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Numerical Simulation of Two Dimensional Transient Water Driven Non-Newtonian Fluid Flow in Porous Media †

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SUMMARY

Numerical simulation of two dimensional transient water driven non-Newtonian fluid flow in porous media has been performed. The hyperbolic non-Newtonian fluid model was used to describe the characteristics of non-Newtonian fluid flow. Governing equations were first approximated by implicit finite difference, and then solved by a stabilized bi-conjugate gradient (Bi-CGSTAB) approach. A comparison of the numerical results for the case of water driven Newtonian fluid was made to validate the numerical method. For water driven Newtonian fluid flow, it was found that the numerical results are satisfactorily consistent with those obtained by commercial software **VIP** which is the abbreviation of Vector Implicit Procedure for numerical simulation of Newtonian fluid flow in porous media. The maximum deviation for average pressure is less than 1.5%; the distribution of water saturation is almost the same as that obtained by **VIP**. For water driven non-Newtonian fluid flow in porous media, it was found that the limit of pressure gradient of the non-Newtonian fluid has significant effects on the process of oil recovery. The correction of numerical simulation based on the global mass balance plays an important role in oil reservoir simulation.

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1. INTRODUCTION

With the development of computer science and technology, numerical simulation of oil reservoir has become an important tool in petroleum engineering. Thus the implementation of efficient numerical methods for this purpose is of great significance.

For non-Newtonian fluid flow in porous media, series models have been proposed, such as bi-linear model (Mirzadjanzade, 1959), hyperbolic model(Molokovich, 1971), power law model (Bird, 1960), and Bingham model (Entov *et al.*, 1975, Wu *et al.*, 1990), among which the latter two models have been extensively used. For example, for one dimensional immiscible displacement of the Newtonian fluid by a non-Newtonian one in porous media, an analytical solution of Buckley-Leverett type was obtained and validated by the numerical results based on power law model.(Wu *et al.*, 1991) A general simulator–TOUGH2 for multiphase flow in porous media has been developed by Wu and Pruess (1998), where both the power-law and Bingham non-Newtonian fluid models were employed. An attempt to explain the non-Darcy effects of non-Newtonian fluid flow has been presented by Ma and Ruth (1997). Recently, the challenges and approaches for multiphase flow and transport in heterogeneous porous media have been reported in detail by Miller *et al.*(1998).

In this study, the hyperbolic non-Newtonian fluid model, which has been justified by Mychidiniv *et al.* (1989), is employed. Finite difference approximation was used to obtain the

discretised equations, which were solved by the Bi-CGSTAB algorithm developed by Von Der Vorst (1992). For Newtonian fluid flow, it was found that the numerical results are satisfactorily consistent with those given by **VIP**, a commercial software developed by using Vector Implicit Procedure has been widely used in oil reservoir simulation. For the problem on hand, it was found that the limit of pressure gradient has pronounced influences on the water fraction in the liquid of production and the amount of residual oil in the reservoir.

2. THE GOVERNING EQUATIONS

2.1. The Governing Equations

Consider water driven non-Newtonian fluid flow in porous media, it is postulated that:

1. Water is Newtonian fluid, oil is visco-plastic non-Newtonian fluid;
2. The two-phase system is isothermal and under a pressure beyond the bubbling point of pressure of oil thase;
3. Both fluids are micro-compressible, but the porous medium is heterogeneous.

Let q_α denote the production or injection rate under standard storage condition, B_α denote volumetric coefficient, $u_{j\alpha}$ denote velocity of αth phase in the porous medium. From mass conservation law, the continuity equation can be written as

$$\frac{\partial}{\partial t}\left[\frac{S_\alpha \phi}{B_\alpha}\right] + q_\alpha = \frac{\partial u_{j\alpha}}{\partial x_j} \tag{1}$$

where the flow velocity is given by

$$\mathbf{u}_\alpha = -\frac{k_{ij} kr_\alpha}{\mu_\alpha} f_\alpha (\bigtriangledown p_\alpha + \gamma_\alpha \bigtriangledown \lambda_j x_j)(\bigtriangledown p_\alpha), \quad \alpha = 1, 2 \tag{2}$$

and $\lambda = (0, 0, 1)$ is the unit vector in the vertical direction, and $\mathbf{x}$ is the positional vector. Here $\alpha = 1$ represents water phase and $\alpha = 2$ represents oil phase. From hyperbolic non-Newtonian

model, the modification factor is

$$f_\alpha(\bigtriangledown p_\alpha + \gamma_\alpha \bigtriangledown \lambda_j x_j) = \begin{cases} 1 & \text{for } \alpha = 1 \\ \frac{|\bigtriangledown p_\alpha + \gamma_\alpha \bigtriangledown \lambda_j x_j|}{\beta + \sqrt{\beta^2 + |\bigtriangledown p_\alpha + \gamma_\alpha \bigtriangledown \lambda_j x_j|^2}} & \text{for } \alpha = 2 \end{cases} \tag{3}$$

Substituting for $\mathbf{u}_\alpha$ into the continuity equation(1), we have the governing equation

$$\frac{\partial}{\partial t}\left[\frac{S_\alpha \phi}{B_\alpha}\right] + q_\alpha = \frac{\partial}{\partial x_j}\left[\sigma_{ij;\alpha}\left(\frac{\partial p_\alpha}{\partial x_j} + \gamma_\alpha \frac{\partial(\lambda_k x_k)}{\partial x_j}\right)\right], \quad \alpha = 1, 2 \tag{4}$$

where ϕ is the porosity of the porous medium, S_α is the saturation of αth phase. For two dimensional flow, it is clear that $\partial/\partial x_3 = 0$. The transmissibility $\sigma_{ij;\alpha}$ of phase α is given by

$$\sigma_{ij;\alpha} = \frac{k_{ij} kr_\alpha}{\mu_\alpha B_\alpha} f_\alpha(\bigtriangledown p_\alpha) \tag{5}$$

2.2. The Supplementary Relations

The constrain condition for saturation is

$$\sum_{\alpha=1}^{2} S_\alpha = 1 \tag{6}$$

The relative permeability, the capillary pressure are assumed to be functions of water saturation ($\alpha = 1$)

$$kr_\alpha = kr_\alpha(S_1), \quad p_c = p_2 - p_1 = p_c(S_1), \quad \alpha = 1, 2 \tag{7}$$

Finally, the micro-compressible property for both fluids requires

$$B_\alpha = B_\alpha^0/(1 + C_\alpha(p_\alpha - p_\alpha^0)), \quad \alpha = 1, 2 \tag{8}$$

$$\rho_\alpha = \rho_\alpha^0(1 + C_\alpha(p_\alpha - p_\alpha^0)), \quad \alpha = 1, 2 \tag{9}$$

$$\mu_1 = \mu_1^0(1 + C_{\mu 1}(p_\alpha - p_\alpha^0)), \tag{10}$$

$$\mu_2 = \mu_2(p_b)(1 + C_{\mu 2}(p_\alpha - p_b)), \tag{11}$$

$$\phi = \phi^0(1 - C_r(p_{av} - p_{av}^0)) \tag{12}$$

where the superscript 0 denotes the state at the pressure for reference point $\mathbf{x}^0 = (x_1, x_2, x_3)^0$. C_α and $C_{\mu\alpha}$ are the compressibility of fluid and the visco-pressure index of phase α. C_r is the compressibility of porous medium. The subscript av indicates the arithmetic mean, e.g. $p_{av} = (p_1 + p_2)/2$. Additionally, p_b is the bubbling point of pressure for oil phase.

2.3. Initial and Boundary Conditions

Solutions of the governing equations (4) must be sought which satisfy the initial and boundary conditions described as follows.

1. Initial Conditions

$$p_2 \mid_{t=0} = p_2(\mathbf{x}, 0), \quad S_1 \mid_{t=0} = S_1(\mathbf{x}, 0) \tag{13}$$

2. Boundary Conditions

The inner condition has the form:

$$\int_0^{2\pi} [u_{r\alpha}(\theta)h]_m d\theta = [Q]_{\alpha m} \tag{14}$$

where $m = 1, 2, \cdots, M$ denotes the well number in the considered oil reservoir, with M to be the total well number. u_r is the magnitude of radial velocity of flow in the porous media at a well with number m. $\alpha = 2$ denotes the second phase. and h is the perforation thickness of oil layer at the location of an oil well.

The outer condition is written as

$$[\frac{\partial p_\alpha}{\partial n} + \gamma_\alpha \frac{\partial}{\partial n}(\lambda_j x_j)]\Big|_{\mathbf{\Gamma}} = 0 \tag{15}$$

where Γ is the boundary of the domain for simulation with n to be its unit normal vector on the boundary of the domain.

3. THE NUMERICAL METHOD

3.1. The Discretisation of the Governing Equations

Since the choice of both pressures as the mandatory variables leads to a difficulty in the determination of water saturation field in the latter stage of oil recovery. Thus, S_1 is taken as the mandatory variable. Taking the pressure potential as an alternative of pressure p_2 gives rises to a choice to simplify the governing equation for the non-Newtonian phase. Accordingly, by defining $\bigtriangledown\psi = \frac{\bigtriangledown p_2}{\gamma_2} + \bigtriangledown\lambda_k x_k$, and after some algebraic operations, we obtain

$$\begin{aligned} \frac{\partial}{\partial t}\left[\frac{S_1\phi}{B_1}\right] + q_1 &= \frac{\partial}{\partial x_j}(\sigma_{ij,1}\gamma_2\frac{\partial}{\partial x_j}\psi) \\ &\quad -\frac{\partial}{\partial x_j}(\sigma_{ij,1}p_c'\frac{\partial}{\partial x_j}S_1) + \frac{\partial}{\partial x_j}[\sigma_{ij,1}(\gamma_1-\gamma_2)\frac{\partial}{\partial x_j}(\lambda_k x_k)] \qquad (16) \\ \frac{\partial}{\partial t}\left[\frac{S_2\phi}{B_2}\right] + q_2 &= \frac{\partial}{\partial x_j}(\sigma_{ij,2}\gamma_2\frac{\partial}{\partial x_j}\psi) \qquad (17) \end{aligned}$$

which are discretised by a finite difference approximation(see, Aziz et al.,1979). The change of capillary pressure in a time interval is neglected. This implies that p_c is very small as compared with the pressure p_2 in porous media. To maintain the physical meaning of the numerical solution, the relative permeability is upstream weighted.

3.2. The Bi-CGSTAB Algorithm

The discretised equations of the governing equations can be written as

$$A\mathbf{X} = \mathbf{B} \qquad (18)$$

Since both the relative permeability and capillary pressure are closely related to the water saturation, and the flow velocity in the porous media is related to the pressure gradient, the problem considered is strongly coupled with high non-linearity. The convergence history of general conjugate gradient method is not better than that of Bi-CGSTAB which was used to

perform the inner iteration. Due to the non-linearity of the problem, the outer iteration is required. Assuming $\mathbf{X}^{\nu}, \nu = 0$, let ϵ equal a positive small number, say 10^{-6}, we can write the outer iteration procedure as the following pseudo code:

1. Evaluate $\mathbf{X}^{\nu+1}$, by solving equation (18) in term of Bi-CGSTAB.

2. Update A and $\mathbf{B}$ based on $\mathbf{X}^{\nu+1}$.

3. Check $\| A\mathbf{X}^{\nu+1} - \mathbf{B} \| / \| \mathbf{B} \| \leq \epsilon$,

   If (CONVER.TRUE.)

   Terminate the outer iteration.

   Else

   let $\nu = \nu + 1$ and return to step 1.

   Endif.

The inner iteration based on the Bi-CGSTAB algorithm can be written as:

1. Select $E = \mathrm{diag}\{A\}$ as a pre-conditioner, let iteration level $s = 0$, and $\mathbf{X}^{(s)} = \mathbf{X}^{\nu}$, then calculate residual $r^{(s)} = \mathbf{B} - A\mathbf{X}^{(s)}$, and let $\tilde{r} = r^{(s)}$.

2. For $s = 1, 2, 3, \cdots$

   $\theta^{(s-1)} = \tilde{r}^T r^{(s-1)}$

   if $\theta_{s-1} = 0$, method fails.

   if $s = 1$

   $v_1^{(s)} = r^{(s-1)}$

   else

   $\Omega^{(s-1)} = \frac{\theta^{(s-1)}}{\theta^{(s-2)}} \frac{\alpha^{(s-1)}}{\omega^{(s-1)}}$

   $v_1^{(s)} = r^{(s-1)} + \Omega^{(s-1)} \left(v_1^{(s-1)} - \omega^{(s-1)} v_2^{(s-1)} \right)$

   endif.

3. solve $E\hat{v_1} = v_1^{(s)}$

   $v_2^{(s)} = A\hat{v_1}$

   $\alpha^{(s)} = \frac{\theta^{(s-1)}}{\tilde{r}^T v_2}$

   $v_3 = r^{(s-1)} - \alpha^{(s)} v_2^{(s)}$

   Check if $\mid v_3 \mid \leq \epsilon$, if hold, $\mathbf{X}^{(s)} = \mathbf{X}^{(s-1)} + \alpha\hat{v_1}$, iteration terminated, otherwise, continue step 4.

4. Solve $E\hat{v_3} = v_3$

   $v_4 = A\hat{v_3}$

   $\omega^{(s)} = \frac{v_4^T v_3}{v_3^T v_3}$

   $\mathbf{X}^{(s)} = \mathbf{X}^{(s-1)} + \alpha^{(s)}\hat{v_1} + \omega^{(s)}\hat{v_3}$

   $r^{(s)} = v_3 - \omega^{(s)} v_4$

   Check the convergence, continue to step 2 if necessary.

The evaluated $\mathbf{X}$ must satisfy the global mass balance equation, based on which a correction term $\Delta\mathbf{X}$ can be obtained to improve the numerical results.

4. NUMERICAL VALIDATION

To validate the numerical method described above, an isolate reservoir in Jiang Han Oil Field was selected. Exploration of fossil resources in this region began at the end of 1969. Since then, it has produced petroleum for about forty years by using water injection. It was found that the oil viscosity in this region is low, and the thermal effects can be neglected. Depth of oil layer is about 1580m, and the original pressure is about 179 atm. The reason to do this choice is that the production data has been fitted by the commercial software **VIP** which has been

widely used in petroleum engineering, where Newtonian fluid model and conjugate gradient method were incorporated. The evolution of average pressure during the time range from the end of 1969 to the end of June in 1996 are illustrated in Figure 1. It is found that the curve for p_{mean} obtained by present method coincides completely with the results given by **VIP** in the first three years of oil recovery, deviation appears at the latter stage. The primary reason arises from the different treatment of wells. It is seen that the largest deviation is about 20 atm, which occurs at $t = 5200$ Days. This is less than 1.5%. On the other hand, a satisfactory agreement is also observed from the distribution of water saturation shown in Figure 2 (a) and (b), at the instant of $t = 4015$ Days.

5. THE APPLICATION TO SIMULATE THE RESERVOIR AT BA- MIAN- HE

The numerical method including Bi-CGSTAB was used for the simulation of water driven non-Newtonian fluid flow in porous media. The rock parameters and fluid properties were chosen with respect to the reservoir at Ba Mian He.

The parameters required for the numerical simulation of non-Newtonian reservoir are illustrated in Table 1, where the over-bar of a parameter implies the volumetric average over the whole reservoir, while $S_{2\,\max}$ and $S_{2\,\min}$ are respectively the connate and the irreducible saturation for oil phase.

In accordance with the balance of vertical forces, the initial pressure and water saturation field can be obtained, as seen in Figure 3, where the values of pressure and water saturation in the non-porous region are assigned to be zero, and water saturation in the pure water region is of course unity.

Figure 4 (a) and (b) show the dependence of relative permeability capillary pressure p_c on

water saturation. These curves are important in the numerical simulation of an oil reservoir. The capillary pressure in the sub-surface system is much smaller than the initial pressure, indicating that it is permissible to neglect the time variation of p_c during a time interval.

The variations of the comprehensive water fraction and the amount of residual oil are shown in Figure 5 (a) and 5 (b). It is found that a relatively large value of f_w occurs when the limit of pressure gradient β is large, corresponding to more residual oil in the reservoir.

Figure 6 (a),(b), (c), and (d) show respectively the time evolutions of water injection rate Q_w, oil production rate Q_o, liquid production rate $Q_l = Q_o + Q_w$, and the average pressure p_{mean} for four cases when $\beta = 0, 10^4, 5 \times 10^4, 10^5$Pa/m. Note that $\beta = 0$ means that the oil is Newtonian fluid. The evolution of average pressure is dominated by the operating conditions of oil recovery and the flow performances of the two-phase system. It is observed from Figure 6 that the dependence of evolutions on β is significant.

The pressure fields for $\beta = 0, 10^5$ Pa/m in the reservoir at Ba-Mian-He at the instant of $t = 2008$ Day are illustrated in Figure 7 (a) and (b). From the comparison of shaded areas between Figure 7 (a) and 7 (b), it is observed that there is a pronounced difference between the pressure field in a Newtonian reservoir and that in a non-Newtonian reservoir. More dense contours occurs in the non-Newtonian reservoir indicating there exists a larger pressure gradient field.

6. CONCLUSIONS

The Bi-CGSTAB method developed by Von Der Vorst was used to simulate the two dimensional transient two-phase flow of water driven non-Newtonian fluid flow in porous media. The role of the global mass balance was stressed and used to correct the solution of

discretised equations. For a definite small reservoir with Newtonian oil, the results obtained by the method introduced were compared with that given by **VIP** (i.e. Vector Implicit Procedure). A satisfactory agreement was obtained. The non-Newtonian property shows significant effects on water driven non-Newtonian fluid flow in porous media. The application of the numerical method indicates that it has a potentiality in oil reservoir simulation.

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Nomenclature

B_α = volumetric coefficient

C_α = compressibility of fluid

$C_{\mu\alpha}$ = visco-pressure index

f_α = factor of nonlinearity

f_w = water fraction

h layer thickness

$\mathbf{k}$ = = temsor of permeability

kr_α = relative permeability

$\mathbf{n}$ = unit normal vector on the boundary of the domain

p_b = bubbling pressure of oil

p_α = pressure of αth phase

q_α = source term

p_c = capillary pressure

$[Q]_{\alpha m}$ = production or injection rate for αth phase by a mth well

Q_o = oil production rate

Q_w = water production rate

Q_l = production rate of liquid

Q_{mro} = amount of residual oil

S_α = saturation for αth phase

$\mathbf{u}_\alpha$ = velocity vector

GREEK SYMBOLS

β = limit of pressure gradient

$\gamma_\alpha = \rho_\alpha g$ = specific gravity

$\nabla \lambda_j x_j$ = gradient of depth;

μ_α = viscosity of αth phase fluid

ψ = pressure potential for oil

ρ = fluid density

ρ_0 = initial fluid density

$\sigma_{ij;\alpha}$ = tensor of transmissibility

SUPERSCRIPT

0 = referred point, initial;

b = bubbling point

l = time level, or liquid

s = iteration level

$\prime$ = derivative

SUBSCRIPTS

b ; well-bore

α αth phase;

ij grid node number

m well number;

$o, 2$ oil;

r remained, radial;

$w, 1$ water

Table I. Parameters Used for Reservior Simulation

$\overline{k}^{\dagger} = 161.68 \times 10^{-3} \mu\mathrm{m}^2$	$\overline{\phi}$ =0.29
$\overline{h} = 7.68$ m	$S_{2\,\max}$=0.7
$p_b = 80. \times 10^5$ Pa	$S_{2\,\min}$=0.4
$\mu_1 = 0.58 \times 10^{-3}$ Pas	r_w=0.065 m
$C_{\mu 1} = 0 \quad 1/10^5$Pa	$C_{\mu 2} = 0.16 \times 10^{-5} \quad 1/10^5$Pa
$C_1 = 3.5 \times 10^{-5} \quad 1/10^5$Pa	$C_2 = 8.4 \times 10^{-5} \quad 1/10^5$Pa
$C_r = 4.6 \times 10^{-5} \quad 1/10^5$Pa	

† Here the over bar for k, ϕ and h means the average over the whole volume of the reservoir considered.

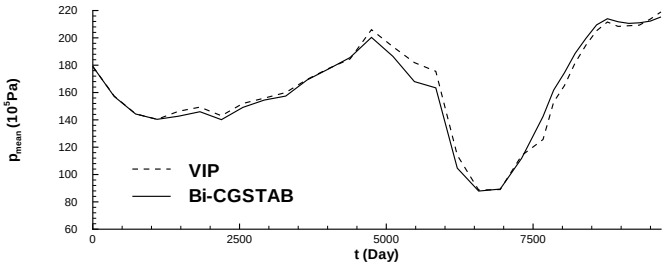

Figure 1. A comparison of average pressure; where dashed curve is given by **VIP**, solid curve is obtained by making use of the present method.

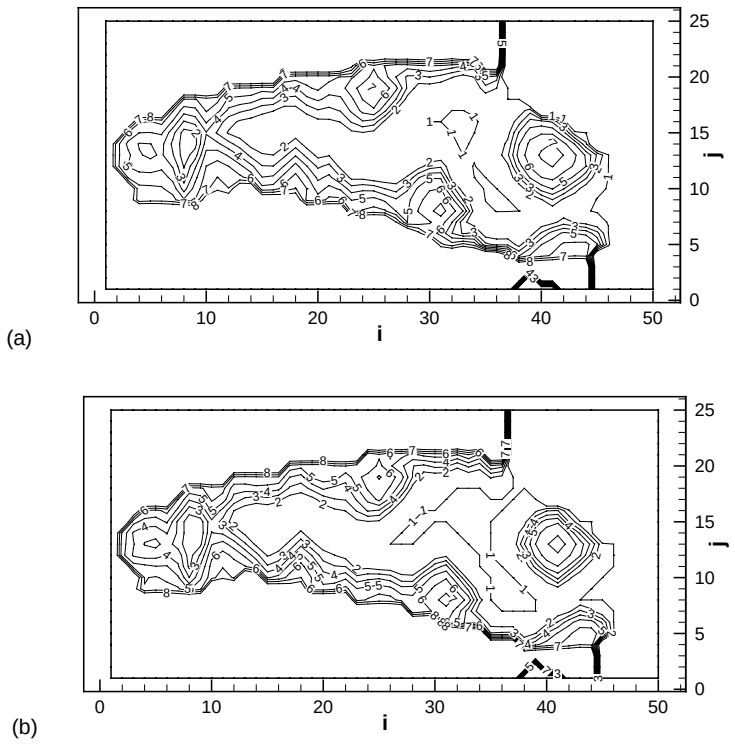

Figure 2. A comparison of water saturation at the moment of $t = 4015$ (Days) from the beginning of oil recovery. (a) Obtained by **VIP**, (b) Obtained by Bi-CGSTAB. The curves labelled 1,2,3...,8 are for the values of $S_1 = 0.065, 0.1875, 0.3125, 0.4375, 0.5625, 0.6875, 0.8125, 0.9375$.

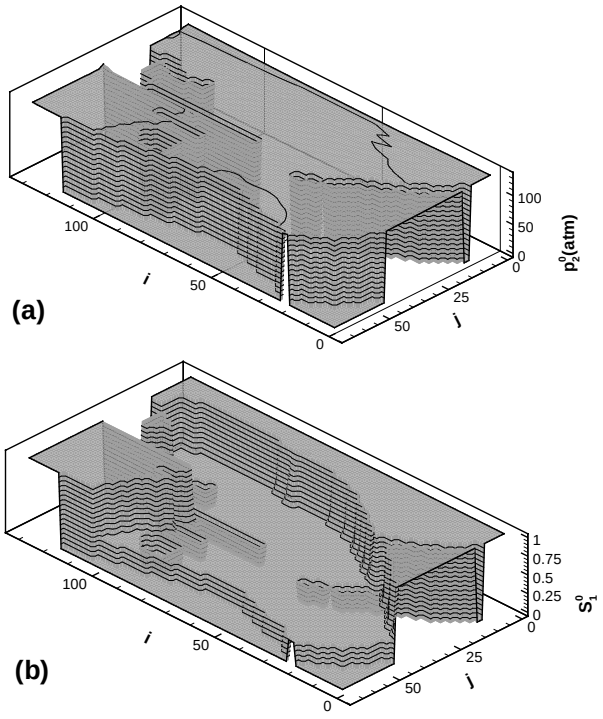

Figure 3. The initial fields of (a) pressure, and (b) water saturation.

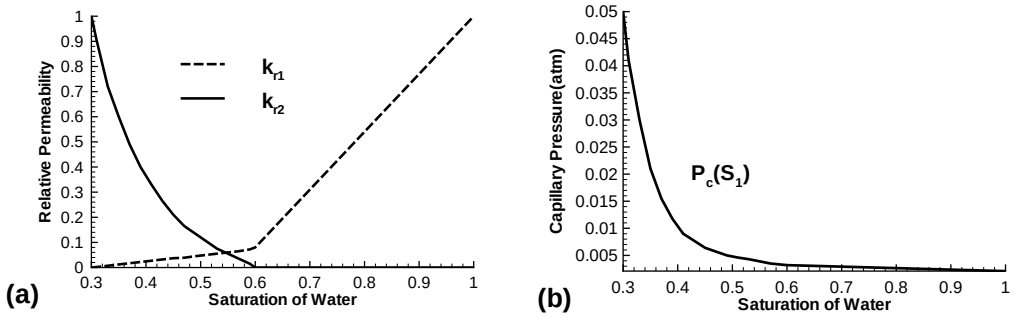

Figure 4. The dependence of (a) the relative permeability for each phase, and (b) the capillary pressure p_c on water saturation.

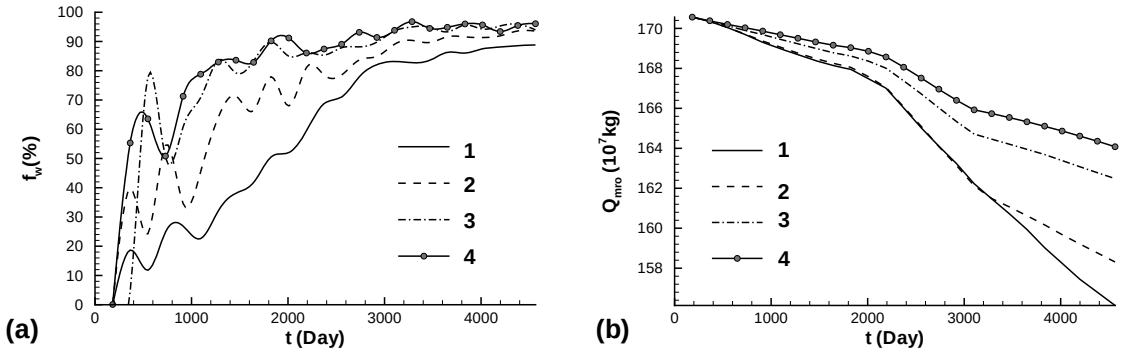

Figure 5. The time variations of (a) water fraction; (b) residual oil. The curves labelled 1,2,3, and 4 are appropriate for values of the limit of pressure gradient $\beta = 10^4, 5 \times 10^4, 10 \times 10^4$(Pa/m), respectively.

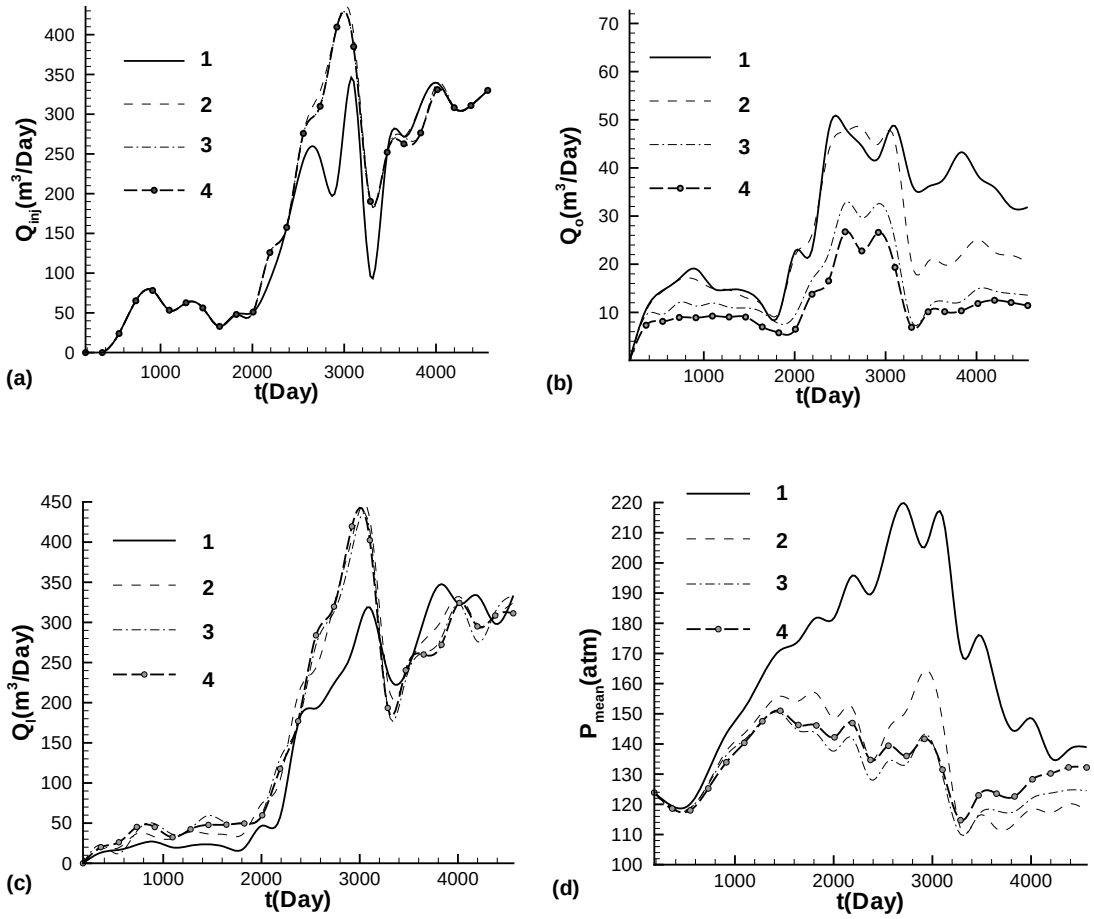

Figure 6. The evolutions of (a) the water injection rate Q_w; for the case of $\beta = 0, 10^4, 5 \times 10^4, 10^5$Pa/m; (b) oil production rate Q_o; (c) liquid production Q_l; (d) the average pressure in the porous media. The curves labelled 1,2,3, and 4 are appropriate for the cases of $\beta = 0, 10^4, 5 \times 10^4, 10^5$Pa/m, respectively.

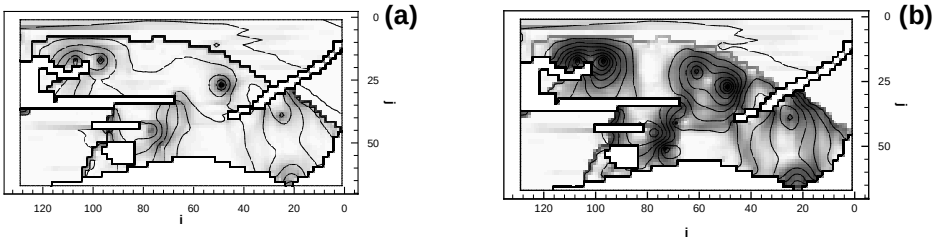

Figure 7. The predicted contours of pressure at the moment of five years' simulation. (a) for Newtonian oil; (b) for non-Newtonian oil when $\beta = 10^5$Pa/m. The curves labelled 1,2,3,...,15 are for values of $p_2 = 80, 80+\delta p, 80+2\delta p, ..., 190-\delta p, 190$atm, where $\delta p = 7.8571$atm.