Two-Phase Fluids Model for Freeway Traffic Flow and Its Application to Simulate Evolution of Solitons in Traffic

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Abstract: A two-phase fluids model for mixing traffic flow on freeways has been proposed, where vehicles are decomposed into two parts (i.e., phases)—slow moving and fast moving–denoted by subscripts $i=1$ and 2, respectively. Based on the fact that both phases should be at rest under traffic jam conditions, it is assumed that the vehicular speeds for both phases are functions of the global traffic density, so that traffic flux can be expressed explicitly, considering that the speed of the second phase may be decreased when the mass fraction of the first phase becomes large. In addition to the relation to global density of traffic, it is assumed that the speed of vehicles in the second phase also depends on the mass fraction for the first phase. By neglecting the traffic generation rate, the governing equations from the mass conservation law were solved numerically with the Yee-Roe-Davis second-order symmetrical total variable diminishing algorithm. Two cases were considered: first, that there exists a soliton in the initial distribution of global density; second, that there is an initial uniform global density. Both cases were allowed to have a soliton in the initial density for the slowly moving vehicles. The numerical results indicate that the evolution of a soliton in traffic is quite different from those in water wave problems. The initial solitary-wave perturbation has been distorted dramatically. It was found that the presence of a soliton just in the slowly moving vehicles can increase global density, which means that traffic mixing can be viewed as a source of density wave production. Under very congested traffic flow, the speed for moving vehicles fast approaches that of vehicles in the first part.

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Introduction

So far, it is clear that the early work associated with hydrodynamic analogy to traffic flow performed by Lighthill and Whitham (1955) and Richards (1956) has had an extensive impact within transportation engineering, where the single phase shock wave equation was solved by making use of the so-called traffic state equation. Such a traffic model is usually called the LWR kinematic model, generalizations of which were carried out by De (1956) and Bick and Newell (1960) just after its publication. It has generated certain advantages in the treatment of traffic shock on freeways. For example, recently, Daganzo (1995a,b, 1997) has conducted a finite-difference approximation starting from the LWR model for the traffic on freeways, where the accuracy and the transfer of approximating error were expressed and discussed in detail.

In fact, due to the nature of mixing traffic flow on freeways, even for the evaluation of the simplest solution, the original LWR model for traffic can not be used directly without corresponding modifications related to the inherent nature of the traffic.

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A two-fluid (moving and stopped vehicles) model of urban traffic has been developed by Herman and Prigogine (1979). Based on this model, trip time versus stop time and fuel consumption characteristics of traffic in cities have been studied by Chang and Herman (1981). Due to the work of Herman and Ardekani (Herman and Ardekani 1984, 1985; Ardekani and Herman 1987), the reasonableness of the two assumptions in the two-fluid (moving and stopped vehicles) model has been found by a series of experiments conducted in Austin, Texas, and by comparisons with relevant data of various cities around the world. Further, not only have they studied the influence of stops on vehicle fuel consumption in urban traffic, but they have measured the variables of network-wide traffic such as speed, flow, and the fraction of vehicles stopped by using aerial photographic surveys. These works are of great theoretical and practical significance. Nevertheless, considering the behaviors of moving vehicles and proposing a two-phase fluids model (for fast and slow vehicles) is also of great importance, since it may be useful for the understanding of the mechanism of density fluctuation arising from vehicular interactions. In this regard, it will be used to simulate the evolution of traffic solitons on freeways, where stopped vehicles hold a very small percentage.

The two-phase fluids model is based on the mass conservation law, where vehicles on freeways are first decomposed into two parts reflecting the relevant running character. Then the governing equations for the density of the first part and the global traffic density are presented, by assuming that the speed for each part is an explicit function of the mass fraction and the global density. Since, the traffic flow is at rest when a jam is encountered, the nature of the mixing traffic flow on freeways can be manifested by examining the evolution of a soliton initially appearing in the mixing traffic flow.

Table 1. Initial Parameters Used in Simulation

Figure numbers	ρ (vh/km)	ρ_1 (vh/km)	s	ρ_{1j} (vh/km)	ρ_{2j} (vh/km)
2(a), 3, 4	see Eq. (25)	see Eq. (25)	(ρ_1/ρ)	200	300
2(b), 5, 6	see Eq. (26)	see Eq. (26)	(ρ_1/ρ)	200	300

A soliton, as discovered numerically by Zabusky and Kruskal (1965), is a large amplitude coherent pulse or very stable solitary wave, the exact solution of a wave equation, whose shape and speed are not changed by a collision with other solitary waves. Large amplitude wave motions have been observed in various fields ranging from fluids and plasma to solid state, chemical, biological, and geological systems (Kortewg and Devries 1895; Chen 1988; Remoissenet 1999). Due to the wide existence of nonlinearity in the real world, the character of solitons has been used in optical communication systems and electrical networks. For instance, Hirota (1973) proposed a pulse soliton modulation technique, and Singer (1996) has recently developed a technique considering a soliton train. Nevertheless, in the field of traffic, there seems to be no published reports involved with solitons. Not only for a soliton but with a general density wave as well, as long as it is propagating along the corresponding characteristic line, its shape remains unchanged. Intuitively, this may be the primary reason that solitons have not been appreciated so far in the traffic field. The reason for applying the two-phase fluids model to simulate the evolution of solitons is mainly due to smooth property of its large amplitude wave, which might occur on multilane freeways.

Two-Phase Fluids Model for Freeway Traffic

By considering mixing traffic flow on freeways, we divide the vehicles into two parts. The first part is the vehicular cluster running slowly; the second is the cluster running fast. In other words, we expect to study a two-phase fluid traffic system. Thus, it is convenient to assume that vehicles can pass themselves but the macroscopic hypothesis is made postulating that flow can be represented by global variables not taking into account the different lanes.

Let s denote the mass fraction for the first vehicular phase, while u_{1f} and u_{2f} represent, respectively, the two free speeds for the two phase system. Further, let u_1 and u_2 denote the relevant vehicular speeds in the form

$$\begin{cases} u_1 = u_{1f}\cdot K_1(s) f_1(\rho/\rho_m) \\ u_2 = u_{2f}\cdot K_2(s) f_2(\rho/\rho_m) \end{cases} \tag{1}$$

where $f_i(\rho/\rho_m)$ for $i=1,2$ are relations to be obtained from the observation of freeway traffic; $s=$mass fraction of the first phase; and $K_1(s)$ and $K_2(s)=$monotonic functions of s used to reflect the vehicular interaction. Since the free speed for vehicles in the first phase must not be enlarged by vehicles in the second phase even under the condition when s approaches zero, we suppose that $K_1(s)=1$. On the other hand, the increase of mass fraction s must lead to the decrease of the speed for vehicles in the second phase due to the impeding effects arising from the former. Thus, we assume that

$$K_2(s) = \frac{u_{1f}}{u_{2f}}\left[1+\left(\frac{u_{2f}}{u_{1f}}-1\right)(1-s)^2\right] \tag{2}$$

For simplicity, we choose the one-parameter family polynomial speed-density model as mentioned by Zhang (1999):

$$f_i = 1-\left(\frac{\rho}{\rho_m}\right)^{n_i}, \quad i=1,2 \tag{3}$$

where $\rho_m=$maximum global density allowed for the traffic, which is defined by

$$\rho_m = \rho_{1j}s+\rho_{2j}(1-s) \tag{4}$$

This is a linear function of the mass fraction s, where ρ_{ij} for $i=1,2$ is the jam density for the case of s equals 1 or 0, respectively. It is assumed that the average length of slow moving vehicles $(=\overline{l}_{vh1})$ is just about 1.5 times that of those moving fast $(=\overline{l}_{vh2})$. Thus, the assigned value of ρ_{1j} shown in Table 1 is less than the value of ρ_{2j}, and it gives rise to the ratio $\rho_{1j}/\rho_{2j} \approx (\overline{l}_{vh1}/\overline{l}_{vh2})^{-1} = 1/1.5$. Note that the average length of the mixing traffic flow is proportional to the reciprocal of ρ_m.

From the continuity equations for the two-phase fluids system, by neglecting the traffic generation rates and recalling that s is the mass fraction for the first phase, we have the governing equations for freeway traffic in the form

$$\frac{\partial(s\rho)}{\partial t}+\frac{\partial(s\rho u_1)}{\partial x}=0 \tag{5}$$

$$\frac{\partial[(1-s)\rho]}{\partial t}+\frac{\partial[(1-s)\rho u_2]}{\partial x}=0 \tag{6}$$

By taking the summation of Eqs. (5) and (6) as a governing equation for the global traffic density, we have the alternative form of the governing equations for the two-phase fluids traffic system:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0 \tag{7}$$

$$\frac{\partial(s\rho)}{\partial t}+\frac{\partial(s\rho u_1)}{\partial x}=0 \tag{8}$$

where

$$\begin{aligned} \rho_1 &= s\rho, \qquad u = s u_{1f}K_1(s) f_1(\rho/\rho_m) \\ &\qquad\qquad + (1-s)u_{2f}K_2(s) f_2(\rho/\rho_m) \\ \rho &= \rho_1+\rho_2, \quad u_1 = u_{1f}K_1(s) f_1(\rho/\rho_m) \end{aligned} \tag{9}$$

Solutions for Eqs. (7) and (8) must be sought that satisfy the boundary conditions

$$\begin{aligned} &s|_{x=0}=s(0,t), \quad s|_{x=L}=s(L,t) \\ &\rho|_{x=0}=\rho(0,t), \quad \rho|_{x=L}=\rho(L,t) \quad t\in[0,\infty) \end{aligned} \tag{10}$$

and the initial conditions

$$s|_{t=0}=s(x,0), \quad \rho|_{t=0}=\rho(x,0), \quad x\in[0,L] \tag{11}$$

Numerical Algorithm

We shall now use the second-order symmetrical (total variation diminishing TVD) algorithm of (Roe 1981; Yee-Roe-Davis Yee 1987; Davis 1988) to solve a traffic problem described by the two-phase fluids model given in the previous section. Shui (1998) has introduced this algorithm in detail. For convenience, we choose a uniform time space grid system for the finite-difference approximation. From Eqs. (7) and (8), we have the alternative vector form

$$\frac{\partial \rho}{\partial t}+\frac{\partial \mathbf{F}(\rho)}{\partial x}=0 \tag{12}$$

whose Jacobian matrix is given by

$$\mathbf{A}=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}=\begin{pmatrix} \dfrac{\partial F_1}{\partial \rho} & \dfrac{\partial F_1}{\partial \rho_1} \\ \dfrac{\partial F_2}{\partial \rho} & \dfrac{\partial F_2}{\partial \rho_1} \end{pmatrix} \tag{13}$$

The two characteristic values for the Jacobian matrix **A** can be expressed as

$$a_1=\frac{a_{11}+a_{22}-\sqrt{(a_{11}-a_{22})^2+4a_{12}a_{21}}}{2}$$
$$a_2=\frac{a_{11}+a_{22}+\sqrt{(a_{11}-a_{22})^2+4a_{12}a_{21}}}{2} \tag{14}$$

According to the speed-density relation for f_1, and f_2, we have

$$\begin{aligned} a_{21}&=F_2\left\{\frac{1}{\rho}-\frac{n_1(\rho/\rho_m)^{n_1}}{1-(\rho/\rho_m)^{n_1}}\left[\frac{1}{\rho}+\frac{\rho_{1j}-\rho_{2j}}{\rho_m}\frac{\rho_1}{\rho^2}\right]\right\} \\ a_{22}&=F_2\left\{\frac{1}{\rho_1}+\frac{n_1(\rho/\rho_m)^{n_1}}{1-(\rho/\rho_m)^{n_1}}\left[\frac{\rho_{1j}-\rho_{2j}}{\rho_m}\frac{1}{\rho}\right]\right\} \\ a_{11}&=a_{21}+(F_1-F_2)\left\{\Delta_1+\frac{1}{\rho-\rho_1}-\frac{n_2(\rho/\rho_m)^{n_2}}{1-(\rho/\rho_m)^{n_2}}\right. \\ &\quad\left.\times\left[\frac{1}{\rho}+\frac{\rho_{1j}-\rho_{2j}}{\rho_m}\frac{\rho_1}{\rho^2}\right]\right\} \\ a_{12}&=a_{22}+(F_1-F_2)\left\{\Delta_2-\frac{1}{\rho-\rho_1}-\frac{n_2(\rho/\rho_m)^{n_2}}{1-(\rho/\rho_m)^{n_2}}\right. \\ &\quad\left.\times\left[\frac{\rho_{1j}-\rho_{2j}}{\rho_m}\frac{1}{\rho}\right]\right\} \end{aligned} \tag{15}$$

where

$$\Delta_1=-\frac{2(u_{2f}/u_{1f}-1)(1-\rho_1/\rho)/\rho_1}{[1+(u_{2f}/u_{1f}-1)(1-\rho_1/\rho)^2]} \tag{16}$$

$$\Delta_2=\frac{2(u_{2f}/u_{1f}-1)(1-\rho_1/\rho)\rho_1/\rho^2}{[1+(u_{2f}/u_{1f}-1)(1-\rho_1/\rho)^2]} \tag{17}$$

are, respectively, the terms arising from $K_2(s)$.

By making use of the Jacobian matrix, we obtain the right characteristic matrix in the form

$$\mathbf{R}=[\mathbf{r}_1(\rho),\mathbf{r}_2(\rho)]=\begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}=\begin{bmatrix} 1 & -\dfrac{a_{12}}{a_{11}-a_2} \\ -\dfrac{a_{21}}{a_{22}-a_1} & 1 \end{bmatrix} \tag{18}$$

with its inverse, the left characteristic matrix

$$\mathbf{L}=\begin{bmatrix} \mathbf{l}_1(\rho) \\ \mathbf{l}_2(\rho) \end{bmatrix}=\mathbf{R}^{-1}=\frac{1}{1-r_{12}r_{21}}\begin{bmatrix} 1 & -r_{12} \\ -r_{21} & 1 \end{bmatrix} \tag{19}$$

Now we shall briefly describe the Yee-Roe-Davis symmetrical TVD algorithm in the second order. First let $\mathbf{l}_k$ and $\mathbf{r}_k$, where $k=1,2$, denote the left and right characteristic vectors corresponding to the kth characteristic value of the Jacobian matrix **A**, respectively, and let the space grid and time level be denoted by subscript and superscript distinctively—then define $\lambda=\Delta t/\Delta x$ as the ratio between time interval and grid space, which should satisfy the condition for computational stability as follows:

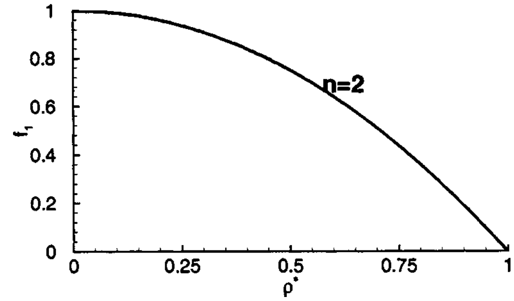

Fig. 1. Diagram of traffic speed-density for $n_1=n_2=n=2$, where ρ^* and f_1 are, respectively, normalized global density and speed in unit of free speed u_{1f} for vehicles in first phase

$$\lambda \max_{k,j}|a_{k,j+1/2}|<1$$

where $a_{k,j+(1/2)}$ is the kth characteristic value for **A** located at $x_{j+(1/2)}$. By defining the coefficient of the viscous term as

$$Q_k(x)=\begin{cases} |z|, & \text{if } |z|\geqslant\epsilon_k \\ \dfrac{z^2+\epsilon_k^2}{2\epsilon_k}, & \text{otherwise} \end{cases} \tag{20}$$

for the second-order symmetrical TVD algorithm due to Yee-Roe-Davis, we have

$$\rho_j^{m+1}=\rho_j^m-\lambda(\hat{\mathbf{F}}_{j+1/2}-\hat{\mathbf{F}}_{j-1/2}) \tag{21}$$

where

$$\begin{aligned} \hat{\mathbf{F}}_{j+1/2}&=\frac{1}{2}\left[\mathbf{F}(\rho_j)+\mathbf{F}(\rho_{j+1})+\sum_{k=1}^{2}\psi_{k,j+1/2}\mathbf{r}_{k,j+1/2}\right] \\ \psi_{k,j+1/2}&=-\frac{1}{\lambda}[(\lambda a_{k,j+1/2})^2 g_{k,j+1/2} \\ &\quad+Q_k(\lambda a_{k,j+1/2})(\alpha_{k,j+1/2}-g_{k,j+1/2})] \end{aligned} \tag{22}$$

and

$$\begin{aligned} \alpha_{k,j+1/2}&=\mathbf{l}_{k,j+1/2}(\rho_{j+1}-\rho_j) \\ g_{k,j+1/2}&=\min\operatorname{mod}(\alpha_{k,j-1/2},\alpha_{k,j+1/2},\alpha_{k,j+1/2}) \end{aligned} \tag{23}$$

while the minimum modification function is given by

$$\min\operatorname{mod}(z_1,z_2,z_3)=\begin{cases} \operatorname{sgn} z_1\cdot\min(|z_1|,|z_2|,|z_3|), \\ \quad \text{if } \operatorname{sgn} z_1=\operatorname{sgn} z_2=\operatorname{sgn} z_3 \\ 0, \text{ otherwise} \end{cases} \tag{24}$$

where $\operatorname{sgn} z$=sign function whose value is 1, 0, or -1, if z is positive, zero, or negative, respectively. The minimum modification function plays the role of providing monotonic treatment for the numerical solution.

The relation between traffic speed-density, $f_1(\rho/\rho_m)$ and $f_2(\rho/\rho_m)$ will dominate the flow patterns for freeway flow. The relations under definite circumstances with indices $n_1=n_2=2$ are illustrated in Fig. 1. In addition, the flow patterns are also closely dependent upon the initial and boundary conditions considered. We shall discuss some numerical results obtained from the Yee-Roe-Davis second-order symmetrical algorithm in the next section, where evolution of solitons in traffic will be conveyed with respect to the results, the traffic speed is measured by u_{1f}, and the unit of distance is Δx, from which it is seen that the time unit is $\Delta x/u_{1f}$.

Results and Discussions

The following numerical results are obtained by using the two-phase fluids model for traffic flow, for which the initial parameters are given in Table 1, and which contain some new phenomena on the evolution of a soliton in traffic flow on freeways. A traffic soliton on freeways may occur in the region near a ramp, where local density on the road may change due to the presence of the ramp.

The speed-density relation used in the present calculation is shown in Fig. 1, where the index is $n=2$. This seems to be compatible with the traffic situation on freeways.

Two kinds of specific cases were considered. The expressions given below are the initial density-distributions for the first case:

$$\begin{aligned}\rho(x)|_{t=0} &= \rho_{2j}\left[0.5+0.3\,\mathrm{sech}^2\frac{0.3(x-x_0)}{2}\right]\\ \rho_1(x)|_{t=0} &= \rho_{2j}\left[0.2+0.2\,\mathrm{sech}^2\frac{0.2(x-x_0)}{2}\right], \quad x_0/\Delta x=100\end{aligned} \tag{25}$$

and for the second case:

$$\begin{aligned}\rho(x)|_{t=0} &= 0.5\rho_{2j}\\ \rho_1(x)|_{t=0} &= \rho_{2j}\left[0.2+0.2\,\mathrm{sech}^2\frac{0.2(x-x_0)}{2}\right], \quad x_0/\Delta x=100\end{aligned} \tag{26}$$

For the first case, solitary waves of global density and density of the first phase occurred under initial conditions. But for the second case, there was merely an initial soliton of density for the first phase, as the initial global density was uniform.

We shall examine the evolutions of such solitary-wave perturbations with respect to the proposed two-phase fluids model to reveal the vehicular interactions in freeway traffic. It should be noted that the following results are obtained under the condition of $u_{2f}/u_{1f}=2$.

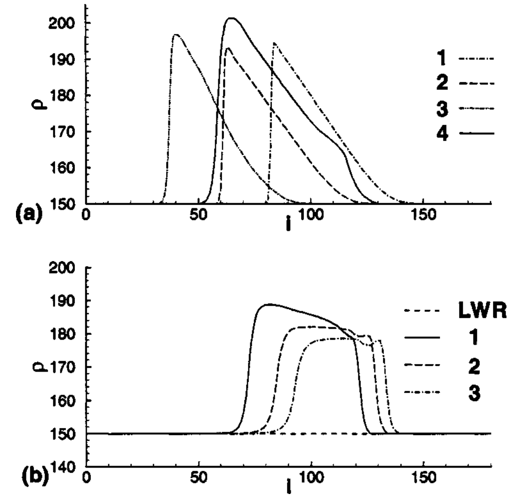

Fig. 2. Comparison with solution of LWR model at instant of $t=40$: (a) first case, where curves labeled with 1, 2, and 3 are appropriate for LWR solutions when value of uniform mass fraction $s=0$, 0.4, and 1, while curve labeled with 4 corresponds to solution of present model; (b) second case, where solution of LWR model is same as initial distribution, while curves labeled with 1, 2, and 3 are appropriate for solutions of present model when jam densities of first phase are 200, 250, and 300 vh/km, respectively.

Comparison with LWR Model

A comparison of the two-phase fluids model and the LWR model is made using Fig. 2, where the numerical results at the instant of $t=40$ are illustrated. From Fig. 2(a), it is seen that the results of the LWR model depend on the values of s. These are clearly different from the result obtained by using the two-phase fluids model (see curve 4). Interestingly, from Fig. 2(b), since the initial density is uniform, the LWR model can not predict a density wave on the road. However, with a density soliton of the first phase, the two-phase fluids model reveals a wave of global density, indicating that mixing can be viewed as a mechanism of density wave production. Note that Fig. 2(b) also shows that the density wave depends on the jam density of the first phase.

Solutions for First Case

For the problem in hand, any perturbations due to density solitons will spread along their corresponding characteristic lines. Unlike the propagation form of a soliton on free surface for a water wave problem (Chen 1988), where the mechanism of spread can be described by a KDV equation (Korteweg and Devries 1895), the initial soliton in a traffic flow should satisfy the vehicular conservation relations and the relevant state relations and boundary conditions. For this reason, the solitary wave should deform with time and spread back and forth along the road. The frontal part of the soliton alleviates and the rear part becomes steep, showing a wave distortion.

The speed distributions at four instants are illustrated in Fig. 3(a), together with the distributions for the mass fraction s and the global density ρ. It is seen that the speed does decrease to some extent due to the variation of global density and the mass fraction. Fig. 3(b) shows that the wave shape for mass fraction generally propagates downstream for the present case, while its shape gradually adjusts to a single maxima type. It is noted that the perturbation propagated upstream correspondingly leads to a slight drop in s. Fig. 3(c) indicates the wave shape of global density ρ distorts transparently during propagation of the initial solitary-wave type perturbation. The reason for this is that the upstream vehicles belonging to the second phase would have arrived earlier due to their faster running behavior.

From Fig. 4, the 3D presentations for traffic densities, it is seen that the density wave of the first phase deforms dramatically due to the interaction of both phases. The impact of fast running vehicles on the first phase comes indirectly from the variation of the global density, which plays a great role in the determination of the vehicular speeds for both phases.

Solutions for Second Case

To demonstrate the effects of ρ_1 on the traffic flow more clearly, the second case only allows the presence of a solitary-wave perturbation for ρ_1, while the global density remains uniform.

For this case, the numerical results are shown in Figs. 5 and 6. From Fig. 5(a), one can see that the vehicular speed evolutions are closely dependent on the distribution of the global density ρ and the mass fraction s. For the speed u_1 measured by u_{1f}, from the dot-dashed curve, it is found that the minima occurred as the maxima of s appeared. However, for u_2, the minima occurred as the presence of the maxima of ρ arrived. From Fig. 5(b), it is seen that the wave magnitude of s propagating downstream gradually

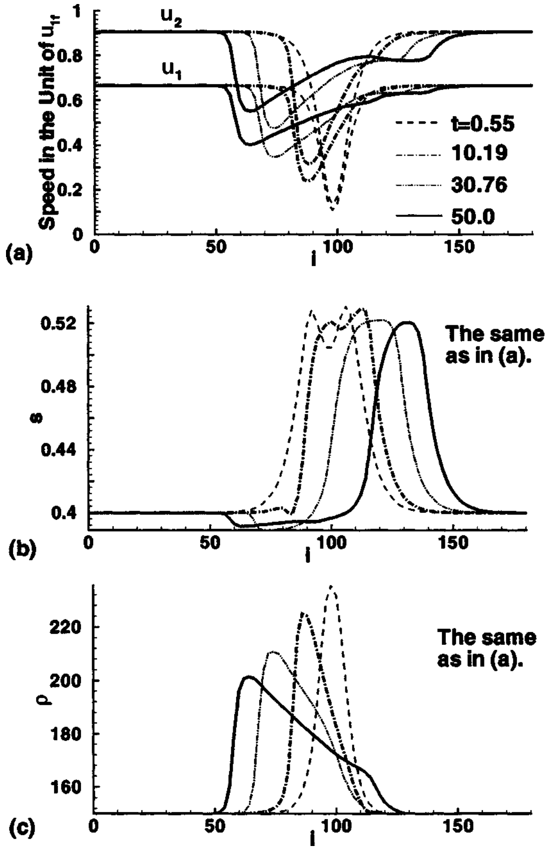

Fig. 3. Evolution of traffic wave in first case for which both ρ and ρ_1 have solitary-wave shapes given in Table 1: (a) for u_1 and u_2 in unit of u_{1f}; (b) for mass fraction s of first vehicular phase; (c) for global density ρ.

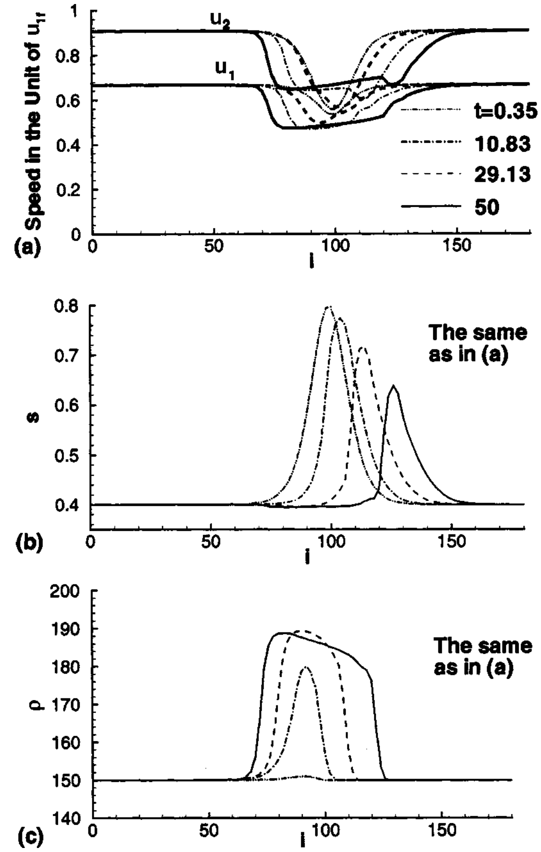

Fig. 5. Evolution of traffic wave in second case where only ρ_1 has a solitary-wave shape as given in Table 1: (a) for u_1 and u_2 in unit of u_{1f}; (b) for mass fraction s of first vehicular phase; (c) for global density ρ.

reduces, together with the distortion at the rear part of the wave, as no bifurcation of the wave was observed. From Figs. 5(a and c), it is seen that in the large ρ value region, the speed of vehicles obviously reduces.

Similar to Fig. 4, Fig. 6 depicts the 3D presentations of ρ and ρ_1 in the $x-t$ plane, from which the behaviors involving with traffic wave deformation and propagation can be observed. The wave influenced region increases with time.

Conclusions

In this work, a two-phase fluids model for the mixing traffic flow on freeways has been proposed from the mass conservation law, since all vehicles are at rest when a traffic jam appears, and the vehicles moving slowly have an impeding effect on the faster running cluster. The vehicular interactions are assumed to be concerned with the mass fraction of the slower running vehicles and the global density ρ.

Two cases involved with initial solitary-wave perturbations in traffic density were studied numerically by making use of the Yee-Roe-Davis second-order symmetrical TVD algorithm. A comparison with the results of the LWR model was performed. It was seen that the two-phase fluids model does have a clear advantage over the LWR model in the prediction of vehicular mixing on freeways. The mixing of traffic flow having different running behaviors provides a mechanism of density wave production in traffic flow. When the initial global density is assumed to be

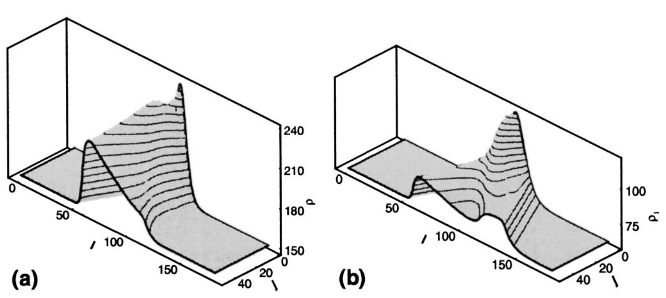

Fig. 4. 3D presentation for ρ and ρ_1 for first case: (a) and (b) denote distributions of ρ and ρ_1 in $x-t$ plane, respectively

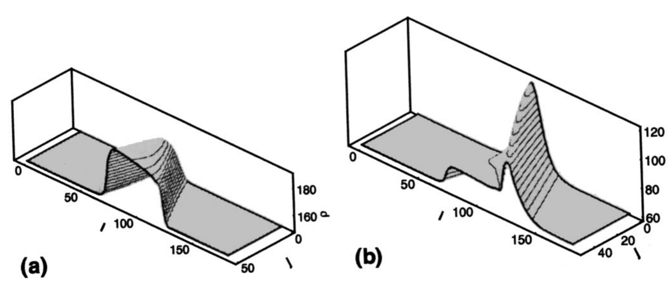

Fig. 6. 3D presentation for ρ and ρ_1 for second case: (a) and (b) denote distributions of ρ and ρ_1 in $x-t$ plane, respectively

uniform, a soliton in the first phase can lead to a wave of global density propagating along the characteristic directions. However, using the LWR model, the density wave does not appear; that is, the density distribution remains unchanged [see Fig. 2(b)].

The numerical results indicate that, in contrast to the results of solitary-wave evolution in water wave problems, the solitary perturbation in traffic has its own propagation properties. At first, such an initial perturbation in slowly moving vehicles leads to a wave of global density ρ [see Figs. 2(b) and 5(c)]. Subsequently, the vehicular speed for fast moving vehicles tends to the speed for the slowly running part when s approaches unity. Finally, the initial soliton will distort dramatically during its propagation.

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Notation

The following symbols are used in this paper:

$\mathbf{A}$ = Jacobian matrix;
a_1, a_2 = wave propagation speeds;
$F=(\rho u, \rho_1 u_1)^T$ = traffic flux, where T denotes transpose of vector;
$\mathbf{l}_k$ = left vector belonging to kth characteristic value;
$\overline{l}_{rmvh1}, \overline{l}_{vh2}$ = average vehicular lengths for both phases;
$\mathbf{L}$ = left characteristic matrix;
Q_k = coefficient of numerical viscous term;
$\mathbf{r}_k$ = right vector for kth characteristic value;
$\mathbf{R}$ = right characteristic matrix;
$\mathbf{u}=(u,u_1)^T$ = vector of vehicular speed;
$s=\rho_1/\rho$ = mass fraction of first phase;
t = time;
v_f = free vehicular speed;
v_{if} = free vehicular for ith group;
x = space;
Δt = time interval;
Δx = space interval;
$\lambda=\Delta t/\Delta x$ = ratio of time and space intervals;
$\boldsymbol{\rho}=(\rho,\rho_1)^T$ = vector of traffic density;
ρ_{ij} = jam density of i phase traffic flow; and
ρ^* = normalized traffic density.

Subscript

f = free;
i = vehicular group number;
j = space grid number, jam;
m = maximum, jam; and
vh = vehicle.

Superscript

m = time level; and
n = speed-density index.

References

Ardekani, S., and Herman, R. (1987). ‘‘Urban network-wide traffic variables and their relations.’’ *Transp. Sci.*, 21(1), 1–16.

Bick, J. H., and Newell, G. F. (1960). ‘‘A continuum model for traffic flow on an undivided freeway.’’ *Q. Appl. Math.*, 18(2), 191–204.

Chang, M. F., and Herman, R. (1981). ‘‘Trip time versus stop time and fuel consumption characteristics in cities.’’ *Transp. Sci.*, 15, 183–209.

Chen, X. N. (1988). ‘‘The diffraction, reflection, and radiation of soliton and its relevant viscous decay.’’ PhD thesis, Shanghai Jiao Tong Univ., Shanghai, P.R. China.

Daganzo, C. (1995a). ‘‘A finite difference approximation of the kinematic wave model of traffic flow.’’ *Transp. Res., Part B: Methodol.*, 29(4), 261–276.

Daganzo, C. (1995b). ‘‘Requiem for second order approximations of traffic flow.’’ *Transp. Res., Part B: Methodol.*, 24(4), 277–286.

Daganzo, C. (1997). *Fundamentals of transportation and traffic flow*, Elsevier, New York, 66–149.

Davis, S. F. (1988). ‘‘Simplified second-order Godunov-type methods.’’ *J. Sci. Statist. Comput.*, 9, 445–473.

De, S. C. (1956). ‘‘Kinematic wave theory of bottlenecks of varying capacity.’’ *Proc. Cambridge Philos. Soc.*, 52(3), 564–572.

Hirota, R. (1973). ‘‘Exact envelope solutions of a nonlinear wave equation.’’ *J. Math. Phys.*, 14, 805–809.

Herman, R., and Ardekani, S. (1984). ‘‘Characterizing traffic conditions in urban areas.’’ *Transp. Sci.*, 18(2), 101–140.

Herman, R., and Ardekani, S. (1985). ‘‘The influence of stops on vehicle fuel consumption in urban traffic.’’ *Transp. Sci.*, 19(1), 1–12.

Herman, R., and Prigogine. (1979). ‘‘A two-fluid approach to town traffic.’’ *Science*, 204, 148–151.

Kortewg, D. J., and Devries, G. (1895). ‘‘On the change of form of long waves advancing in a rectangular channel and on a new type of long stationary waves.’’ *Philos. Mag.*, 39, 422–443.

Lighthill, M. J., and Whitham, G. B. (1955). ‘‘On kinematic waves. II: A theory of traffic flow on long crowded roads.’’ *Proc. R. Soc.*, 229A, 317–345.

Richards, P. I. (1956). ‘‘Shock waves on the freeway.’’ *Oper. Res.*, 4, 42–51.

Remoisenet, P. C. (1999). ‘‘Waves called solitons.’’ *Concepts and experiments*, 3rd Ed., Springer, New York, 1–12.

Roe, P. C. (1981). ‘‘Approximate Riemann solvers, parameter vectors, and difference schemes.’’ *J. Comput. Phys.*, 43, 357–372.

Shui, H. S. (1998). *Finite difference in one dimensional fluid mechanics*, National Defense, Beijing, 343–551.

Singer, P. C. (1996). ‘‘Signal processing and communication with solitons.’’ PhD thesis, Massachusetts Institute of Technology, Cambridge, Mass.

Yee, H. C. (1987). ‘‘Construction of explicit and implicit symmetric TVD schemes and their applications.’’ *J. Comput. Phys.*, 68, 151–179.

Zabusky, N. J., and Kruskal, M. D. (1965). ‘‘Interaction of solitons in a collisionless plasma and the recurrence of initial states.’’ *Phys. Rev. Lett.*, 15, 240–243.

Zhang, H. M. (1999). ‘‘A mathematical theory of traffic hysteresis.’’ *Transp. Res., Part B: Methodol.*, 33, 1–23.