Technical Note

Mathematical Modeling of Air Pressure in a Drainage Stack of a High-Rise Building Test Platform

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Abstract: The air pressure in a drainage stack of a high-rise building test platform (HBTP) is mathematically modeled by unsteady one-dimensional (1 + 1) partial differential equations, in which an additional term is introduced to reflect the gas-liquid interphase interaction, the stack top-base effect. This model is crucial for understanding the characteristics of air pressure variation, which is significant for the recognition of the HBTP's operational performance. A time-splitting based characteristic line method is used to solve the 1 + 1 type governing equations, with the model parameters being calibrated by the measured data obtained on the HBTP. It is concluded that the generally used Saint-Venant equations should be extended appropriately so that the stack air pressure can be satisfactorily predicted. **DOI: 10.1061/(ASCE)AE.1943-5568.0000123.** © *2013 American Society of Civil Engineers.*

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Introduction

The air pressure in a high-rise building drainage stack is dynamically varied, which is similar to that measured in the drainage stack of the Li Ka-Shing (LKS) building in PolyU of Hong Kong (Wong et al. 2011), because there exists the dynamically changed branch air entrainment, gas-liquid interphase interaction, and stack top-base effect (Swaffield et al. 2004).

Previous numerical explorations (Swaffield and Campbell 1995; Swaffield 1996; McDougall and Swaffield 2000; Swaffield et al. 2004; Swaffield and Jack 2004) were generally based on the solution of the Saint-Venant equations (Swaffield and Campbell 1992a, b). The results indicate that the depletion of the trap seals (Kelly et al. 2008) and the bathroom floor drain traps (Gormley 2007) could cause cross-contamination via the drainage system.

However, previous work (Wong et al. 2013) revealed that the Saint-Venant equations should be extended appropriately so that the stack base effect can be properly considered. Differently, the present mathematical modeling further considers the stack top effect, and the air pressure in the stack is calculated by a characteristic line method (CLM) but not by a mixed method based on the CLM and the total variation diminishing (TVD) (Yee 1987; Roe 1981; Davis 1988) algorithm.

In the present mathematical model, an additional term is introduced to reflect the gas-liquid interphase interaction and the stack base-top effects. Based on the model, a time-splitting based CLM (TSCLM) is used to seek numerical solutions. A calibration gave possible values of the model parameters by using data measured in a real-scale high-rise building test platform (HBTP) with a constant water flow from the branch pipes (Zhang and Chen 2006).

Governing Equations

Following the approach of Swaffield and Campbell (1992a, b), the pressure variation process is assumed to be isentropic. For the stack airflow defined by $x \in (0, H)$, the governing equations can be expressed as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{1}$$

and

$$\frac{\partial q}{\partial t} + \frac{\partial (q^2/\rho + p)}{\partial x} + \frac{4f}{2D}\frac{q^2}{\rho} + f_1 \rho v = 0 \tag{2}$$

the mass flow rate is

$$q = \rho(u + v) = \rho\left[u + \sum_k v_k S(x - x_k)\right] \tag{3}$$

where v_k = air speed entrained from the branch pipe located at x_k. The step function $S(x - x_k)$ has zero value for $x < x_k$ and equals unity for $x \geq x_k$. It is used to reflect the effect of water discharge, as seen in Fig. 1(a), with the total entrained air speed $v[= \sum_k v_k S(x - x_k)]$. Different from Saint-Venant equations (Swaffield and Campbell 1992a, b), an additional term with parameter f_1 is introduced to take account of the complex interphase interaction and the stack base effect. Assuming the stack top is at $x = 0$, the stack base part is in the range $x > H_1$, and the time step in simulation is Δt, f_1 can be expressed by

$$f_1 = \begin{cases} -\frac{\sigma_1}{\Delta t} \times \frac{x}{x_N}, & \text{for } x \in [0, x_N] \\ 0, & \text{for } x \in (x_N, H_1] \\ -\frac{\sigma_0}{\Delta t} \times \frac{x - H_1}{H - H_1}, & \text{for } x \in (H_1, H) \end{cases} \tag{4}$$

where σ_0 and σ_1 = model parameters for adjusting airflow velocity. Eq. (4) indicates f_1 is assumed to be linear with coordinate x. The

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larger the σ_0, the more intensive effect is for the interphase interaction and stack base. However, σ_1 should be assigned to be negative, because the branch water discharging can result in the stack top air flowing downward, causing the stack top air pressure be less than the ambient pressure. Hence, the smaller the σ_1, the larger effect is for the stack top. When $\sigma_0 = \sigma_1 = 0$, the form of governing equations returns to the Saint-Venant type. The parameter x_N is used to define the stack top region, and its value is assigned with respect to the water discharge of the HBTP. The value of H_1 is assigned according to the convenience of numerical validation. The entrained airflow speed v is calculated by using the same approach reported elsewhere (Wong et al. 2013).

For the prediction of the stack air pressure in a high-rise building platform, the initial air speed u and relative air pressure in the stack are assumed to be zero. The intermediate height H_1 is used to partition a stack into the upper and base parts. Because the top and base parts are, respectively, associated with the model parameters σ_0 and σ_1, a TSCLM (Abarbanel and Gottlieb 1981; Karniadakis et al. 1991) is used.

The stack air pressure in a drainage system of 17-floor HBTP is predicted with the proposed mathematical model. The flow rates of water discharged from branch pipes are definitely assigned. As shown in Fig. 2, the measured stack air pressure and its peak values (Zhang and Chen 2006) are used to calibrate the model parameters σ_0 and σ_1. The HBTP for the experiments of Zhang and Chen is located in Shiga, Japan. Some details of the experiments are indirectly given elsewhere (Wong et al. 2013).

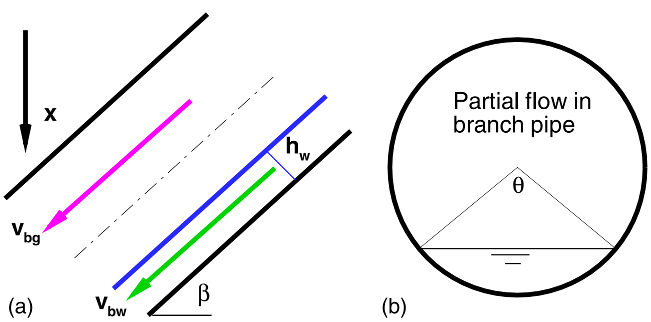

Fig. 1. (a) Schematic of the branch discharging; (b) schematic of partial water flow in a branch pipe

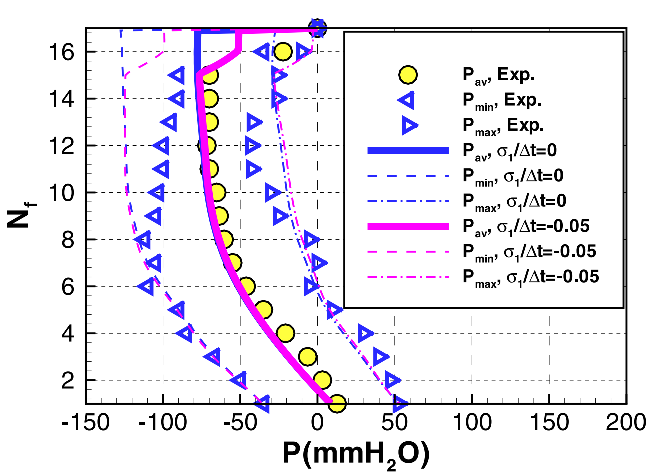

Fig. 2. Comparison of pressures with experimental data: peak pressures ($P_{\min}$, $P_{\max}$) are calculated by $P_{av} \pm \sqrt{2}\sigma_p$, where σ_p is the root mean square value of the pressure fluctuation

The parameters used in the numerical validation are as follows:

- $H_1/L_x = 60$
- $x_1/L_x = 10$
- $x_N = x_2$
- $\sigma_1/\Delta_t = -0.05$
- $D = 100\,\text{mm}$
- $x_2/L_x = 20$
- $L_x = 0.28\,\text{m}$
- $\sigma_0/\Delta_t = 0.01$

The use of parameter σ_1 certainly made the average pressure curve and the pressure peak curves closer to the measured data in the stack top region. The peak pressure values are expected to be $P_{av} \pm \sqrt{2}\sigma_p$, where P_{av} = time-average pressure, and σ_p = its RMS value. This approach of peak value prediction is correct if the stack air pressure really oscillates in a harmonic mode.

The sensitivity of numerical solutions to σ_0 is explored by comparing the distributions of time-averaged relative pressure and the pressure peaks plotted as functions of the floor number $N_f = (H - x)/(10L_x) + 1$, where L_x = space-grid step. The effect of σ_0 on the distributions of pressure peaks can be seen in Fig. 3. The larger the value of σ_0, the smaller the values of pressure peaks, suggesting that σ_0 is a key role parameter in the calibration of stack air pressure.

The distributions of time-averaged peak pressures in the stack are closely related to the velocity and pressure evolutions, as shown in Figs. 4(a and b). For $t > 1$, the time-dependent pressure [Fig. 4(a)] approximately fluctuates in harmonic and quasi-periodic waves, whereas the time-average value and oscillation amplitude are evidently dependent on the stack location x/L_x. The time period of pressure fluctuation is around H/c_0. Because the pressure oscillation is quasi-periodic, the peak pressures can be approximated by multiplying the RMS of pressure fluctuation σ_p with a factor of $\sqrt{2}$ because

$$\frac{1}{2\pi}\int_0^{2\pi}(\sin\alpha)^2 d\alpha = \frac{1}{2}$$

The temporal evolution of u in the range of $t \in (1, 5)$ is shown in Fig. 4(b). It is seen that the $u-$ peaks correspond to the valleys of pressure peaks at $x = H_1$ and H and vice versa.

The spatiotemporal pressure evolution is given in Fig. 5. It is shown in flood mode. The pressure $P(x, t)$ is calculated by $(p - p_0)/10$; thus, it has the unit of $mmH_2O(= 10\,\text{Pa})$. For

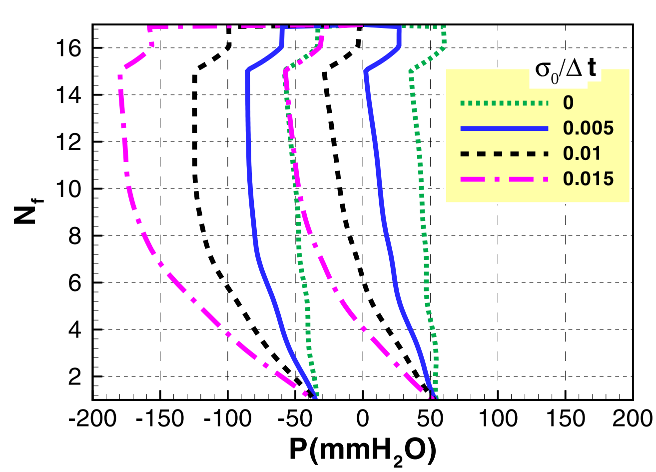

Fig. 3. Parameter σ_0 effect on predicted peak pressures in the stack

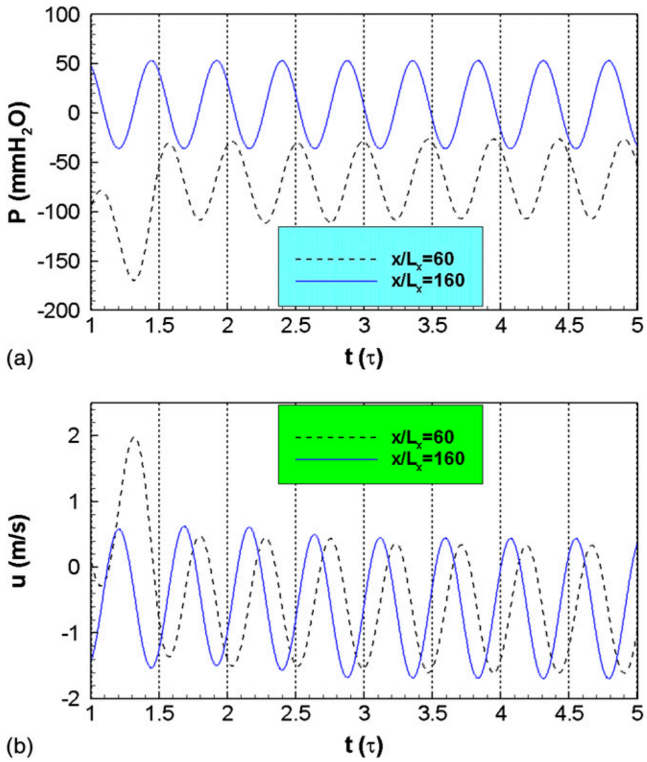

Fig. 4. Evolutions of (a) pressure P and (b) velocity u at different stack locations of $x = H_1$ and H: note that $\tau = L_x/m(s)$

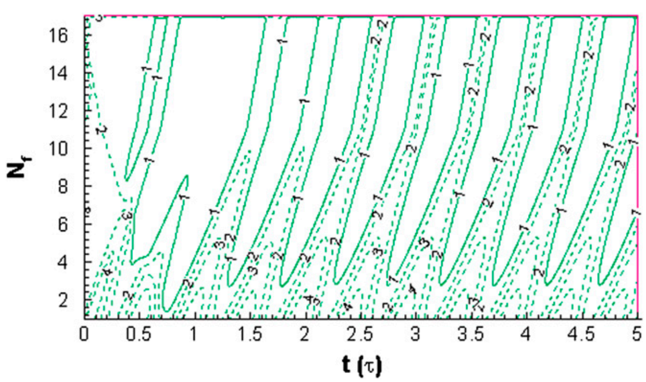

Fig. 5. Spatiotemporal evolution of pressure in the time range of $t \in (0, 5)$; note that the pressure contours are -60, -30, 0, and 30 mmH_2O

$\sigma_0/\Delta t = 0.01$, the air entrainment leads to the decrease in stack air pressure. The pressure contours are labeled -60, -30, 0, and 30, indicating that when $t > 1$, the temporal variation of stack air pressure occurs in a quasi-periodic mode because of the adopted boundary conditions. This spatiotemporal pressure evolution indicates the airflow in the building stack is oscillating, suggesting that the relevant influences of the branch discharging, gas-liquid interphase interaction, and stack top and base are crucial.

Conclusions

Therefore, it can be concluded that the Saint-Venant equations should be extended appropriately for describing stack air pressure, at least in the HBTP.

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