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Extended Doo-Sabin Surfaces ⋆

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Abstract

In this paper we propose a modification of quadratic NURSSes called EDSes (Extended Doo-Sabin Surfaces). EDSes inherit the refinement rules for quadratic NURSSes for four-sided faces but use the refinement rules for Doo-Sabin surfaces for faces with other than four sides. Compared to quadratic NURSSes, if all the knot intervals are positive, EDSes always converge to G^1 continuous limit surfaces with closed-form limit point as well as limit normal rules.

Keywords: Doo-Sabin surfaces; NURBS; Subdivision surfaces

1 Introduction

In 1978, Doo and Sabin [1] and Catmull and Clark [2] generalized the subdivision schemes for uniform biquadratic and bicubic B-splines to arbitrary topological meshes, respectively.

In 1998, Sederberg et al. introduced NURSSes (Non-Uniform Recursive Subdivision Surfaces) [3] that extend non-uniform tensor product B-spline surfaces to arbitrary topologies. Quadratic and cubic NURSSes respectively correspond to non-uniform Doo-Sabin and Catmull-Clark surfaces. Later, they proposed NURCCs [4] by requiring opposing edges of each four-sided face of cubic NURSSes to have the same knot interval. NURCCs have stationary refinement rules, whereas cubic NURSSes have non-stationary subdivision rules.

Limit meshes are shown to be tighter approximations to limit surfaces than corresponding control meshes [5], and are essential for adaptive rendering of subdivision surfaces [6]. For a

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stationary smooth subdivision scheme, an eigenanalysis can be performed for each configuration of knot intervals to compute the limit point and corresponding limit normal vector [7]. However, explicit limit point and limit normal rules for quadratic NURSSes or NURCCs have not yet become available.

Müller et al. recently presented two different approaches to generalize both bicubic NURBS and Catmull-Clark surfaces. ESubs (Extended Subdivision Surfaces) [8] offer limit point rules but are non-stationary. DINUS [9] is a stationary scheme and provides limit point as well as limit normal rules. Both schemes use the refinement rules for Catmull-Clark surfaces [2] near control points with valence unequal to four.

Quadratic NURSSes [3] are the only subdivision surfaces that extend both non-uniform biquadratic B-spline surfaces and Doo-Sabin surfaces [1]. In Doo-Sabin like schemes, the *extraordinary points* are at the "centers" of n-sided faces with $n \neq 4$. In [10], Qin et al. pointed out that quadratic NURSSes converge for $n \leq 12$, but may diverge when $n > 12$. As a stationary scheme, a detailed eigenanalysis has been performed for quadratic NURSSes [11]. But no closed-form limit point or limit normal rules are known for this scheme up to now.

This paper presents a modification of quadratic NURSSes called EDSes (Extended Doo-Sabin Surfaces): For four-sided faces, the quadratic NURSS refinement rules [3] are applied; for n-sided faces with $n \neq 4$, the Doo-Sabin refinement rules [1] are used. As a result, if the knot intervals are all positive, the EDS scheme always generate G^1 continuous limit surfaces. And explicit limit point and limit normal rules are derived as well.

The remainder of the paper is structured as follows. The next section briefly reviews quadratic NURSSes. Sect. 3 describes the refinement rules for EDSes, and derives limit point and limit normal rules. Several examples are given as well. Finally, we conclude the paper with some suggestions for future work.

2 Quadratic NURSSes

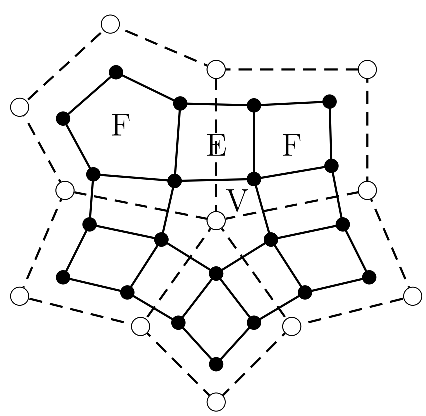


Fig. 1: Doo-Sabin subdivision.

Quadratic NURSSes are non-uniform Doo-Sabin surfaces that generalize non-uniform biquadratic B-spline surfaces to meshes of arbitrary topology [3]. Quadratic NURSSes have the same topological split rules as Doo-Sabin surfaces [1]: At each subdivision step, new vertices are generated, and then connected to form new faces of type F, type E and type V respectively for each face, edge and vertex of the previous mesh (see Fig. 1).

Initial control meshes may consist of vertices of valence other than four, and faces with other than four sides. We call a vertex of valence four *regular*, otherwise, *irregular*. Similarly, a face with four sides is *regular*, otherwise, *irregular*. In Doo-Sabin like schemes, the *extraordinary points* are at the centers of irregular faces. After one subdivision step, every vertex of the refined mesh will be regular, and the number of irregular faces will remain constant.

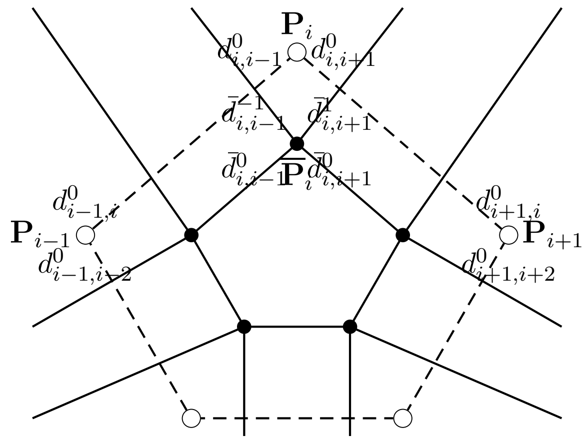


Fig. 2: Non-uniform Doo-Sabin refinement.

For non-uniform Doo-Sabin surfaces, each vertex is assigned a *knot interval* for each edge incident to it [3]. Refer to Fig. 2 for notations. $d_{i,j} = d^0_{i,j}$ indicates the knot interval for edge $\mathbf{P}_i\mathbf{P}_j$. $d^m_{i,j}, m \geq 1$ denotes the knot interval for the m-th edge encountered when rotating the edge $\mathbf{P}_i\mathbf{P}_j$ counter-clockwise about $\mathbf{P}_i$. And a $-m$ refers to rotating clockwise. In general, $d_{i,j} \neq d_{j,i}$.

For a face with n vertices $\mathbf{P}_0, \ldots, \mathbf{P}_{n-1}$, the corresponding new vertices $\overline{\mathbf{P}}_0, \ldots, \overline{\mathbf{P}}_{n-1}$ are computed by [3]

$$\overline{\mathbf{P}}_i = \frac{\mathbf{P}_i + \mathbf{V}}{2} + (c_{i+2} + c_{i-2}) \times \frac{-n\mathbf{P}_i + \sum_{j=0}^{n-1}(1 + 2\cos(\frac{2\pi|i-j|}{n}))\mathbf{P}_j}{8\sum_{k=0}^{n-1} c_k}\,, \tag{1}$$

where $c_k = d_{k-1,k}d_{k+1,k}$ and

$$\mathbf{V} = \frac{\sum_{k=0}^{n-1} c_k\mathbf{P}_k}{\sum_{k=0}^{n-1} c_k}\,.$$

The rules for specifying new knot intervals $\bar{d}^k_{ij}$ are given as follows [3]

$$\begin{array}{l} \bar{d}^0_{i,i+1} = \bar{d}^{-1}_{i,i-1} = d^0_{i,i+1} \\ \bar{d}^0_{i,i-1} = \bar{d}^{1}_{i,i+1} = d^0_{i,i-1} \end{array}. \tag{2}$$

Remark 1 Different from the original form in [3], the new knot intervals are all multiplied by two, since non-uniform Doo-Sabin subdivision rules only rely on ratios of the knot intervals. This simplifies computation and leads to stationary subdivision.

Unfortunately, quadratic NURSSes are not even convergent in some cases. Qin et al. [10] proved that quadratic NURSS refinement converges for n-sided faces with $n \leq 12$, but may diverge when

$n > 12$. On the other hand, for convergent quadratic NURSSes, no closed-form limit point or limit normal rules have been proposed so far.

3 Extended Doo-Sabin Surfaces

We now present a modification of quadratic NURSSes [3] that we call EDSes (Extended Doo-Sabin Surfaces). EDSes are identical to quadratic NURSSes, with one difference: EDSes use the refinement rules for Doo-Sabin surfaces [1] to compute new vertices created for irregular faces. It is for that reason that EDSes are always convergent for all positive knot intervals. It is also for that reason that EDSes have closed-form limit point as well as limit normal rules.

3.1 Regular faces

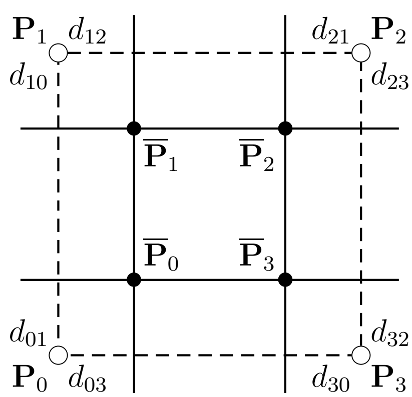


Fig. 3: A regular face with its knot intervals.

The refinement rules for regular faces for EDSes are identical to those for quadratic NURSSes. Let $n = 4$, it follows from Eq. (1) that

$$
\begin{aligned}
\overline{\mathbf{P}}_i &= \frac{\mathbf{P}_i + \mathbf{V}}{2} + \frac{c_{i+2}}{4\sum_{k=0}^{3} c_k}(\mathbf{P}_{i-1} + \mathbf{P}_{i+1} - \mathbf{P}_i - \mathbf{P}_{i+2}) \\
&= (\frac{1}{2} + \frac{w_i}{2} - \frac{w_{i+2}}{4})\mathbf{P}_i + (\frac{w_{i-1}}{2} + \frac{w_{i+2}}{4})\mathbf{P}_{i-1} + (\frac{w_{i+1}}{2} + \frac{w_{i+2}}{4})\mathbf{P}_{i+1} + \frac{w_{i+2}}{4}\mathbf{P}_{i+2}
\end{aligned}
$$

where $w_j = c_j / \sum_{k=0}^{k=3} c_k$, $j = 0, 1, 2, 3$, and indices are taken modulo 4.

We now derive limit point and limit normal rules for regular faces. Let the control point vector of a regular face be $\mathbf{M} = [\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3]^T$ and $\overline{\mathbf{M}}$ be the corresponding control point vector after subdivision. Then $\overline{\mathbf{M}} = \mathbf{S}_4\mathbf{M}$, where the subdivision matrix $\mathbf{S}_4$ is a 4×4 matrix:

$$\mathbf{S}_4 = \begin{bmatrix} \frac{1}{2}+\frac{w_0}{2}-\frac{w_2}{4} & \frac{w_1}{2}+\frac{w_2}{4} & \frac{w_2}{4} & \frac{w_3}{2}+\frac{w_2}{4} \\ \frac{w_0}{2}+\frac{w_2}{4} & \frac{1}{2}+\frac{w_1}{2}-\frac{w_3}{4} & \frac{w_2}{2}+\frac{w_3}{4} & \frac{w_3}{4} \\ \frac{w_0}{4} & \frac{w_1}{2}+\frac{w_0}{4} & \frac{1}{2}+\frac{w_2}{2}-\frac{w_0}{4} & \frac{w_3}{2}+\frac{w_0}{4} \\ \frac{w_0}{2}+\frac{w_1}{4} & \frac{w_1}{4} & \frac{w_2}{2}+\frac{w_1}{4} & \frac{1}{2}+\frac{w_3}{2}-\frac{w_1}{4} \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{l}_0 \\ \mathbf{l}_1 \\ \mathbf{l}_2 \\ \mathbf{l}_3 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} \mathbf{l}_0 \\ \mathbf{l}_1 \\ \mathbf{l}_2 \\ \mathbf{l}_3 \end{bmatrix}$$

Here $\mathbf{l}_0, \mathbf{l}_1, \mathbf{l}_2$ and $\mathbf{l}_3$ are the left eigenvectors of $\mathbf{S}_4$ corresponding to eigenvalues $\lambda_0 = 1, \lambda_1 = \lambda_2 = \frac{1}{2}$, and $\lambda_3 = \frac{1}{4}$ respectively:

$$\begin{aligned} \mathbf{l}_0 =&[\frac{w_0}{3}+\frac{2}{3}(w_0+w_3)(w_0+w_1), \frac{w_1}{3}+\frac{2}{3}(w_0+w_1)(w_1+w_2), \\ &\frac{w_2}{3}+\frac{2}{3}(w_1+w_2)(w_2+w_3), \frac{w_3}{3}+\frac{2}{3}(w_2+w_3)(w_0+w_3)] , \\ \mathbf{l}_1 =&[(w_0+w_1)(1-2(w_0+w_3)), -2(w_0+w_1)(w_1+w_2), \\ &(w_1+w_2)(1-2(w_2+w_3)), w_0-w_2+2(w_1+w_2)(w_2+w_3)] , \\ \mathbf{l}_2 =&[-2(w_0+w_1)(w_0+w_3), -(w_0+w_1)(1-2(w_0+w_3)), \\ &w_1-w_3+2(w_0+w_3)(w_2+w_3), (w_0+w_3)(1-2(w_2+w_3))] , \\ \mathbf{l}_3 =&\frac{1}{3}(w_1-4(w_0+w_1)(w_1+w_2))[1,-1,1,-1] . \end{aligned} \tag{3}$$

Using the approach described in [7], one can get:

Proposition 1 *For a regular face with vertices $\mathbf{M} = [\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3]^T$, its center will converge to the point $\mathbf{l}_0 \cdot \mathbf{M}$ on the limit surface.*

Proposition 2 *The normal vector to an EDS at the limit point $\mathbf{l}_0 \cdot \mathbf{M}$ corresponding to a regular face with vertices $\mathbf{M}$ is the vector $(\mathbf{l}_1 \cdot \mathbf{M}) \times (\mathbf{l}_2 \cdot \mathbf{M})$, here "$\times$" denotes vector cross product.*

3.2 Irregular faces

To solve the divergence problem of quadratic NURSSes, EDSes employ the same refinement rules for irregular faces as Doo-Sabin surfaces [1]. For an irregular face with n vertices $\mathbf{P}_0, \ldots, \mathbf{P}_{n-1}$ (see Fig. 4), the new vertex $\overline{\mathbf{P}}_i$ is computed as

$$\overline{\mathbf{P}}_i = \sum_{j=0}^{n-1} a_{i,j}\mathbf{P}_j , \tag{4}$$

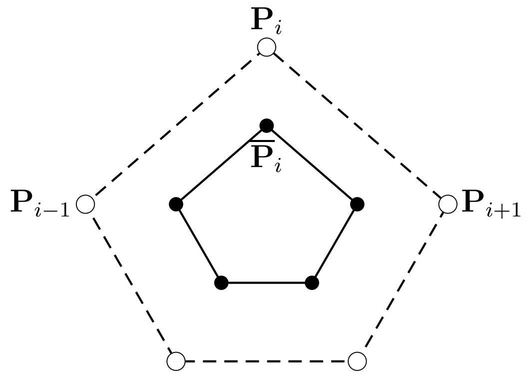


Fig. 4: An irregular face.

where the weights suggested in [1] are

$$a_{ij} = \begin{cases} \frac{n+5}{4n}, & i = j \ ; \\ \frac{3+2\cos(2\pi(i-j)/n)}{4n}, & i \neq j \ . \end{cases}$$

Let $\mathbf{M} = [\mathbf{P}_0, \mathbf{P}_1, \ldots, \mathbf{P}_{n-1}]^T$ denote the control point vector of an irregular n-sided face, and $\overline{\mathbf{M}}$ denote the corresponding control point vector after subdivision. Then it follows that $\overline{\mathbf{M}} = \mathbf{S}_n\mathbf{M}$, where the subdivision matrix $\mathbf{S}_n$ is an $n \times n$ matrix. Using the discrete Fourier transform technique [12], the eigenvalues of $\mathbf{S}_n$ are

$$\lambda_0 = 1 > \lambda_1 = \lambda_2 = \frac{1}{2} > \lambda_3 = \ldots = \lambda_{n-1} = \frac{1}{4} \ ,$$

and the left eigenvectors $\mathbf{l}_0, \mathbf{l}_1$ and $\mathbf{l}_2$ of $\mathbf{S}_n$ corresponding to λ_0, λ_1 and λ_2 respectively are as follows

$$\begin{aligned} \mathbf{l}_0 =& \frac{1}{n}[1, 1, \ldots, 1] \ , \\ \mathbf{l}_1 =& [1, \cos(2\pi/n), \ldots, \cos(2\pi(n-1)/n)] \ , \\ \mathbf{l}_2 =& [0, \sin(2\pi/n), \ldots, \sin(2\pi(n-1)/n)] \ . \end{aligned} \tag{5}$$

Thus we have the following limit point and limit normal rules for irregular faces.

Proposition 3 *For an irregular n-sided face with vertices $\mathbf{M} = [\mathbf{P}_0, \mathbf{P}_1, \ldots, \mathbf{P}_{n-1}]^T$, its center will converge to the point $\mathbf{l}_0 \cdot \mathbf{M} = \frac{1}{n}\sum_{i=0}^{n-1} \mathbf{P}_i$ on the limit surface.*

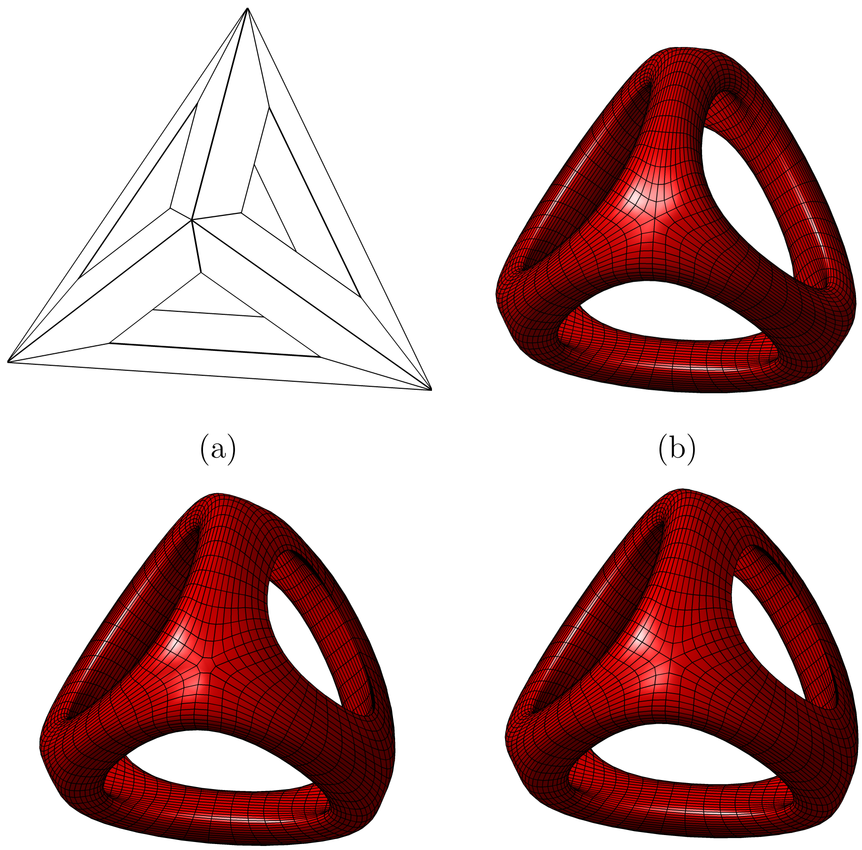
Proposition 4 *The normal vector to an EDS at the limit point $\mathbf{l}_0 \cdot \mathbf{M}$ corresponding to an irregular n-sided face with vertices $\mathbf{M}$ is the vector $(\mathbf{l}_1 \cdot \mathbf{M}) \times (\mathbf{l}_2 \cdot \mathbf{M})$, here "$\times$" denotes vector cross product.*

For Doo-Sabin subdivision scheme [1], Peters and Reif proved C^1-continuity for all valences [12]. Then we get the following continuity result for EDSes.

Theorem 1 *If all knot intervals are positive, the limit surface generated by the EDS refinement scheme is G^1 continuous everywhere.*

3.3 Examples

EDSes are a generalization of non-uniform biquadratic B-spline surfaces and Doo-Sabin surfaces. If all vertices are of valence four, if all faces are four-sided, and if the knot intervals on rows and columns are equal respectively, EDSes reduce to non-uniform biquadratic B-spline surfaces. If all knot intervals are equal, EDSes degenerate to Doo-Sabin surfaces. Thus, a modeling program for EDSes can generate biquadratic NURBS sufaces and Doo-Sabin surfaces as special cases.



(a) (b)

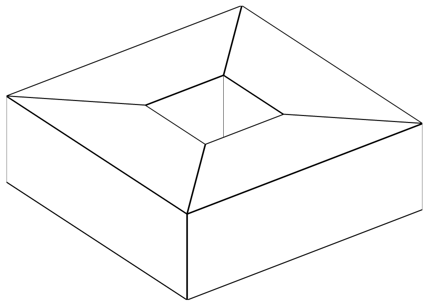
(c) (d)

Fig. 5: Tetrahedral mesh with holes: (a) initial control mesh, (b) Doo-Sabin surface, (c) control mesh after four rounds of EDS subdivision, (d) limit mesh of (c).

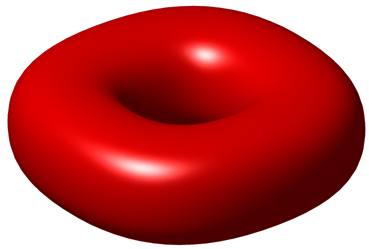
Fig. 5 shows uniform and non-uniform Doo-Sabin surfaces produced from a tetrahedral mesh with holes. Fig. 5(b) depicts a Doo-Sabin surface after four refinement steps on the initial control mesh in (a). Figs. 5(c) and (d) show the control mesh and limit mesh of an EDS whose crease is formed by setting three knot intervals of certain initial vertices to zero. The limit mesh in (d) is rendered using the proposed limit point and limit normal rules.

Fig. 6 is an example of a doughnut model whose initial control mesh is topologically a rectangular grid (i.e. a B-spline control net). Fig. 6(b) shows a uniform biquadratic B-spline surface. A biquadratic NURBS surface with a crease is depicted in (c) by setting one row of the knot intervals to zero. Fig. 6(d) illustrates an EDS with a dart induced by setting to zero the knot intervals of appropriate vertices.

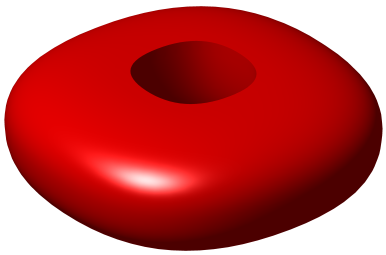
The shapes in Figs. 5(d) and 6(d) cannot be produced using biquadratic NURBS or uniform



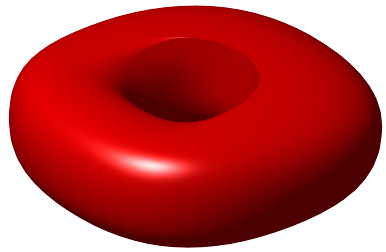
(a)



(b)



(c)



(d)

Fig. 6: Doughnut model: (a) initial control mesh, (b) uniform biquadratic B-spline surface, (c) biquadratic NURBS surface with a crease, (d) EDS with a dart.

Doo-Sabin surfaces.

4 Conclusion

This paper presents EDSes which are a modification of quadratic NURSSes [3]. EDSes retain the refinement rules for quadratic NURSSes for regular faces whereas use the refinement rules for Doo-Sabin surfaces for irregular faces.

Both EDSes and quadratic NURSSes are generalizations of non-uniform biquadratic B-spline surfaces and Doo-Sabin surfaces. In comparison to quadratic NURSSes, if all the knot intervals are greater than zero, EDSes converge at extraordinary points of arbitrary valence while quadratic NURSSes may diverge for valences larger than 12. And closed-form limit point and limit normal rules are available for EDSes as well.

In future work we hope to derive a parametrization for exact and efficient evaluation of EDSes following the method of Stam [13].

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