This article was downloaded by: [China Science & Technology University], [Zuojin Zhu] On: 15 July 2015, At: 23:32 Publisher: Taylor & Francis Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: 5 Howick Place, London, SW1P 1WG



Click for updates



Publication details, including instructions for authors and subscription information: <u>http://www.tandfonline.com/loi/gcfd20</u>

# LES of incompressible heat and fluid flows past a square cylinder at high Reynolds numbers

Yinglin Li<sup>ab</sup>, C.K. Chan<sup>b</sup>, Bowen Mei<sup>a</sup> & Zuojin Zhu<sup>a</sup>

<sup>a</sup> Faculty of Engineering Science, University of Science and Technology of China, Hefei, P.R. China

<sup>b</sup> Department of Applied Mathematics, The Hong Kong Polytechnic University, Kowloon, Hong Kong

Published online: 15 Jul 2015.

To cite this article: Yinglin Li, C.K. Chan, Bowen Mei & Zuojin Zhu (2015): LES of incompressible heat and fluid flows past a square cylinder at high Reynolds numbers, International Journal of Computational Fluid Dynamics, DOI: <u>10.1080/10618562.2015.1058373</u>

To link to this article: <u>http://dx.doi.org/10.1080/10618562.2015.1058373</u>

# PLEASE SCROLL DOWN FOR ARTICLE

Taylor & Francis makes every effort to ensure the accuracy of all the information (the "Content") contained in the publications on our platform. However, Taylor & Francis, our agents, and our licensors make no representations or warranties whatsoever as to the accuracy, completeness, or suitability for any purpose of the Content. Any opinions and views expressed in this publication are the opinions and views of the authors, and are not the views of or endorsed by Taylor & Francis. The accuracy of the Content should not be relied upon and should be independently verified with primary sources of information. Taylor and Francis shall not be liable for any losses, actions, claims, proceedings, demands, costs, expenses, damages, and other liabilities whatsoever or howsoever caused arising directly or indirectly in connection with, in relation to or arising out of the use of the Content.

This article may be used for research, teaching, and private study purposes. Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden. Terms & Conditions of access and use can be found at <a href="http://www.tandfonline.com/page/terms-and-conditions">http://www.tandfonline.com/page/terms-and-conditions</a>



# LES of incompressible heat and fluid flows past a square cylinder at high Reynolds numbers

Yinglin Li<sup>a,b</sup>, C.K. Chan<sup>b</sup>, Bowen Mei<sup>a</sup> and Zuojin Zhu<sup>a,\*</sup>

<sup>a</sup>Faculty of Engineering Science, University of Science and Technology of China, Hefei, P.R. China; <sup>b</sup>Department of Applied Mathematics, The Hong Kong Polytechnic University, Kowloon, Hong Kong

(Received 17 February 2015; accepted 31 May 2015)

This paper presents large eddy simulation (LES) results of incompressible heat and fluid flows around a square cylinder (SC) at zero incident angle at high Reynolds numbers (Re) in the range from  $1.25 \times 10^5$  to  $3.5 \times 10^5$ . LES results are obtained on the basis of swirling strength based sub-grid model, and a higher order upwind scheme developed with respect to the Taylor expansion. It was found that, for the zero incident SC wake flows at a Reynolds number in the range {Re<sub>5</sub> = Re/10<sup>5</sup>  $\in$  [1.25, 3.5]}, the Strouhal number equals to 0.1079, completely independent of the Reynolds number; the coefficient of drag is around 1.835 with an uncertainty of about 1.9%, almost non-sensitive to the Re. When Re is beyond  $3.0 \times 10^5$ , the time-averaged peak value of sub-grid viscosity is over 340, implying that the role of sub-grid model is crucial in some regions where vortex motion is active and vortex interaction is intense. The time–spanwise (*t-z*) averaged sub-grid viscosity ratio profiles of fluctuations of the sub-grid viscosity ratio and velocity components at four locations downstream of the SC are presented. The fields of the *t-z* averaged sub-grid viscosity ratio, and the instantaneous fields of streamwise and spanwise vorticities are also reported and discussed. The predicted mean Nusselt number is compared with empirical correlations, revealing that swirling strength based LES has its potential in predicting natural and industrial flows.

Keywords: square cylinder wake flow; sub-grid viscosity ratio; incompressible heat and fluid flows; swirling strength; vortex interaction

#### 1. Introduction

Incompressible turbulent heat and fluid flows past a square cylinder (SC) involving vortex shedding and convective heat transfer are of great significance in applied mathematics and mechanics. Recently, many studies of wake flows were performed by experiments (Luo, Chew, and Ng 2003; Tamura and Miyagi 1999; Wang and Zhou 2009), stability analysis (Williamson 1996; Robichaux, Balachandar, and Vanka 1999), and numerical simulations (Kato and Launder 1993; Bosch and Rodi 1998; Sohankar, Norberg, and Davidson 1999; Sohankar 2006; Saha, Biswas, and Muralidhar 2003; Zhu, Niu, and Li 2013).

We should mention the heat transfer measurements of Hilpert (1933) and Igarashi (1987), as reported in Sparow, Abraham, and Tong (2004) and Wiesche (2007). The measurements of Igarashi were done in a low-speed wind tunnel with a working section 400 mm high, 150 mm wide, and 800 mm long, with the wind speed ranging from 6 to 20 m/s. The rectangular cylinders with a height of 30 mm were heated under the condition of constant heat flux.

In general, for turbulent flow modelling, three strategies, i.e. the phenomenological model, direct numerical simulation (DNS), and large eddy simulation (LES) are widely used. The first phenomenological model, as reviewed by Hanjalic (2002), is based on Reynolds averaging, without any universal model coefficient, and implies that the coefficient examined in some benchmark cases might be unsuitable to other flows in nature and modern industry.

DNS, as reported in Holmes, Lumley, and Berkooz (1996), Manhart (2004), and Niu and Zhu (2006), in the traditional meaning, requires the use of an extremely fine grid with a total spatial grid number as large as Re<sup>2.25</sup>, where Re is the flow Reynolds number. This indicates that for DNS only supercomputers are possible to provide the computation resources.

LES, as reported in Smagorinsky (1963) and McMillan and Feiziger (1979), uses grid-filtering to simulate the large motion explicitly, with the small eddy effect on the large eddy motion being modelled. The LES work reported previously (Moin and Kim 1982; Madabhushi and Vanka 1991; Metais and Lesieur 1996; Sohankar 2006; Yu, Lau, and Chan 2004; Guo et al. 1995; Wu, Chan, and Zhou 2011; Duwig et al. 2011; Nogenmyr et al. 2013) indicates that this strategy is appropriate for predicting turbulent flows.

For LES, many sub-grid scale (SGS) models have been developed, among which, are the earlier SGS model of Smagorinsky (1963), the dynamic SGS model given by Germano et al. (1991), a subsequent modification given by Lilly (1992), the SGS model based on the Kolmogorov equation for resolved turbulence given by Cui, Xu, and

<sup>\*</sup>Corresponding author. Email: zuojin@ustc.edu.cn

Zhang (2004) and Cui, Zhou, et al. (2004), the model based on algebraic theory by Vreman (2004), the model assuming the sub-grid stress to be proportional to the temporal increment of filtered strain rate by Zhu, Yang, and Chen (2010), and the recent SGS model based on dynamic estimation of Lagrangian time scale given by Verma and Mahesh (2012).

In LES the unclosed stresses are usually modelled by SGS viscosity models to smooth the Navier–Stokes equations. Alternatively, another way is to modify the non-linear convective terms directly as done by Holm (1999). Thus, models based on these regulations occur (Cheskidov et al. 2005; Geurts and Holm 2003; van Reeuwijk, Jonker, and Hanjalic 2008, 2009).

Turbulent flow is intermittent in time and space, indicating that the flow may be locally swirling or non-swirling, depending on the swirling strength. The SGS model used in this paper, as described by Zhu, Niu, and Li (2013), can to some extent characterise the flow intermittency. Results of the LES are compared with existing experimental data.

To show different flow characteristics found in a circular cylinder (CC) flow, this paper presents LES results of incompressible heat and fluid flow around an SC at high Reynolds numbers relevant to the range  $\text{Re}_5 \in [1.25, 3.5]$ , covering the subcritical Reynolds number ( $\sim 2.0 \times 10^5$ ) of circular cylinder flow as reported by Beaudan and Moin (1994). The incoming flow has zero incident angle and has no inlet disturbances. LES demonstrates some characteristics involved in incompressible heat and fluid flows around an SC from four aspects: (1) the Reynolds number dependence; (2) the t-z averaged sub-grid viscosity ratio profiles and the profiles of root-mean-square (rms) values of the sub-grid viscosity ratio and velocity components at four locations downstream of the SC; (3) the field of the t-z averaged sub-grid viscosity ratio, and the instantaneous fields of streamwise and spanwise vorticities; (4) the convective heat transfer from the SC.

#### 2. Governing equations

At high Reynolds numbers, effects of turbulence are rather intense and significant as introduced in the work of Holmes, Lumley, and Berkooz (1996), the influence of SGS on largescale motions should be considered carefully. In previous work (Moin and Kim 1982; Madabhushi and Vanka 1991; Metais and Lesieur 1996; Sohankar 2006; Yu, Lau, and Chan 2004; Guo et al. 1995; Wu, Chan, and Zhou 2011; Duwig et al. 2011; Nogenmyr et al. 2013) the sub-grid stress was postulated to be proportional to the locally filtered strain rate, with a factor called sub-grid viscosity, similar to the Newtonian constitutive relationship, implying that these SGS models have not satisfactorily ascertained the problem: at which Reynolds number, these models are applicable, because the sub-grid viscosity still exists even when the flow is in the laminar regime.

In the earlier model of Smagorinsky (1963), sub-grid viscosity is assumed to be proportional to the modular of locally filtered strain rate. Unfortunately, for threedimensional velocity fields obtained by stability analysis of Chandrasekhar (1961), numerical tests revealed that the peak values of the modular of strain rate still exist at some flow regions without swirling. To avoid this ambiguity, considering turbulent flows having vortical structures and intrinsic intermittency, as reported in Zhu, Niu, and Li (2013) and Zhang et al. (2014), the model used in the present LES study is briefly introduced below.

In the SGS model, sub-grid viscosity is suggested to be proportional to the factor of swirling strength intermittency (FSI) and the local swirling strength,

$$\nu_s = C_{\mu} f_I(\mathbf{x}, t) \lambda_{ci} \delta^2 \tag{1}$$

Here  $C_{\mu}$  (= 0.09) is an artificially defined constant;  $\lambda_{ci}$  is the swirling strength of filtered velocity gradient  $\nabla \mathbf{u}$ , the FSI denoted by  $f_i$ , is defined by the ratio of swirling strength  $\lambda_{ci}$  to the magnitude of the complex eigenvalue  $\lambda(=\lambda_{cr} + i\lambda_{ci})$  of  $\nabla \mathbf{u}$ , i.e.

$$f_I(\mathbf{x}, t) = \frac{\lambda_{ci}}{\sqrt{\lambda_{cr}^2 + \lambda_{ci}^2}}$$
(2)

where  $\delta$  is the length scale, defined by the harmonic average of grid intervals,

$$\frac{1}{\delta} = \sum_{i=1}^{3} 1/\delta_i \tag{3}$$

with  $\delta_i$  being the grid interval in  $x_i$  direction. Let the filtered velocity gradient  $\nabla \mathbf{u}$  be

$$\nabla \mathbf{u} = \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$
(4)

its eigenvalue  $\lambda$  should satisfy the characteristic equation

$$\lambda^3 + b\lambda^2 + c\lambda + d = 0 \tag{5}$$

where

$$\begin{cases} b = -\text{tr}(\mathbf{A}) = -(a_{11} + a_{22} + a_{33}), \\ c = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}, \quad d = -|\mathbf{A}|$$
(6)

with  $|\mathbf{A}|$  being the determinant of matrix **A**. For incompressible flows, with respect to the free divergence of velocity vector ( $\nabla \cdot \mathbf{u} = 0$ ), the coefficient in Equation (5), b = 0. When the coordinates are non-rotating, the Coriolis force should be zero. At the local point where the roots of Equation (5) are real, indicating that the vortices have not arrived at the point, the local flow regime is temporally

laminar. When the local flow is temporally turbulent, the roots have two conjugate complex eigenvalues denoted by  $\lambda_{cr} \pm i\lambda_{ci}$ , here  $i(=\sqrt{-1})$  is the unit of imaginary number. Iso-surfaces of swirling strength have been used to show vortices in turbulent flows previously (Zhou et al. 1999; Ganapathisubramani, Longmire, and Marusic 2006; Lin and Zhu 2010).

To maintain a certain parametric consistency in turbulence modelling, as reported in Zhu, Niu, and Li (2013) and Zhang et al. (2014), the coefficient  $C_{\mu} = 0.09$ , which is assigned to be the same as that used for turbulent eddy viscosity of phenomenological models (Hanjalic 2002). In the sub-grid viscosity model of Vreman (2004), the relevant coefficient is set as  $2.5C_S^2$ , where  $C_S = 0.17$  is the theoretical value for homogeneous isotropic turbulence (Lilly 1967), giving a value of 0.07. Furthermore, to obtain robust simulations in complex cases the practical value of  $C_S$  is sometimes higher than the theoretical value. For example,  $C_{\rm S} = 0.2$ , which has frequently been used in literature, corresponds to a value of 0.1, in the sub-grid model. In the sub-grid model of Germano et al. (1991), the relevant coefficient is obtained by double control volumes averaging, and the coefficient is temporally and spatially changing, hence such kind of model is called dynamic model.

In order to define the length scale of the grid interval  $\delta$ , the harmonic average is used, in which any smaller grid interval can get a larger weight of average. This has an influence on the calculation of SGS viscosity, but just in the value. In principle, we suggest that the local SGS viscosity be omitted when  $\lambda_{ci} = 0$ , therefore, leading to LES' natural transition to DNS.

Considering that the sub-grid viscosity  $v_s$  already has a factor  $f_l$ , the decaying factor of length scale  $[1 - \exp(-y^+/25)]$  near the wall, as used previously in the work of Moin and Kim (1982), is cancelled in the present model. Here  $y^+ = yu_\tau/v$ , y denotes the normal distance to the wall, with  $u_\tau$  and v representing the mean friction velocity and the kinematic viscosity of fluid, respectively.

It is noted that swirling strength does not vanish completely when flow is laminar under a rotating coordinate system. The difference of the present sub-grid model with that of Vreman (2004) was given by Zhu, Niu, and Li (2013), from which it can be ascertained that the sub-grid viscosity of the present model is zero if the local flow is non-swirling, while the sub-grid viscosity of the Vreman's model does not hold this peculiarity.

Consider the turbulent SC flow in a Cartesian coordinate system, as schematically shown in Figure 1(a) and 1(b), in which  $x(x_1)$  is the horizontal coordinate,  $y(x_2)$  and  $z(x_3)$  are the vertical and spanwise directions. The SC's spanwise length is given by *B*, and its cross-sectional side length is set as *d*.  $x_u$ ,  $x_d$ ,  $y_b$ , and  $y_t$  are geometric parameters of the computational domain, which can be seen in Table 1. The coordinate origin is set at the left-bottom corner of the SC. It is seen that the distance from the SC front face to



Figure 1. Schematic of flow past a square cylinder. (a) x-y plane; (b) x-z plane.

the inlet section  $(x_u)$ , the distance from the top face to the top boundary of the domain  $(y_t)$ , and the distance from the bottom face to the bottom boundary of the domain  $(y_b)$ , in the unit of *d*, have the same value 8.5, implying that the blockage of the SC flow is  $1/18 \ \approx 5.56\%$ ). While the distance of the outlet section to the SC rear face has a value 15, which was found to be sufficiently long to conduct DNS of the SC flow as reported by Niu and Zhu (2006). For all cases in this LES, the spanwise length of the domain *B* is set as 4.

Denoting the incoming flow velocity by  $u_{in}$ , the Reynolds number of the SC flow should be  $\text{Re} = u_{in}d/v$ ; the incoming fluid is air at ambient condition, hence the fluid Prandtl number (Pr = v/a) is set as 0.71. It is assumed that the temperature influence on fluid thermodynamic properties is negligible. The SC wall surfaces have a constant temperature  $T_w$ , which is different from the ambient  $T_\infty$ .

Let the fluid temperature be *T*, the normalised temperature is given by  $\Theta = (T - T_{\infty})/\Delta T_w$ , where  $\Delta T_w = T_w$  $- T_{\infty}$ . With the incoming flow speed  $u_{in}$  and SC crosssectional side length *d*, express the fluid density by  $\rho$ , the time and pressure scales are expressed as  $t_0 = d/u_{in}$  and  $\rho u_{in}^2$ . Therefore, for the incompressible heat and fluid flow past an SC, the normalised governing equations have the following form:

$$\nabla \cdot \mathbf{u} = 0 \tag{7}$$

$$\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot [\gamma_1 \nabla \mathbf{u}] + \mathbf{R}$$
(8)

$$\Theta_t + \mathbf{u} \cdot \nabla \Theta = \nabla \cdot [\gamma_2 \nabla \Theta] \tag{9}$$

For simplicity, the over bar for the filtering of flow variables is omitted. The normalised total viscosities  $\gamma_1$  and  $\gamma_2$  are

Table 1. Computational parameters.

$x_u/d$	$x_d/d$	$y_b/d$	$y_t/d$	$N_1$	$N_2$	$N_3$	$R_S^{\rm a}$
8.5	15	8.5	8.5	191	145	41	0.819

<sup>a</sup>S<sup>max</sup>  $\approx 0.5\%$ , with actual number of time steps n = 13,800 relevant to the final time t = 170.

given by

choose

$$\gamma_1 = \frac{1}{\text{Re}} \left( 1 + \frac{\nu_s}{\nu} \right), \quad \gamma_2 = \frac{1}{\text{RePr}} \left( 1 + \frac{\nu_s}{\sigma_\theta \nu} \right) \quad (10)$$

where the turbulent Prandtl number  $\sigma_{\theta}$  is assumed to be 0.9. **R** is a force related to the gradient of viscosity  $\gamma_1$ , given by

$$\mathbf{R} = (\nabla \mathbf{u})^T \cdot \nabla \gamma_1 \tag{11}$$

The use of  $\sigma_{\theta}$  in Equation (9) indicates that the temperature-velocity coupling induced heat flux has been considered by a gradient diffusion assumption. In mathematical modelling of the turbulent heat flux, there are other kinds of methods such as those described by Uddin et al. (2009) and Smirnov and Nikitin (2014). An assessment of some turbulent heat flux models was done by Uddin et al. (2009) in the LES of heat transfer induced by a turbulent impinging jet. For hydrogen combustion in engines, in the recent modelling and simulation encompassed by Smirnov and Nikitin (2014), the turbulent heat flux model was simply given by a gradient diffusion hypothesis, and the temperature fluctuation was governed by a partial differential equation derived from energy conservation, where the Prandtl numbers for the turbulent diffusion of species were also used.

In this paper, we focus on the swirling strength based model in describing the effect from SGS eddies, for temperature in the SC flow field, we choose the constant turbulent  $\sigma_{\theta}$  model, so that the turbulent heat flux is expressed as

$$-\overline{u_j'\theta} = \frac{v_s}{\sigma_\theta} \frac{\partial\Theta}{\partial x_j}$$

where  $u'_{j}$  denotes velocity fluctuation in  $x_{j}$  direction,  $\theta$  being the temperature fluctuation. The solutions of Equations (7)–(9) are sought under appropriate conditions. For the boundary conditions (BC) on the SC walls, constant wall temperature and non-slip BC are used, such that

$$u = 0, \quad v = 0, \quad w = 0, \quad \Theta = 1$$
 (12)

while for the BC at the outlet, similar to the treatment of Sohankar, Norberg, and Davidson (1999) and Saha, Biswas, and Muralidhar (2003), we use the Olanski (1976) type, for  $x = 1 + x_d$ 

$$\varphi + u_c \varphi = 0 \tag{13}$$

where in the normalised form, we choose  $u_c = 1$ ,  $\varphi = (u, v, w, \Theta)^T$ , with the superscript 'T' denoting the transpose of matrix  $(u, v, w, \Theta)$ . At the inlet section, for  $x = -x_u$  we

 $u = 1, v = 0, w = 0, \Theta = 0$ 

On the bottom and top boundaries of the computational domain, where  $y = -y_b$ , or  $1 + y_t$ , we use

$$u_{y} = 0, \quad v = 0, \quad w = 0, \quad \Theta_{y} = 0$$
 (15)

(14)

On the spanwise boundaries, where z = 0, or z = B, we use the periodic condition such that

$$\varphi(x, y, z, t) = \varphi(x, y, z + B, t)$$
(16)

where *B* is the spanwise length of the computational domain (see in Figure 1(b)]. Finally, the initial condition, at t = 0, is given by

$$u = 1, \quad v = 0, \quad w = 0, \quad \Theta = 0$$
 (17)

# 3. Numerical method

#### 3.1. Solution procedure

The governing equations (7)–(9) of the heat and fluid flow problem are discretised by a finite difference method in a non-uniform staggered gird system, where the convective terms are treated by a fourth-order upwind scheme reported by Yang, Chen, and Zhu (2009). It is noted that by some proper changes, the existing numerical methods in DNS, such as those described in Patankar (1980), Harlow and Welch (1995), Nikitin (2006), Papanicolaou and Jaluria (1992), Khanafer, Vafai, and Lightstone (2002), are also applicable. The simple approach recently reported by Trias et al. (2013), which can be implemented easily on any structured or unstructured grid, is of course also suitable for LES of incompressible flows.

The solutions procedure of the governing equations is designed on the basis of the accurate projection algorithm PmIII of Brown, Cortez, and Minion (2001). Hence, just a brief description is given below.

Using  $\overline{\mathbf{u}}$ ,  $\phi$ , and *n* to represent intermediate velocity vector, pressure potential, and time level, respectively, defining a vector  $\mathbf{H} = (\mathbf{u} \cdot \nabla)\mathbf{u} - \mathbf{R}$ , then the velocity at the time level n + 1 can be written as

$$\mathbf{u}^{n+1} = \overline{\mathbf{u}} - \Delta t \nabla \phi \tag{18}$$

where the intermediate velocity vector  $\overline{\boldsymbol{u}}$  is calculated by means of

$$\frac{\overline{\mathbf{u}} - \mathbf{u}^n}{\Delta t} + \mathbf{H}^{n+1/2} = \nabla \cdot \left\{ \gamma_1 \nabla \left[ \mathbf{u}^n + \frac{1}{2} (\overline{\mathbf{u}} - \mathbf{u}^n) \right] \right\}$$
(19)

with pressure p predicted by

$$p^{n+1/2} = \{1 - 0.5(\Delta t)\nabla \cdot [\gamma_1 \nabla]\}\phi$$
(20)

The pressure potential  $\phi$  satisfies the Poisson's equation

$$\nabla^2 \phi = \nabla \cdot \overline{\mathbf{u}} / \Delta t \tag{21}$$

The temperature  $\Theta^{n+1}$  is calculated by

$$\frac{\Theta^{n+1} - \Theta^n}{\Delta t} + H_4^{n+1/2} = \nabla \cdot \left\{ \gamma_2 \nabla \left[ \Theta^n + \frac{1}{2} (\Theta^{n+1} - \Theta^n) \right] \right\}$$
(22)

where  $H_4 = (\mathbf{u} \cdot \nabla)\Theta$ , and the terms at time level of (n + 1/2) are calculated explicitly by the second-order Adams– Bashforth formula. The non-linear convective terms in the governing equations are spatially discretised by the fourthorder upwind finite difference scheme of Yang, Chen, and Zhu (2009), with the viscous diffusion terms by a secondorder central difference scheme.

The pressure potential Poisson's equation is initially solved by the approximate factorisation one (AF1) method of Adrian (1981), and then corrected by the stabilised biconjugate gradient method (Bi-CGSTAB) given by Vorst (1992) to improve the solution accuracy. In spatial discretisation, the implicit second-order Crank–Nicolson method is used to solve the right-hand side diffusion terms. It is noted that the present numerical approach treats the diffusion term in a non-standard way, where the part depending on the viscosity gradient ( $\nabla \gamma_1$ ) [see in Equation (10)] is calculated explicitly, with the remaining part denoted by [ $\nabla \cdot (\gamma_1 \nabla \mathbf{u})$ ] calculated implicitly.

#### 3.2. Accuracy improvement

For the finite difference of the term  $\nabla \cdot \overline{\mathbf{u}}$  and  $\nabla^2 \phi$  at time level *n*, the effect of heterogeneous stagger grid (i.e.  $\phi_i$  is positioned at  $x_i$  and arranged between  $u_i$  and  $u_{i+1}$ ) should be considered carefully. For convenience of description, we present here the relevant expressions for  $\overline{u}_x$  and  $\phi_{xx}$ , other formulae can be derived similarly. To cancel the unexpected numerical errors arisen from grid heterogeneity, let  $\delta_1 = (x_i - x_{i-1})$ ,  $\delta_2 = (x_{i+1} - x_i)$ , and  $\delta = (\delta_1 + \delta_2)/2$ , by means of Taylor expansion, we have

$$\begin{cases} \overline{u}_{x} = (\overline{u}_{i+1} - \overline{u}_{i})/\delta + R_{u} + o(\epsilon^{3}), \\ \phi_{xx} = [(\phi_{i+1} - \phi_{i})/\delta_{2} + (\phi_{i-1} - \phi_{i})/\delta_{1}]/\delta + R_{\phi} + o(\epsilon^{2}) \end{cases}$$
(23)

where

$$\begin{cases} R_{u} = -\left[(\delta_{2} - \delta_{1})\overline{u}_{x}^{(2)}/2! + (\delta_{1}^{2} + \delta_{2}^{2} - \delta_{1}\delta_{2})\overline{u}_{x}^{(3)}/3!\right], \\ R_{\phi} = -\left[(\delta_{2} - \delta_{1})\phi_{x}^{(3)}/3\right] \end{cases}$$
(24)

with  $o(\epsilon^2)$  and  $o(\epsilon^3)$  denoting the second- and third-order cut-off errors. In Equation (24),  $\overline{u}_x^{(2)}$ ,  $\overline{u}_x^{(3)}$  are respectively the

second- and third-order partial derivatives of  $\overline{u}$  with respect to x at  $x_i$ , and  $\phi_x^{(3)}$  is the third-order partial derivative of  $\phi$ with respect to x at  $x_i$ . The terms  $\overline{v}_y$  and  $\overline{w}_z$  in  $\nabla \cdot \overline{\mathbf{u}}$ , and the terms  $\phi_{yy}$  and  $\phi_{zz}$  in  $\nabla^2 \phi$  at time level *n* can be written similarly. Tracing the previous numerical works published in the literature, it is seen that very few studies have been conducted on the heterogeneous grid effect in dealing with the Poisson's equation of pressure potential.

In the recent numerical study of Smirnov et al. (2014), involving hydrogen fuel rocket engines simulation using LOGOS simulator (Betelin et al. 2014), an approach for estimating the accumulated error in numerical work based on Navier-Stokes equations was reported in detail. As the diffusive terms are treated with the second-order central scheme, using the values of  $N_1$ ,  $N_2$ , and  $N_3$  as listed in Table 1, and with the assumed allowable total error  $S^{max} =$ 0.5%, we can estimate the ratio of maximal allowable number of time steps for the problem and the actual number of time steps used to obtain the result,  $R_s = 0.819$ , as shown in Table 1. As reported by Smirnov et al. (2014), the ratio  $R_{\rm s}$  characterises reliability of results, i.e. how far below the limit the simulations were finalised. Indirectly it characterises the accumulated error. The higher the value  $R_s$  is, the lower is the error. On tending  $R_s$  to unity the error tends to maximal allowable value.

In the present study, using the strategy of accuracy improvement, results for  $\text{Re}_5 \in [1.25, 3.5]$  reveal that Strouhal number is Reynolds number independent. Evidence can be seen in the next section.

#### 4. Results and discussion

Using the solution method described in Section 3, LES of the heat and fluid flows past an SC (see schematically in Figure 1) was carried out in a personal computer with a memory of 3.2 GB and CPU frequency 3.30 GHz. Geometric parameters of the computational domain are given in Table 1. The scenarios are classified by  $Re_5$  (=  $Re/10^5$ ), each requiring a CPU time about 85.8 hours. As shown in Tables 2–4, eight scenarios are studied and presented.

In the z-direction, with uniform grid, the grid number  $N_3$  is set as 41. However, in the x- and y- directions the grids are non-uniform, and the grid numbers are  $N_1 = 191$  and  $N_2 = 145$ , respectively, with grid arrangement based on a geometric series algorithm. In this LES, the assignment of grids on the SC section walls uses two key parameters called amplifying factor (1/0.847) and series termination number (14). The total number of vertical grid lines through the SC cross-sectional side is 34.

In the bottom and top sides of the SC, the *y*-grid is assigned with an amplifying factor of 1/0.9 and a series termination number of 37. For the *x*-grid assignment, the relevant values are 1/0.817 and 17 for the region upstream

Re <sub>5</sub>	CL	CD	$(v_{sr})_{pm}$	CL'	CD'	$(v_{sr})'_p$	St
1.25	-0.0369	1.825	146.23	0.1679	0.1013	34.14	0.1079
1.5	-0.0365	1.834	172.91	0.1721	0.1016	39.98	0.1079
2.0	-0.0373	1.832	228.38	0.1708	0.1024	43.92	0.1079
2.5	-0.0370	1.833	290.74	0.1710	0.1022	71.81	0.1079
2.75	-0.0369	1.845	292.59	0.1789	0.1040	60.42	0.1079
3.0	-0.0369	1.841	342.65	0.1817	0.1081	65.17	0.1079
3.25	-0.0367	1.843	366.75	0.1795	0.1046	81.55	0.1079
3.5	-0.0374	1.834	404.23	0.1725	0.1042	89.92	0.1079

Table 2. Re dependence of CL, CD,  $(v_{sr})_{pm}$ , their RMS values, and Strouhal number.

Table 3. Re dependence of Nusselt numbers.

Re <sub>5</sub>	Nu <sub>b</sub>	Nu <sub>t</sub>	Nuf	Nu <sub>r</sub>	Nu <sub>m</sub>
1.25	65.03	79.04	356.69	73.97	143.68
1.5	72.79	89.86	405.63	85.21	163.37
2.0	89.45	111.66	508.36	114.43	205.97
2.5	100.36	127.41	623.69	142.75	248.55
2.75	107.02	139.74	690.60	162.30	274.91
3.0	113.17	148.76	764.71	175.91	300.64
3.25	117.09	154.58	878.87	192.40	335.73
3.5	118.77	158.32	1134.57	197.64	402.32

of the SC, but 1/0.927 and 47 for the region downstream of the SC. Using the geometric series algorithm for grid partition, it can be shown that the finest grid distance to each SC wall in the unit of *d* is  $2.733 \times 10^{-3}$ . Such a grid arrangement leads to a time step of  $\Delta t = 1.25 \times 10^{-3}$ .

To show the distribution of sub-grid viscosity ratio in x-y plane, time-spanwise averaging (t-z) is used. A subscript (m) is used to label the *t-z* averaged variables, with a superscript (') used to represent the rms values of the variables. In particular, the subscript 'm' is omitted for *t-z* average lift (CL) and drag (CD), for simplicity. Furthermore, grid spacings and time step for the eight scenarios are the same. To remove the influence from initial condition, merely the data in the period of  $t \in [68, 170]$  are processed.

As reported by Beaudan and Moin (1994), experimental results of Roshko (1961) suggested that in the extreme case of very rough circular cylinder, some mean flow statis-

Table 4. Re dependence of rms values of Nusselt numbers.

Re <sub>5</sub>	$\mathrm{Nu}_b'$	$\mathbf{N}\mathbf{u}_t'$	$\mathrm{Nu}_f'$	$\mathrm{Nu}_r'$	$\operatorname{Nu}_m'$
1.25	11.56	12.66	3.04	20.86	8.84
1.5	13.34	15.66	3.51	25.29	10.16
2.0	17.61	22.20	4.92	62.71	20.30
2.5	18.30	42.65	7.18	93.93	32.37
2.75	19.53	49.19	8.67	113.78	37.85
3.0	20.07	53.48	12.66	120.66	40.39
3.25	21.71	55.67	34.69	139.73	47.20
3.5	25.66	60.95	65.24	147.66	50.02

tics become Reynolds number independent and vary only with wall-surface roughness when Reynolds number is beyond  $2 \times 10^6$ . Hence, the SC surface roughness effect in the present LES is neglected in the range of Re<sub>5</sub>  $\in$  [1.25, 3.5].

#### 4.1. Turbulent flow statistics

For fluid flow past an SC, it is characterised mainly by vortex shedding whose normalised primary frequency is usually depicted by Strouhal number (St =  $fd/u_{in}$ ), and by the flow-induced force acting on the SC, usually decomposed into drag and lift. The vortex shedding naturally generates retrograde/prograde spanwise vortices as described by Natrajan, Wu, and Christensen (2007), moving downstream, mutually interact, and sometimes could be engulfed into the recirculation zone near the SC rear wall. On the contrary, the incoming flow impinging directly on the SC front wall is less impacted by the vortex shedding and flow separation.

As SC flows have fixed vortex separation points, the vortex shedding would be different from that past a circular cylinder. For circular cylinder flows, there is sensitivity to disturbances in the critical range as reported by Schewe (1986); whereas this remains a question in SC flows.

To show Reynolds number dependence, the *t-z* averaged CL, CD, *t*-averaged  $(v_{sr})_{pm}$ , the corresponding rms values CL', CD',  $(v_{sr})'_p$ , and St are given in Table 2. As shown in the second column of Table 2, the mean CL is close to zero. As shown in the third column, for Re<sub>5</sub>  $\in$  [1.25, 3.5], the mean CD is about 1.835 ( $\pm$ 0.035), with a maximum relative deviation less than 1.9%, suggesting that in the range Re<sub>5</sub>  $\in$  [1.25, 3.5], CD is not sensitive to Re, and almost Re independent. As indicated in the fourth column, the time average of the peak of sub-grid viscosity ratio, denoted by  $(v_{sr})_{pm}$ , becomes large with the increase of Re. The Re dependence of these averaged variables is shown in Figure 2(a) and 2(c),where the abscissa is Re<sub>5</sub>.

For corresponding rms values, the Re dependence can be found in the fifth, sixth, and seventh columns of Table 2. For CL', CD', their variations with Re are plotted as functions of Re<sub>5</sub> in Figure 2(b). Within the range of Re<sub>5</sub>  $\in$  [1.25, 3.5], it can be seen that CL' is around 0.1748 ( $\pm$ 0.0069), with a maximum relative deviation of less than 3.95%.



Figure 2. Re dependence of mean lift (CL) and drag (CD) (a), their root mean square (rms) values of CL and CD (b), and the timeaveraged viscosity ratio  $(v_{sr})_{pm}$  as well as its relevant rms  $(v_{sr})'_p$  (c). Note that  $(v_{sr})_{pm}$  is calculated merely by time-averaging for peak value of  $v_{sr} (= v_s/v)$  in the computational domain, as can be seen in Figure 5.

The CD' is approximately equal to 0.1047 ( $\pm 0.0034$ ), with a maximum relative deviation of less than 3.25%. For  $(v_{sr})'_p$ , its relation to Re is shown in Figure 2(c), from which it can be seen that the  $(v_{sr})'_p$  has a similar trend as  $(v_{sr})_{pm}$ , but the growth rate of  $(v_{sr})'_p$  with the increase of Re is much lower than that of  $(v_{sr})_{pm}$ .

It is noted that the values of the mean CD and its rms value agree quite well with the existing experimental data of Luo, Yazdani, and Lee (1994) as well as the LES results of Sohankar (2006), but the rms value of CL is less well predicted. Our LES results show that the CL' value is about twice the CD' value. Considering the experiment of Tamura and Miyagi (1999), the effect of inlet turbulence on aerodynamic forces on an SC with various corner shapes indicates that the reason requires further studies. It is noted that the peak value of viscosity ratio  $(v_{sr})_p$ is sought from the entire flow field. As turbulent SC flow involves vortex shedding, the peak value must be changed temporally. The *t*-averaged peak value of viscosity ratio denoted by  $(v_{sr})_{pm}$ , as shown in Figure 2(c), has a larger growth rate. For instance, in the range of Re<sub>5</sub>  $\in$  [1.25, 3.5], the  $(v_{sr})_{pm}$  grows from 146.23 to 404.23 when Re<sub>5</sub> increases from 1.25 to 3.5. Quantitatively, with respect to the parameter Re<sub>5</sub>, the mean growth rate  $[\partial(v_{sr})_{pm}/\partial Re_5]$ is about 115, with  $[\partial(v_{sr})'_p/\partial Re_5] \approx 25$ .

For  $\text{Re}_5 \in [1.25, 3.5]$ , the Strouhal number (St) is independent of Re. As shown in the eighth column of Table 2, St is 0.1079 for each scenario. This value of St is obtained by discrete Hilbert transform (see Hahn 1996) of the evolution data of lift coefficient. The power-spectra diagrams for



Figure 3. Power spectra of the evolutions of CD and CL at (a)  $Re_5 = 1.25$ , (b)  $Re_5 = 2.5$ , (c)  $Re_5 = 3.0$ , (d)  $Re_5 = 3.5$ .



Figure 4. Evolutions of peak value of viscosity ratio  $((v_{sr})_p \equiv (v_s/v)_p)$  at Re<sub>5</sub> = Re/10<sup>5</sup> = 1.25, 2.5, and 3.5.

Re<sub>5</sub> = 1.25, 2.5, 3.0, 3.5 are given in Figure 3(a)–(d). It can be seen that for the SC flows at the four cases, the primary frequency of vortex shedding in the unit of  $(u_{in}/d)$  labelled by St is 0.1079.

This St is very close to the predicted value 0.124 for the case of Re =  $2.2 \times 10^4$  in the numerical work of Bosch and Rodi (1998). The prediction of Bosch and Rodi was based on the two-dimensional flow assumption and the two-layer approach with standard  $k - \varepsilon$  turbulence model, and the inlet viscosity ratio ( $r_{\mu} = v_t/v$ ) was set as 100. Here  $v_t$  is the eddy viscosity which depends on kinetic energy k and its dissipation rate  $\varepsilon$ . As reported by Bosch and Rodi (1998), the St measured by Bearman and Trueman (1972) is 0.123 for the SC flow at Re  $\approx 5 \times 10^4$  when the inlet flow turbulence level  $Tu(=\sqrt{u'^2})$  was less than 1.2%. This comparison with existing numerical and experimental results, to some extent indicates the reliability of the present LES work.

Figure 4 shows the evolution of the peak value of viscosity ratio  $(v_{sr})_p$ , where the three dash-dotted purple horizontal lines are used to show the mean peak value of viscosity ratio  $(v_{sr})_{pm}$ , with the dash-dotted green, dashed blue, and solid lines illustrating the evolution curves of  $(v_{sr})_p$  for the three scenarios of Re<sub>5</sub> = 1.25, 2.5, and 3.5, respectively. Both the mean peak value of viscosity ratio, i.e.  $(v_{sr})_{pm}$ , and its rms have a growing tendency with Re<sub>5</sub>. The values of  $(v_{sr})_{pm}$  as shown in Figure 4 are consistent with those given by Table 2 for the three scenarios.

#### 4.2. Nusselt numbers

More conveniently, to describe the temperature  $\Theta$  profile near the SC wall surface, we use y to denote the normal distance to the wall surface, and assume

$$\Theta(y) = A(1+y)^m \tag{25}$$

Hence, we have

$$\Theta_w = \Theta(0) = A; \quad \Theta_1 = A(1+y_1)^m \tag{26}$$



Figure 5. Re dependence of Nu<sub>2d</sub> and the relevant relative deviations  $\sigma_1$  and  $\sigma_2$ . Note that  $\sigma_1 = (Nu_{2d} - Nu_{exp1})/Nu_{exp1} \times 100\%$ ,  $\sigma_2 = (Nu_{2d} - Nu_{exp2})/Nu_{exp2} \times 100\%$ , where for Re<sub>2d</sub> = 2Re, Nu<sub>exp1</sub> =  $0.085 \text{Re}_{2d}^{0.675}$  in Hilpert (1933); Nu<sub>exp2</sub> =  $0.14 \text{Re}_{2d}^{0.66}$  in Igarashi (1987).



Figure 6. Profiles of *t-z* averaged velocity rms values at four locations (i.e. x = 3, 6, 9, and 12) for Re<sub>5</sub> = 1.25 (a–d), and Re<sub>5</sub> = 2.5 (e–h).



Figure 7. Profiles of *t*-*z* averaged viscosity ratio and the relevant rms value at four locations (i.e. x = 3, 6, 9, and 12) for Re<sub>5</sub> = 1.25 (a–d), and Re<sub>5</sub> = 2.5 (e–h).



Figure 8. Contours of streamwise vorticity component ( $\omega_1$ ) at t = 170, and four locations (i.e. x = 3, 6, 9, and 12) for Re<sub>5</sub> = 1.25 (a–d), and Re<sub>5</sub> = 2.5 (e–h). Note that the contours are labelled by -0.3, -0.2, -0.1, 0, 0.1, 0.2, and -0.3.

where  $y_1$  denotes the normal distance of the near-wall grid point. Accordingly, the local Nusselt number

$$\operatorname{Nu} = \dot{q}_w / [\rho C_p \cdot (\nu / \operatorname{Pr}) \cdot \Delta T_w] = \left[ \partial \Theta / \partial y \big|_{y \to 0} \right] = Am$$
(27)

where  $m = \log (\Theta_1 / \Theta_w) / \log (1 + y_1)$ . The expression (27) for Nu is used to calculate the *t*-*z* averaged face Nusselt numbers (Nu<sub> $\beta$ </sub>,  $\beta = b$ , *t*, *f*, *r*) and their arithmetic mean (Nu<sub>m</sub>), as shown in Table 3. The subscript ' $\beta$ ' can represent the bottom (b), top (t), front (f), and rear (r) faces, respectively. It can be seen that the *t*-*z* averaged Nu<sub> $\beta$ </sub> increases with Reynolds number, but the growth rate of Nu<sub> $\beta$ </sub> is different as the local fluid flow near the faces has different vortex structures. Totally, as shown in Table 4, the normalised rms of Nu<sub> $\beta$ </sub> also has a growing tendency.

Following the work of Wiesche (2007), we further choose 2d as the length scale for the mean Nusselt number and compare its value with experimental results. In comparison with empirical correlations of Hilpert (1933) and Igarashi (1987), respectively, the Re dependence of Nu<sub>2d</sub>

and the corresponding relative deviations ( $\sigma_1$  and  $\sigma_2$ ) are shown in Figure 5(a) and 5(b). It is shown that the mean Nusselt number is slightly under-predicted as compared with the earlier experiment of Hilpert, with a maximum relative deviation less than 23.2%. As Igarashi (1987) applied constant heat flux in his wind tunnel measurement, which may result in the measured mean Nusselt number higher than that obtained under constant wall temperature, as described by Mihiev (1956).

### 4.3. Flow profiles

As shown in Figures 6 and 7(a)–(h), flow profiles are obtained by *t-z* averaging, and then treated by assuming that they are symmetrical to the mid plane y = 0.5[see in Figure 1(a)].

Different from the illustration of Bosch and Rodi (1998), which showed the profiles of mainstream velocity and the turbulent kinetic energy along the line of symmetry, and compared with existing numerical and experimental



Figure 9. Contours of spanwise vorticity component ( $\omega_3$ ) at t = 170, and z = 2.0 for Re<sub>5</sub> = 1.25 (a), and Re<sub>5</sub> = 2.5 (b). Note that contours of  $\omega_3$  are labelled by -2, -1, -0.5, 0, 0.5, 1, and 2.

results, the present LES obtained the relevant data to calculate the rms profiles of velocity at the four locations x =3, 6, 9, and 12. The rms profiles are shown in Figure 6(a)– (h). It is found that the wake width gradually broadens in the mainstream direction. Defining the wake core region by { $x \ge 1$ , and  $y \in [0, 1]$ }, the profiles of u', v', and w'indicate that when  $x \ge 6$ , v' maintains a relative large value which varies with the downstream distance to the SC, and the order of relative value (v' > u' > w') holds in the core region, depending on the downstream distance. In the near cylindrical wake region, when x = 3, the influence of flow recirculation can lead to the change of the order of relative value, as can be seen in parts (a) and (e) of Figure 6. It can also be seen that the Re dependence of the normalised velocity fluctuation profiles is not significant.

In contrast, the influence of Re on mean sub-grid viscosity ratio and its rms value is significant. Especially in the wake region, the  $(v_{sr})_m$  and its rms value  $(v_{sr})'$  in general take relatively smaller values in comparison with those in the outer wake regions simply denoted by  $y \in (1, 3)$  and  $y \in$ (-2, 0). This suggests that vortex interaction in the outer wake region is more intense, causing larger values of  $(v_{sr})_m$ and  $(v_{sr})'$ .

#### 4.4. Instantaneous flow patterns

Due to vortex shedding, the SC flows have patterns varying temporally as reported in Shraiman and Siggia (2000), Adrian (2007), and Natrajan, Wu, and Christensen (2007). It is useful to show the instantaneous flow patterns for a deeper understanding of SC flows at high Reynolds numbers.

At the four *x*-locations x = 3, 6, 9, 12, and time t = 170, contours of  $\omega_1$  are labelled by values -0.3, -0.2, -0.1, 0, 0.1, 0.2, and 0.3, the instantaneous secondary flow patterns for Re<sub>5</sub> = 1.25 are shown in Figure 8(a)–(d), with those for Re<sub>5</sub> = 2.5 shown in Figure 8(e)–(h). A comparison between the parts (a)–(d) and the parts (e)–(h) indicates that secondary flow patterns are not only Re dependent but also different at various *x*-locations.

Close correlation with the secondary flows [Figure 8(a)-(h)], the flow patterns in the mid plane

![](_page_11_Figure_10.jpeg)

Figure 10. Contours of the (*t-z*) averaged sub-grid viscosity ratio ( $v_{sr}$ ) at Re<sub>5</sub> = 1.25 (a), and 2.5 (b). Note that the contours of  $v_{sr}$  are labelled by 0.05, 1, 10, and 20.

z = 2 as depicted by the contours of  $\omega_3$  can be seen in Figure 9(a) and 9(b), from which one can capture an impression of the Karman vortex street. It can be seen that the calculation of flow vorticity components has adopted the staggered grid speciality.

# 4.5. t-z averaged field of sub-grid viscosity ratio

For the peak value of  $(v_{sr})_{pm}$ , its occurrence position varies temporally as a result of large eddy motion and interaction. Exploring the occurrence of peak value would be helpful for the understanding of the SC flows. Therefore, for Re<sub>5</sub> = 1.25, and 2.5, the *t*-*z* averaged field of sub-grid viscosity ratio  $v_{sr}$  is shown by contours labelled by 0.05, 1, 10, and 20, in Figure 10(a) and 10(b). In the red coloured zone, the *t*-*z* averaged  $v_{sr}$  is over 20. While in the blue coloured region, its value is less than 0.05, whereas the cyan, green, and yellow coloured regions correspond to  $v_{sr} \in (0.05, 1)$ , (1, 10), and (10, 20) respectively. As seen in Figure 10(a) and 10(b), for Re<sub>5</sub> = 1.25, and 2.5, the  $(v_{sr})_{pm}$  occurs in the two downstream regions of SC, where the vortex interaction is more intense.

#### 5. Conclusions

A swirling strength based LES was performed to explore the characteristics of incompressible turbulent heat and fluid flow past an SC at Reynolds numbers in the range of  $Re_5 \in (1.25, 3.5)$ . The LES was based on a finite difference method, where the numerical scheme accuracy was improved by means of Taylor expansion. The present LES for eight scenarios reveals several findings as listed below:

- Different from circular cylinder flows, for Re<sub>5</sub> ∈ (1.25, 3.5), the numerically simulated mean CD is about 1.835 (±0.035), with maximum relative deviation of 1.9%; calculated Strouhal number is 0.1079, independent of the high Reynolds number. Both agree well with existing experimental and numerical data. Downstream the SC, the effect of Reynolds number on the *t-z* averaged profiles of normalised velocities and their rms values is not significant.
- (2) Both mean peak of viscosity ratio, i.e. (v<sub>sr</sub>)<sub>pm</sub>, and its rms value increase with Reynolds number, but the growth rate of the mean peak is much larger as compared to that of its rms value. This may be attributed to the fact that when the Cartesian coordinate origin is set at the bottom flow separation point of the SC flow, the peak of sub-grid viscosity ratio occurs in the outer wake region denoted by {x ≥ 1, y ∈ (1, 3)} and {x ≥ 1, y ∈ (-2, 0)}. The iso-surfaces of the FSI and the sub-grid viscosity ratio in a sub-domain reflect to some extent the

reliability of *t-z* averaged sub-grid viscosity ratio field.

- (3) The SC's wake width gradually broadens in the mainstream direction. For the rms of velocities, in the far core wake region, for instance, when the distance from the SC's rear face is beyond 5, among the fluctuations of velocity components, the fluctuation of vertical component is the highest, with the fluctuation of spanwise component being the lowest. Within the near cylindrical wake, flow recirculation significantly influences the velocity fluctuations.
- (4) The mean Nusselt number agrees well with the earlier experiment of Hilpert (1933). The *t-z* averaged top Nusselt number is not identical to the bottom one, suggesting that the *t-z* averaged flow field is asymmetrical about the symmetry line y = 0.5 in the *x*-*y* plane.

#### Acknowledgements

We thank the anonymous referees for their invaluable suggestions and Dr. J.L. Niu working at PolyU of Hong Kong for his helpful private communications.

#### **Disclosure statement**

No potential conflict of interest was reported by the authors.

#### Funding

This work is financially supported by NSFC [grant number 11372303] and the Research Grant Council of HKSAR [grant numbers PolyU 5109/10E and B-Q21J].

## References

- Adrian, R. J. 2007. "Hairpin Vortex Organization in Wall Turbulence." *Physics of Fluids* 19: 041301.
- Baker, T. J. 1981. "Potential Flow Calculation by the Approximate Factorization Method." *Journal of Computational Physics* 42: 1–19.
- Bearman, P. W., and D. M. Trueman. 1972. "An Investigation of the Flow Around Rectangular Cylinders." *Aeronautical Quarterly* 23: 229–237.
- Beaudan, P., and P. Moin. 1994. Numerical Experiments on the Flow Past a Circular Cylinder at Sub-critical Reynolds Number. Tech. Rep. TF-62. California: Stanford University.
- Betelin, V. B., R. M. Shagaliev, S. V. Aksenov, I. M. Belyakov, Yu. N. Deryuguin, D. A. Korchazhkin, A. S. Kozelkov, V. F. Nikitin, A. V. Sarazov, and D. K. Zelenskiy. 2014. "Mathematical Simulation of Hydrogen–Oxygen Combustion in Rocket Engines Using LOGOS Code." *Acta Astronautica* 96: 53–64.
- Bosch, G., and W. Rodi. 1998. "Simulation of Vortex Shedding Past a Square Cylinder with Different Turbulence Models." *International Journal for Numerical Methods in Fluids* 28: 601–616.
- Brown, D. L., R. Cortez, and M. L. Minion. 2001. "Accurate Projection Methods for the Incompressible Navier-Stokes Equations." *Journal of Computational Physics* 168: 464–499.
- Chandrasekhar, S. 1961. *Hydrodynamic and Hydromagnetic Stability*. London: Oxford University Press.

- Cheskidov, A., D. D. Holm, E. Olson, and E. S. Titi. 2005. "On a Leray-Model of Turbulence." *Proceedings of the Royal Soci*ety of London A 461(2055): 629–649.
- Cui, G. X., C. X. Xu, and Z. S. Zhang. 2004. "Progress in Large Eddy Simulation of Turbulent Flows." Acta Aerodynamica Sinica 22(2): 121–129 (in Chinese).
- Cui, G. X., H. B. Zhou, Z. S. Zhang, and L. Shao. 2004. "A New Subgrid Eddy Viscosity Model and Its Application." *Chinese Journal of Computational Physics* 21(3): 289–293 (in Chinese).
- Duwig, C., K. J. Nogenmyr, C. K. Chan, and M. Dunn. 2011. "Large Eddy Simulations of a Piloted Lean Premixed Jet Flame Using Finite-Rate Chemistry." *Combustion Theory and Modelling* 15 (4): 537–568.
- Ganapathisubramani, B., E. K. Longmire, and I. Marusic. 2006. "Experimental Investigation of Vortex Properties in a Turbulent Boundary Layer." *Physics of Fluids* 18: 155105.
- Germano, M., U. Piomelli, P. Moin, and W. H. Cabot. 1991. "A Dynamic Subgrid-Scale Eddy Viscosity Model." *Physics of Fluids A* 3(7): 1760–1765.
- Geurts, B. J., and D. D. Holm. 2003. "Regularization Modeling for Large-Eddy Simulation." *Physics of Fluids* 15(1): L13–L16.
- Guo, Y. C., P. Jiang, C. K. Chan, and W. Y. Lin. 2011. "Large Eddy Simulation of Coherent Structures in Rectangular Methane Non-premixed Flame." *Journal of Computational and Applied Mathematics* 235(13): 3760–3767.
- Hahn, S. L. 1996. *Hilbert Transforms in Signal Processing*. Boston, MA: Artech House.
- Hanjalic, K. 2002. "One-Point Closure Model for Buoyancy-Driven Turbulent Flows." Annual Review of Fluid Mechanics 34: 321–347.
- Harlow, F. H., and J. E. Welch. 1965. "Numerical Calculation of Time Dependent Viscous Incompressible Flow of Fluid with Free Surfaces." *Physics of Fluids* 8: 2182–2188.
- Hilpert, R. 1933. "Warmeabgabe von beheizten draten und rohren im luftstrom." *Ingenieurwesen* 4: 215–224.
- Holm, D. D. 1999. "Fluctuation Effects on 3d Lagrangian Mean and Eulerian Mean Fluid Motion." *Physica D* 133(1–4): 215– 269.
- Holmes, P., J. L. Lumley, and G. Berkooz. 1996. Turbulence, Coherent Structures, Dynamical Systems and Symmetry. New York: Cambridge University Press.
- Igarashi, T. 1987. "Fluid Flow and Heat Transfer Around Rectangular Cylinders (the Case of a Width/Height Ratio of a Section of 0.33-1.5)." *International Journal of Heat and Mass Transfer* 30: 893–901.
- Kato, M., and B. E. Launder. 1993. "The Modelling of Turbulent Flow Around Stationary and Vibrating Square Cylinders." In *Proceedings of the 9th Symposium on Turbulent Shear Flows*, edited by F. Durst, N. Kassagi, and B. E. Launder, Berlin: Springer, 10-4-1. Kyoto.
- Khanafer, K., K. Vafai, and M. Lightstone. 2002. "Mixed Convection Heat Transfer in Two Dimensional Open-Ended Enclosures." *International Journal of Heat and Mass Transfer* 45: 5171–5190.
- Lilly, D. K. 1967. "The Representation of Small-Scale Turbulence in Numerical Simulation Experiments." In *Proceedings* of *IBM Scientific Computing Symposium on Environmental Sciences*, edited by H.H. Goldstine, 195–210. New York: Yorktown Heights.
- Lilly, D. K. 1992. "A Proposed Modification of the Germano Subgrid-Scale Closure Model." *Physics of Fluids A* 4(3): 633– 635.
- Lin, C. T., and Z. J. Zhu. 2010. "Direct Numerical Simulation of Incompressible Flows in a Zero-Pressure Gradient Turbu-

lent Boundary Layer." Advances in Applied Mathematics and Mechanics 2(4): 503–517.

- Luo, S. C., Y. T. Chew, and Y. T. Ng. 2003. "Strouhal Numbers of Rectangular Cylinders." *Journal of Fluid Mechanics* 15: 2549–2559.
- Luo, S. C., Y. T. Yazdani, and T. S. Lee. 1994. "Effects of Incidence and Afterbody Shape on Flow Past Bluff Cylinders." *Journal* of Wind Engineering and Industrial Aerodynamics 53: 375– 399.
- Madabhushi, R. K., and S. P. Vanka. 1991. "Large Eddy Simulation of Turbulence-Driven Secondary Flow in a Square Duct." *Physics of Fluids A* 11: 2734–2745.
- Manhart, M. 2004. "A Zonal Grid Algorithm for DNS of Turbulent Boundary Layers." *Computers and Fluids* 33: 435–461.
- McMillan, O. J., and J. H. Feiziger. 1979. "Direct Testing of Subgrid-Scale Models." *AIAA Journal* 17(12): 1340–1346.
- Metais, O., and M. Lesieur. 1996. "New Trend in Large Eddy Simulation of Turbulence." Annual Review of Fluid Mechanics 28: 45–82.
- Mihiev, M. 1956. *Fundamentals of Heat Transfer*. Moscow: National-Thermodynamic Publisher (In Russian).
- Moin, P., and J. Kim. 1982. "Numerical Investigation of Turbulent Channel Flow." *Journal of Fluid Mechanics* 118: 341–377.
- Natrajan, V. K., Y. Wu, and K. T. Christensen. 2007. "Spatial Signatures of Retrograde Spanwise Vortices in Wall Turbulence." *Journal of Fluid Mechanics* 574: 155–167.
- Nikitin, N. 2006. "Finite-Difference Method for Incompressible Navier-Stokes Equations in Arbitrary Orthogonal Curvilinear Coordinates." *Journal of Computational Physics* 217: 759– 781.
- Niu, J. L., and Z. J. Zhu. 2006. "Numerical Study of Three-Dimensional Flows Around Two Identical Square Cylinders in Staggered Arrangements." *Physics of Fluids* 18: 044106.
- Nogenmyr, K. J., H. J. Cao, C. K. Chan, and R. K. Cheng. 2013. "Effects of Confinement on Premixed Turbulent Swirling Flame Using Large Eddy Simulation." *Combustion Theory* and Modelling 17(6): 1003–1019.
- Olanski, I. 1976. "A Simple Boundary Condition for Unbounded Flows." Journal of Computational Physics 21: 251–269.
- Papanicolaou, E., and Y. Jaluria. 1992. "Transition to a Periodic Regime in Mixed Convection in a Square Cavity." *Journal of Fluid Mechanics* 239: 489–509.
- Patankar, S. V. 1980. *Numerical Heat Transfer and Fluid Flow*. London: Hemisphere.
- Robichaux, J., S. Balachandar, and S. P. Vanka. 1999. "Three-Dimensional Floquet Instability of the Wake of Square Cylinder." *Physics of Fluids* 11: 560–578.
- Roshko, A. 1961. "Experiments on the Flow Past a Circular Cylinder at Very High Reynolds Number." *Journal of Fluid Mechanics* 10: 345–356.
- Saha, A. K., G. Biswas, and K. Muralidhar. 2003. "Three-Dimensional Study of Flow Past a Square Cylinder at Low Reynolds Numbers." *International Journal of Heat and Fluid Flow* 24: 54–66.
- Schewe, G. 1986. "Sensitivity of Transition Phenomena to Small Perturbations in the Flow Around a Circular Cylinder." *Jour*nal of Fluid Mechanics 172: 33–46.
- Shraiman, B. I., and E. D. Siggia. 2000. "Scalar Turbulence." Nature 405: 639–646.
- Smagorinsky, J. S. 1963. "General Circulation Experiments with the Primitive Equations, the Basic Experiment." *Monthly Weather Review* 91(3): 99–164.
- Smirnov, N. N., V. B. Betelin, R. M. Shagaliev, V. F. Nikitin, I. M. Belyakov, Yu. N. Deryuguin, S. V. Aksenov, and D. A. Korchazhkin. 2014. "Hydrogen Fuel Rocket Engines

Simulation Using LOGOS Code." International Journal of Hydrogen Energy 39: 10748–10756.

- Smirnov, N. N., and V. F. Nikitin. 2014. "Modeling and Simulation of Hydrogen Combustion in Engines." *International Journal* of Hydrogen Energy 39(2): 1122–1136.
- Sohankar, A. 2006. "Flow over a Bluff Body from Moderate to High Reynolds Numbers Using Large Eddy Simulation." *Computers and Fluids* 35: 1154–1168.
- Sohankar, A., C. Norberg, and L. Davidson. 1999. "Simulation of Three-Dimensional Flow Around a Square Cylinder at Moderate Reynolds Numbers." *Physics of Fluids* 11: 288–306.
- Sparrow, E. M., J. P. Abraham, and H. C. K. Tong. 2004. "Archival Correlations for Average Heat Transfer Coefficients for Noncircular Cylinders and for Spheres in Cross-flow." *International Journal of Heat and Mass Transfer* 47: 5285–5296.
- Tamura, T., and T. Miyagi. 1999. "The Effect of Turbulence on Aerodynamic Forces on a Square Cylinder with Various Corner Shapes." *Journal of Wind Engineering & Industrial Aerodynamics* 83: 135–145.
- Trias, F. X., A. Gorobets, and A. Oliva. 2013. "A Simple Approach ro Discretize the Viscous Terms with Spatially Varying (Eddy-) Viscosity." *Journal of Computational Physics* 253: 405–417.
- Uddin, N., S. O. Neumann, B. Weigand, and B. A. Younis. 2009. "Large Eddy Simulation and Heat Flux Modeling in a Turbulent Impinging Jet." *Numerical Heat Transfer, Part A* 55: 906–930.
- van Reeuwijk, M., H. J. J. Jonker, and K. Hanjalic. 2008. "Wind and Boundary Layers in Rayleigh–Benard Convection. I. Analysis and Modelling. *Physical Review E* 77: 036311.
- van Reeuwijk, M., H. J. J. Jonker, and K. Hanjalic. 2009. "Leray-Simulations of Wall-Bounded Turbulent Flows." *International Journal of Heat and Fluid Flow* 30: 1044–1053.
- Verma, A., and K. Mahesh. 2012. "A Lagrangian Subgrid-Scale Model with Dynamic Estimation of Lagrangian Time Scale for Large Eddy Simulation of Complex Flows." *Physics of Fluids* 24: 085101.
- Vreman, A. W. 2004. "An Eddy-Viscosity Subgrid-Scale Model for Turbulent Shear Flow: Algebraic Theory and Applications." *Physics of Fluids* 16(10): 3670–3681.

- Vorst, H. V. D. 1992. "Bicgstab: A Fast and Smoothly Converging Variant of Bicg for the Solution of Non-symmetric Linear System." SIAM Journal on Scientific and Statistical Computing 13: 631–644.
- Wang, H. F., and Y. Zhou. 2009. "The Finite-Length Square Cylinder Near Wake." *Journal of Fluid Mechanics* 638: 453– 490.
- Wiesche, S. 2007. "Large-Eddy Simulation Study of an Air Flow Past a Heated Square Cylinder." *Heat and Mass Transfer* 43: 515–525.
- Williamson, C. H. K. 1996. "Vortex Dynamics in the Cylinder Wake." Annual Review of Fluid Mechanics 28: 477–525.
- Wu, Y. Y., C. K. Chan, and L. X. Zhou. 2011. "Large Eddy Simulation of an Ethylene-Air Turbulent Premixed v-Flame." *Journal* of Computational and Applied Mathematics 235(13): 3768– 3774.
- Yang, H. X., T. Y. Chen, and Z. J. Zhu. 2009. "Numerical Study of Forced Turbulent Heat Convection in a Straight Square Duct." *International Journal of Heat and Mass Transfer* 52: 3128–3136.
- Yu, K. F., K. S. Lau, and C. K. Chan. 2004. "Large Eddy Simulation of Particle-Laden Turbulent Flow over a Backward-Facing Step." *Communications in Nonlinear Science and Numerical Simulation* 9(2): 251–262.
- Zhang, Z., W. Chen, Z. J. Zhu, and Y. L. Li. 2014. "Numerical Exploration of Turbulent Air Natural Convection in a Differentially Heated Square Cavity at Ra = 5.33109." *Heat and Mass Transfer* 50(12): 1737–1749.
- Zhou, J., R. J. Adrian, S. Balachandar, and T. M. Kendall. 1999. "Mechanisms of Generating Coherent Packets of Hairpin Vortices in Channel Flow." *Journal of Fluid Mechanics* 387: 353– 396.
- Zhu, Z. J., J. L. Niu, and Y. L. Li. 2013. "Swirling-Strength Based Large Eddy Simulation of Turbulent Flows Around a Single Square Cylinder at Low Reynolds Numbers." *Applied Mathematics and Mechanics (English Edition)* 35(8): 959–978.
- Zhu, Z. J., H. X. Yang, and T. Y. Chen. 2010. "Numerical Study of Turbulent Heat and Fluid Flow in a Straight Square Duct at Higher Reynolds Numbers." *International Journal of Heat* and Mass Transfer 53: 356–364.