n \times \widetilde{f}_i^n(u,v)/\widetilde{t}_i^n = \widetilde{f}_i^n(u,v)$. So we have $E[f_i^n(u,v)] = 1\times\widetilde{f}_i^n(u,v) + 0\times(1-\widetilde{f}_i^n(u,v)) = \widetilde{f}_i^n(u,v)$. The expected number of entanglement segment (u,v) that can be created successfully is $\sum_i\sum_n[E[f_i^n(u,v)] + E[f_i^n(v,u)] = \sum_i\sum_n[\widetilde{f}_i^n(u,v) + \widetilde{f}_i^n(v,u)] \leq \sum_k p_{uv}^k\widetilde{x}_{uv}^k\sqrt{q_u q_v}$. Since we assume there is only one segment between u and v, according to Constraint (2d'), we have $\sum_i\sum_n[\widetilde{f}_i^n(u,v) + \widetilde{f}_i^n(v,u)] \leq p_{uv}\widetilde{x}_{uv}\sqrt{q_u q_v}$.

From the definition of y_{uv}, we have $y_{uv} = \sum_i\sum_n[\widetilde{f}_i^n(u,v)+\widetilde{f}_i^n(v,u)] \leq p_{uv}\widetilde{x}_{uv}\sqrt{q_u q_v}$. Since $p_{uv} \leq 1$, $q_u \leq 1$ and $q_v \leq 1$, We have $y_{uv} < \widetilde{x}_{uv}$. Thus, according to Constraint (2e'), we can acquire $\sum_{u,v:(i,j)\in C_{uv}} y_{uv} < \sum_{u,v:(i,j)\in C_{uv}} \widetilde{x}_{uv} \leq c_{ij}$. Based on the Chernoff Bound, we have $\mathbf{Pr}[\sum_{u,v:(i,j)\in C_{uv}} y_{uv} \geq (1+\epsilon)c_{ij}]] \leq e^{-\frac{\epsilon^2}{2+\epsilon}c_{ij}/2}$. □

This theorem shows that the solution derived by Algorithm EPI will satisfy the link capacity constraint with a high probability.

Theorem 4: $\mathbf{Pr}[\sum_v y_{uv} \geq (1+\epsilon)m_u] \leq e^{-\frac{\epsilon^2}{2+\epsilon}m_u/2}$ for all quantum node $u \in \boldsymbol{V}$.

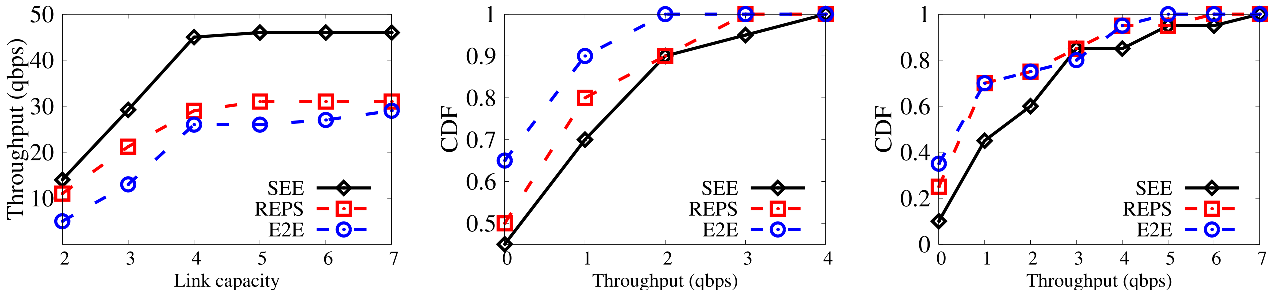
Proof: According to Theorem 3 and the definition of y_{uv}, we have $y_{uv} < \widetilde{x}_{uv}$. Thus, according to Constraint (2f'), we can acquire $\sum_v y_{uv} < \sum_v \widetilde{x}_{uv} < m_u$. Based on the Chernoff Bound, we have $\mathbf{Pr}[\sum_v y_{uv} \geq (1+\epsilon)m_u] \leq e^{-\frac{\epsilon^2}{2+\epsilon}m_u/2}$. □

Theorem 4 shows that the quantum memory capacity can be satisfied with a high probability.

In conclusion, Theorem 2 shows that the EPI algorithm can derive a throughput close to the optimal value with a high probability. Theorem 3 and Theorem 4 show that it can derive a feasible solution with a high probability.

F. Discussions

Physical segments to create entanglement links: In SEE, we have to prepare several physical segments to create each specific entanglement segment. The more physical segments we prepared for each entanglement segment, the better solution, *i.e.*, the higher network throughput, we will expect to



(a) Link capacity vs. throughput (b) Throughput CDF when link capacity is 2. (c) Throughput CDF when link capacity is 7.

Fig. 4. How link capacity impacts network throughput.

achieve. However, it will significantly increase the problem complexity when we increase the number of physical segments prepared for entanglement segment. In SEE, we will find out K physical segments for every node pair with Yen's algorithm [30]. However, the segments consisting of too many hops or with a low probability to create an entanglement segment will be removed. This is to reduce the time complexity of our algorithms.

Time complexity: Though SEE can leverage the same algorithms in REPS to calculate how many entanglement segments should be created over each physical segment, it will incur an extremely large time complexity as there are much more physical segments than physical links in a network. In addition, REPS uses progressive rounding to determine the number of entanglement segments that should be created over each physical segment. With this method, we have to solve plenty of LP models, though the scale of LP models will decrease with the progress of the algorithm, it is still time consuming when there are lots of quantum links in the network. Accordingly, we design a set of new algorithms for SEE, in which the LP model will be solved for only once. Since we introduce larger search space into the SEE by creating entanglement segments through different physical segments, we will see in the simulations that SEE outperforms REPS by 2x in term of the network throughput. In general, the data transmission volume of a quantum data network is relatively small. Usually, the transmission would take a long-time span. Thus, the time complexity of running algorithms can be neglected.

V. Performance Evaluation

In this section, we evaluate the performance of SEE through extensive simulations using a custom in-house simulator built on Python. The LP solver used in our simulator is PuLP. Simulations involve randomly generated networks with a certain amount of quantum resources, a set of randomly chosen SD pairs and success probabilities of creating entanglement segments and quantum swapping. For the network throughput (measured in qubits per time slot, *i.e.*, qbps) shown in the simulations, each data is averaged by 100 trails. Since the SD pairs and network topology in different trails are different, it is not reasonable to show the CDF of average throughput. Accordingly, the throughput CDFs, which show the throughput distributions among all SD pairs, are randomly picked up from one trail. Hereby, in the following figures, the sum of each SD pair's throughput (in Figs. x(b) & x(c)) is not equal to the network throughput (in Fig. x(a)), where x is a wildcard that can be a value from 5 to 10.

A. Simulation Methodology

Network Topology Generation. As in [9], we randomly place a given number of nodes into a 10,000 km by 10,000 km square area. Quantum links are determined following the Waxman model [31]. On the generated topology, we prepare several physical segments for each node pair. The success probability to create an entanglement segment (u, v) over the k^{th} physical segment between node u and v is [19]

$$p^k_{uv} = e^{-\alpha l^k_{uv}} + \delta \tag{3}$$

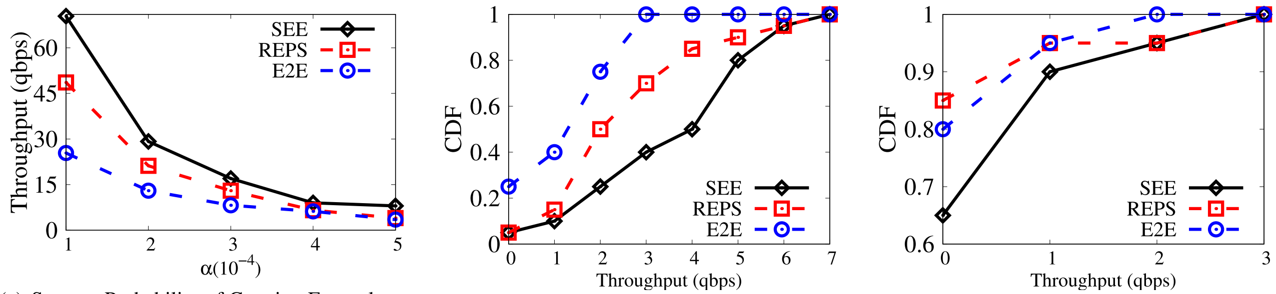
where l^k_{uv} is the length (measured in kilometers) of the corresponding segments and α is a predefined parameter reflecting the signal attenuation along a fiber. A larger α means a larger signal attenuation and results in a smaller success probability to create an entanglement segment along the same physical path. δ is a random variable uniformly distributed on $[-0.05, 0]$.

Default Parameters. In the default settings, there are 200 nodes and 20 SD pairs in the network. According to [32], the success probability for quantum swapping can reach near 100%. Thus, in this paper, similar to [8], [9], we also choose a high probability (*e.g.*, 0.9) as the default success probability for quantum swapping in our simulations. The number of quantum channels supported by each edge is 3; and the parameter that determines the success probability to create an entanglement link, *i.e.*, α in (3), is 0.0002, with which, the average external link success probability is about 0.8. By default, there are 10 units of quantum memory hosted by each quantum node.

Comparison Scheme. We compare SEE with two entanglement establishment schemes. One is REPS, which is the state-of-the-art technique. The other is to establish entanglement connections only by all-optical switching, which is labeled as E2E in all figures. In fact, REPS and E2E are the two extreme cases of SEE. The former one only uses the quantum swapping, while the later only uses all-optical switching.

B. Evaluation Results

Main observations. From our simulations, we observe that SEE outperforms REPS and E2E by up to 100% and

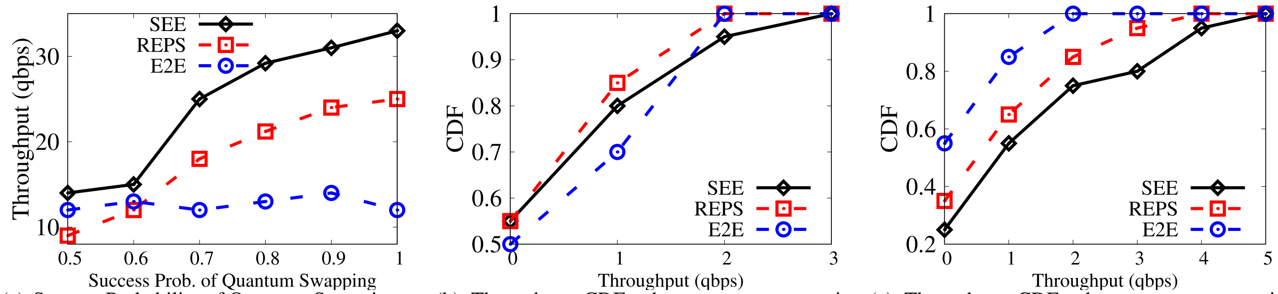


(a) Success Probability of Creating Entanglement Links vs. Throughput

(b) Throughput CDF when α is 1×10^{-4}

(c) Throughput CDF when α is 5×10^{-4}

Fig. 5. How the success probability to create entanglement segments impacts network throughput.



(a) Success Probability of Quantum Swapping vs. Throughput.

(b) Throughput CDF when quantum swapping success probability is 0.5.

(c) Throughput CDF when quantum swapping success probability is 1.

Fig. 6. How the success probability of internal swapping impacts network throughput.

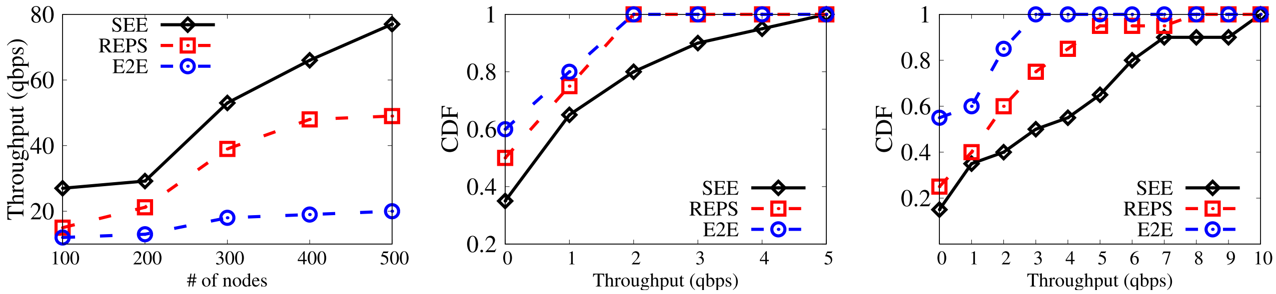
180%, respectively, in throughput. E2E performs the worst since it is difficult to establish an entanglement for a SD pair far away from each other via only all-optical switching. Compared with REPS, SEE can leverage all-optical switching to create longer (but not too long) entanglement segments and save the quantum memory resources. Though this will result in a smaller probability to create an entanglement segment, it enables us trying to create more entanglement segments. Even if it is not resource efficient to try to create entanglement segments over a multi-hop physical segments, it is still an option to create entanglement links and connects them via quantum swapping. Accordingly, SEE is the most feasible scheme to optimize the QDN throughput.

Effect of physical link capacity. We keep the default parameter settings, expect the capacity of each link varying from 2 to 7. The simulation results are shown in Fig. 4. From Fig. 4(a), we can observe that the SEE outperforms REPS and E2E by 27.27% – 55.17% and 58.62% – 180.00%, respectively. The network throughput increases with the link capacity regardless of which algorithm is adopted. This is intuitive since larger link capacity provides more resources to establish entanglement connections. However, when the capacity of each physical link exceeds 4, the network throughput will only slightly increase with the link capacity since the system bottleneck becomes the amount of quantum memory.

Figs. 4(b)&4(c) show the throughput CDF of all SD pairs when the link capacity is 2 and 7, respectively. From these figures, we can see that with SEE, more SD pairs will achieve a higher throughput, and the largest throughput that can be achieved with SEE is also larger than other two algorithms. This coincides with the observation that SEE will achieve a higher throughput than REPS and E2E.

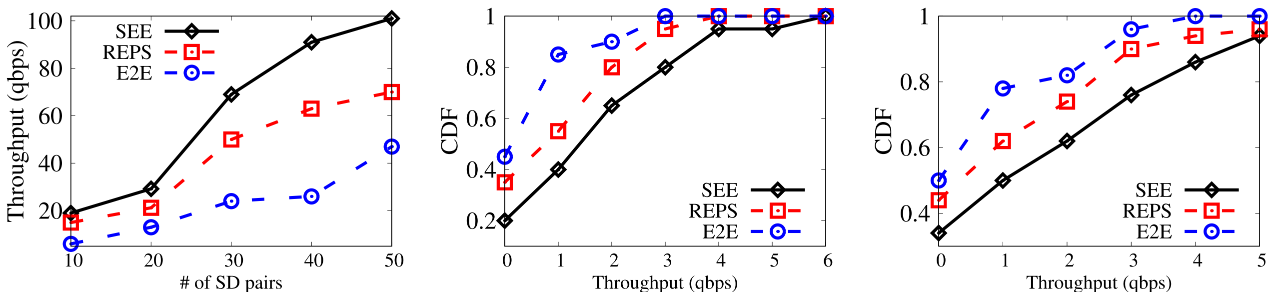
Effect of entanglement segment success probability. To investigate how the success probability affects the performance of SEE, we vary the α in (3) from 1×10^{-4} to 5×10^{-4} and show the simulation results in Fig. 5. Generally, the larger the α is, the smaller the success probability it will be to create an entanglement segment, and so will the network throughput be. In Fig. 5(a), with the varying of the success probability to create an entanglement segment, SEE will achieve a network throughput 30.77% – 100.00% and 45.16% – 177.17% higher than that with REPS and E2E, respectively. Besides, with the decrease of the success probability to create an entanglement segment, the network throughput achieved by SEE decreases much faster than that achieved by other two algorithms, and finally, the performance of SEE will degrade to be the same as REPS. This is because that with the increase of α in (3), it will be more difficult to create an entanglement segment over a long segment. Therefore, fewer entanglement segments will be created via multi-hop physical segments. Thus, SEE will converge to solution similar to REPS. This is also verified in Figs. 5(b)&5(c). In Fig. 5(c), the throughput CDF curves of SEE and REPS is closer to each other than that in Fig. 5(b).

Effect of quantum swapping success probability. Fig. 6 shows how the quantum swapping success probability affects the performance of SEE. In Fig. 6(a), we can see that though the network throughput will increase with the quantum swapping success probability with SEE and REPS, the increase rate will be slower and slower. This is because when the swapping success probability is large enough, the main ingredient that determines the network throughput is



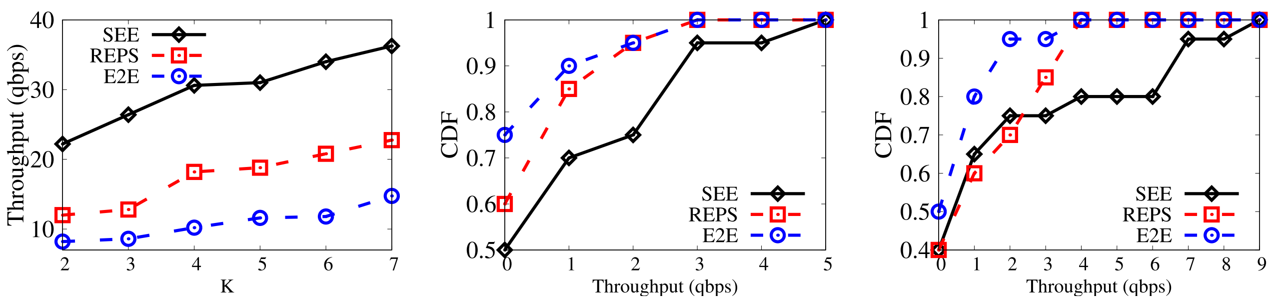
(a) Network Scale vs. Throughput (b) Throughput CDF when there are 100 nodes. (c) Throughput CDF when there are 500 nodes.

Fig. 7. How the network scale impacts network throughput.



(a) Workload vs. Throughput (b) Throughput CDF with 20 SD pairs. (c) Throughput CDF with 50 SD pairs.

Fig. 8. How the workload impacts network throughput.



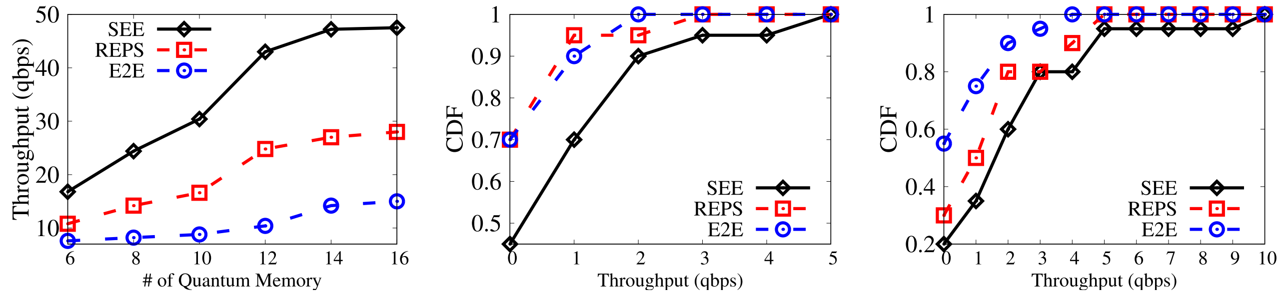
(a) K vs. Throughput (b) Throughput CDF with K=2. (c) Throughput CDF with K=7.

Fig. 9. How the workload impacts network throughput.

the number of entanglement segments (and the entanglement paths accordingly) that can be created. In addition, the quantum swapping success probability almost does not impact the network throughput with E2E since it does not use the quantum swapping to connect multiple entanglement segments. The most interesting observation is that when the success probability of quantum swapping is smaller than 0.6 in our simulations, the E2E outperforms REPS as it is difficult for REPS to connect entanglement segments with quantum swapping. In this case, all-optical switching would be the better option to establish long entanglements.

Effect of network scale. We evaluate the scalability of SEE by varying the number of nodes from 100 to 500. Fig. 7 shows how the throughput changes with the network scale. Generally, SEE outperforms REPS and E2E by 35.90% – 80.00% and 124.62% – 280.00%, respectively. The network throughput will become larger with the increase of the network scale. This is because that there will be more resources to support generating more entanglements and also we will be able to prepare more physical segments to create entanglement segments. Compare Fig. 7(b) with Fig. 7(c), we can see that with more resources and available physical segments to create entanglement segments, the throughput of each SD pair will also significantly increase. In a network with 100 nodes, an SD pair can establish at most 5 entanglement connections in each time slot, while some SD pairs can establish up to 10 entanglement connections in each time slot in a network with 500 nodes.

Effect of number of SD pairs. When the number of SD pairs in the network varies from 10 to 50, the throughput under different schemes are shown in Fig. 8. In this figure, we can see that the network throughput first significantly increases with the number of SD pairs, and then increases in a slower pace. This is because that there are resource contentions among different SD pairs when the network suffers a heavy workload. However, from Figs. 8(b)& 8(c), we can see that the largest



(a) Quantum memory vs. Throughput (b) Throughput CDF with 6 quantum memories. (c) Throughput CDF with 16 quantum memories.

Fig. 10. How the workload impacts network throughput.

throughput that can be achieved by an SD pair will not be significantly affected by the workload, since this is mainly determined by the maximum amount of resources that can be allocated to an SD pair, which is mainly determined by the network topology and has only slight relationship with the number of SD pairs in the network.

Effect of number of feasible physical segments. We keep the default parameter settings, except the number of feasible physical segments between two nodes, varying 2 to 7. The simulation results are shown in Fig. 9. In general, we can learn from Fig. 9(a) that SEE outperforms REPS and E2E by 59.34%–85.00% and 145.76%–170.73%, respectively. Overall, the network throughput will become larger with the increasing number of feasible physical segments between two nodes. This is because with more physical paths, entanglement segments for SD pairs have more opportunities to be created. Compare Fig 9(b) and Fig. 9(c), we can draw the conclusion that with more available physical segments to create entanglement segments, the throughput of each SD pair will also increase. In a network, when we set K as 2, the SD pair with maximum throughput can establish only 5 entanglement connections, while it is 9 if we set $K = 7$.

Effect of quantum memory amount. Fig. 10 demonstrates how the number of quantum memory units affects network throughput. We can observe from Fig.10(a) that with different amount of available quantum memory, SEE outperforms REPS and E2E by 55.55%–69.64% and 121.05%–216.67%, respectively. Generally, the network throughput increases when given more quantum memory regardless of which algorithm is adopted. This is obvious since more memory can support establishing entanglement connections. However, when the number of quantum memory units exceeds 12, the throughput only slightly increases due to the link capacity limitation. Figs. 10(b) and 10(c) show the CDF of all SD pairs when give 6 and 16 units of quantum memory, respectively. From the two figures, we know that more SD pairs can achieve a higher throughput with more quantum memory.

VI. Conclusion

SEE is a framework which optimizes the throughput of Quantum Data Networks (QDNs) by deploying segmented entanglement establishment which integrates all-optical switching and quantum swapping. To the best of our knowledge, SEE is the first work that introduces segmented entanglement establishment into QDNs. We have formulated the throughput maximization problem and proposed efficient algorithms to slove it. Through extensive simulations, we demonstrate that SEE works well in networks with different features, *i.e.*, the success probability to create entanglement segments, the success probability to perform quantum swapping, the network scale, *etc.*., and preserves remarkable performance advantages over the quantum-swapping-only or all-optical-switching-only solutions.

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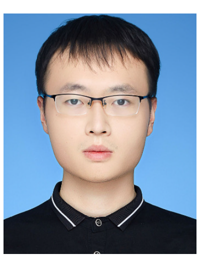
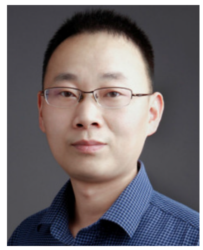
Gongming Zhao (Member, IEEE) received the Ph.D. degree in computer software and theory from the University of Science and Technology of China in 2020. He is currently an Associate Professor with the University of Science and Technology of China. His current research interests include software-defined networks and cloud computing.



Jingzhou Wang (Graduate Student Member, IEEE) is currently pursuing the Ph.D. degree in computer science with the University of Science and Technology of China. His main research interests include software-defined networks and cloud computing.

Yangming Zhao (Member, IEEE) received the B.S. degree in communication engineering and the Ph.D. degree in communication and information system from the University of Electronic Science and Technology of China in 2008 and 2015, respectively. He is currently a Research Professor with the School of Computer Science and Technology, University of Science and Technology of China. Before that, he was a Research Scientist with University at Buffalo. His research interests include network optimization, quantum networks, edge computing, and machine learning.



Hongli Xu (Member, IEEE) received the B.S. degree in computer science and the Ph.D. degree in computer software and theory from the University of Science and Technology of China (USTC), China, in 2002 and 2007, respectively. He is currently a Professor with the School of Computer Science and Technology, USTC. He has published more than 100 papers in famous journals and conferences, including IEEE/ACM TRANSACTIONS ON NETWORKING, IEEE TRANSACTIONS ON MOBILE COMPUTING, IEEE TRANSACTIONS ON PARALLEL AND DISTRIBUTED SYSTEMS, the International Conference on Computer Communications (INFOCOM), and the International Conference on Network Protocols (ICNP). He has held more than 30 patents. His research interests include software-defined networks, edge computing, and the Internet of Things. He received the Outstanding Youth Science Foundation of NSFC in 2018. He received the best paper award or the best paper candidate in several famous conferences.

Liusheng Huang (Senior Member, IEEE) received the M.S. degree in computer science from the University of Science and Technology of China in 1988. He is currently a Senior Professor and a Ph.D. Supervisor with the School of Computer Science and Technology, University of Science and Technology of China. He has authored or coauthored six books and over 300 journals/conference papers. His research interests include the Internet of Things, vehicular ad-hoc networks, information security, and distributed computing. He is a member of ACM.

Chunming Qiao (Fellow, IEEE) is currently a SUNY Distinguished Professor and also the current Chair of the Computer Science and Engineering Department, University at Buffalo. His current focus is on connected and autonomous vehicles. He has been serving as a consultant for several IT and telecommunications companies since 2000. His research has been funded by a dozen major IT and telecommunications companies, including Cisco and Google, and more than a dozen NSF grants. He has published extensively with an H-index of over 69 (according to Google Scholar). He holds seven U.S. patents. He was elected to IEEE Fellow for his contributions to optical and wireless network architectures and protocols. Two of his papers have received the best paper awards from IEEE and Joint ACM/IEEE venues.