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Discrete Hilbert transformation and its application to estimate the wind speed in Hong Kong

Zuojin Zhu^{a,*}, Hongxing Yang^b

^a Department of Thermal Science and Energy Engineering, Institute of Engineering Science, University of Science and Technology of China, Hefei, Anhui, People's Republic of China ^b Department of Building Services Engineering, The Hong Kong Polytechnic University, Hong Kong

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Abstract

Discrete Hilbert Transform (DHT) has been applied to estimate the wind speed with the sample data sequence selected from the data record observed by the observatory in Hong Kong in June 1989, during which the data pertain to deep valleys and sharp crests due to manifold weather conditions in this region. To confirm the performance of the discrete Hilbert transformer, two harmonic input sequences were used to inspect the output signals, whether good agreement with the theoretical results is obtained. It was found that the energy spectrum and the outputs for the two different harmonic discrete waves are certainly correct. After the inspection of the DHT filter, the sample data for wind speed in Hong Kong were used for wind speed forecasting. For zero mean input sequence, the variance of the output is the same as that of the input signals, and so is the energy spectrum. The DHT of an individual input sample can really reflect the local variation performance, since it is the convolution with the reciprocal of time and the input data sequence, but there exists phase shift. For harmonic signals, the output signal holds a 90° phase delay. © 2002 Published by Elsevier Science Ltd.

Keywords: Discrete Hilbert transform; Wind speed; Energy spectrum

1. Introduction

The Hilbert transformation, similarly used as Hilbert transformer which deals with signals in time domain, was first introduced to signal theory by Denis Gabor [1].

^{*}Corresponding author. Tel.: +86-551-3603803; fax: +86-551-3631760.

E-mail address: zuojin@ustc.edu.cn (Z. Zhu).

Nomenclature	
	impulse response of linear analog time invariant system transfer function of linear time invariant system impulse response of the discrete linear time invariant transfer function of discrete linear time invariant defined by Eq. (14) energy spectrum of $u(i)$ at frequency $f(k)$ the DHT of $u(i)$ input sequence, sample wind speed, m/s estimated wind speed, m/s discrete Fourier transform of $u(i)$
Greek symbols	
$ar{lpha}{ar{eta}}{\delta_{ m K}}$	mean of $\ln (u(i)/u_{max})$ mean of phase $\beta(i)$ Kronecker delta sample

Today it plays a significant role in signal processing. The theory on this has been introduced by Hahn [2] in detail. In 1999, the popular approach for signal processing was employed to propose a new view of non-linear water waves by Huang et al. [3], accompanied with the intrinsic mode decomposition method. The original data were decomposed into several modes, and then by analyzing the energy spectrum and the time variation of frequency, the interpretations with more physical meaning were presented. However, it might be incredible to indicate that there exists significant difference between the Hilbert spectrum and the Fourier spectrum. Since signals from a discrete Hilbert transformer can be derived directly from the convolution of the input and the impulse response, mathematical manipulation has indicated that the energy spectrum of the Hilbert-transformed signal remains the same as that of the input, except for the zero component.

In wind and building services engineering, it is of great importance to recover data from sample values by a causal filter. For instance, it is often necessary to predict the cooling loads of buildings where the solar irradiation is needed, but only the data for the past have been known. Similarly, the wind speed estimation is useful for the prediction of the potentials of wind energy of a given region. The wind speed was usually regarded to be satisfied with the Weibull distribution [4,5]. More researches have focused on the parameters for the probability density distribution. As an alternative, the output from a DHT filter is quite possible to be applied in engineering.

Recently, the forecasting of time data sequence (e.g. the global solar irradiation) has been reported by using several auto-regressive moving average (ARMA)

methods and the method of multi-layer perceptron neural networks. These filters can be used to estimate the wind speed as well [6–8]. However, the ARMA method demands the evaluation of auto-regressive parameters and moving average parameters, where numerical stability and non-linearity exist, while a key problem encountered with the multi-layer perceptron for the neural network is the specification of the interconnection weights from a training data set.

The modeling of wind field of nuclear accident consequences for Hong Kong has been conducted by Yeung and Lui [9]. They found that the wind field model, which increases the frequencies of early fatality, early injuries and latent cancers in this district, has a drastic effect on the puff trajectory. A mesoscale numerical model for the simulation field and other meteorological parameters in the complex terrain of Hong Kong has been proposed by using MC2 [10]. In addition, the wind climate in this region has been examined by Jeary [11], and a study of Weibull parameters using long-term wind observations has been carried out by Lun and Lam [5]. However, none of the previous work has been concerned with the application of the discrete Hilbert transformer to the treatment of the observed data for wind speed in Hong Kong. In the context, such a data filter has been employed to forecast the wind speed in this district.

2. The Hilbert transformation

As shown in Fig. 1, the impulse response of linear analog time invariant system and its transfer function are given by the Fourier pair

$$h(t) \stackrel{\mathrm{F}}{\Leftrightarrow} H(j\omega),$$
 (1)

where $H(j\omega)$ is the transfer function which is defined as the ratio of output signal to the input signal. The discrete equivalent of the delta pulse is the Kronecker delta



Fig. 1. (a) The illustration of the excitation of a linear time invariant system by the Dirac delta pulse, where h(t) is the impulse response and (b) the illustration of the excitation of the discrete linear time invariant system by the Kronecker unit sample, where h(i) is the impulse response.

sample of the form

$$\delta_{\mathbf{K}}(i) = \begin{cases} 1 & \text{for } i = 0, \\ 0 & \text{for } i \neq 0. \end{cases}$$
(2)

The impulse response of the discrete linear time invariant, denoted by h(i), is defined as the response to the δ_{K} sample, analogously,

$$h(i) \stackrel{\mathrm{F}}{\Leftrightarrow} H(k). \tag{3}$$

The Hilbert transform of the delta pulse is $H(\delta(t)) = 1/(\pi t)$, which is the impulse response of a non-causal, physically unrealizable Hilbert filter of the transfer function given by the Fourier pair

$$h(t) = \frac{1}{\pi t} \stackrel{\mathrm{F}}{\Leftrightarrow} H(j\omega) = -j \operatorname{sgn}(\omega).$$
(4)

The discrete equivalent of the transfer function of an ideal discrete Hilbert filter is given by

$$H(k) = \begin{cases} -j & \text{for } k = 1, 2, ..., (N-1)/2, \\ 0 & \text{for } k = 0, \\ j & \text{for } k = (N+1)/2, (N+1)/2 + 1, ..., N-1, \end{cases}$$
(5)

where N is odd. If N is even, one has

$$H(k) = \begin{cases} -j & \text{for } k = 1, 2, ..., N/2 - 1, \\ 0 & \text{for } k = 0, \\ j & \text{for } k = N/2 + 1, N/2 + 2, ..., N - 1. \end{cases}$$
(6)

The discrete transfer function may be written in the closed form

$$H(k) = -j \operatorname{sgn}\left(\frac{N}{2} - k\right) \operatorname{sgn}(k).$$
⁽⁷⁾

The impulse response of this transfer function is given by the inverse discrete Fourier transform (DFT) of H(k)

$$h(i) = -\frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{jw} = \frac{2}{N} \sum_{k=1}^{(N-1)/2} \sin 2\pi \, ik/N$$
(8)

with $w = 2\pi i k/N$, whose closed form is

$$h(i) = -\frac{2}{N} \sin^2(\pi i/2) / \tan(\pi i/N).$$
(9)

Consequently, by using the notion of a shifted unit delta sample, any input sequence u(i) may be written in the form

$$u(i) = \sum_{m=0}^{N-1} a(m)\delta_{\rm K}(i-m)$$
(10)

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which is a linear sum of successive samples of values given by the coefficients a(m). Since the discrete Hilbert transform (DHT) is a linear operation, the sequence consisting of a single delta sample and zeros and its DHT is

$$\delta_{\rm K}(i-m) \stackrel{\rm DHT}{\longleftrightarrow} \frac{2}{N} \sin^2[\pi(i-m)/2]/\tan[\pi(i-m)/N]. \tag{11}$$

This yields the DHT of the input signals u(i)

$$Hu(i) = u(i) \oplus \delta_{\rm K}(i-m)$$

= $\sum_{m=0}^{N-1} a(m) \frac{2}{N} \sin^2[\pi(i-m)/2] / \tan[\pi(i-m)/N],$ (12)

no matter whether N is odd or even. For more details, it is suggested to see Ref. [4].

3. Wind speed forecasting

It is assumed that the data sequence of a sample for wind speed forecasting is u(i) (i = 0, 1, 2, ..., N) with wind phase angle denoted by $\beta(i)$, (i = 0, 1, 2, ..., N). The wind speed predicted in terms of this sample by using the DHT filter is

$$\hat{u}(i) = u_{\max} e^{\bar{\alpha} + Hp(i) + j(\beta + Hq(i))},\tag{13}$$

where u_{max} is the maximum value of the sample and $\bar{\alpha}$ and $\bar{\beta}$ are the mean values of the data sequences of $\ln (u(i)/u_{\text{max}})$ and $\beta(i)$, respectively, while Hp(i) and Hq(i) are, respectively, the discrete Hilbert transform of the input signals, p(i) and q(i), which can be represented by the following Hilbert pairs:

$$p(i) = \ln\left(\frac{u(i)}{u_{\max}}\right) - \bar{\alpha} \stackrel{\text{DHT}}{\Leftrightarrow} Hp(i),$$
$$q(i) = \beta(i) - \bar{\beta} \stackrel{\text{DHT}}{\Leftrightarrow} Hq(i).$$
(14)

However, the effectiveness of a discrete Hilbert transformer should be confirmed before its application to estimate wind speed. For this purpose, it is appropriate to use the harmonic input sample, whose DHT can be directly derived from theoretical analysis. Thus, two sequences generated by two harmonic functions with different periods are employed. The input signal for Fig. 2(a) is

$$\cos(6 \times 2\pi/N), \quad i = 1, 2, ..., N$$
 (15)

with

$$\sin(20 \times 2\pi/N), \quad i = 1, 2, ..., N$$
 (16)

for Fig. 2(b). In Fig. 2(c), the energy spectrums of the above sequences have been illustrated. It is seen that the energy is concentrated to both of the corresponding frequencies of the inputs. The energy spectrum at N/2 + 1 is



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Fig. 2. (a) The cosinoidal input sample with six periods and its discrete Hilbert transform; (b) the sinusoidal input sample with 20 periods and its discrete Hilbert transform and (c) the energy spectrums for both harmonic input samples and the corresponding Hilbert transforms.

defined as

$$S(f(0)) = \frac{1}{N^2} \left| \sum_{i=0}^{N-1} U(i) \right|^2,$$

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$$S(f(k)) = \frac{1}{N^2} \Biggl\{ \Biggl| \sum_{i=0}^{N-1} U(i) e^{jw_k} \Biggr|^2 + \Biggl| \sum_{i=0}^{N-1} U(i) e^{jw_{N-k}} \Biggr|^2 \Biggr\},$$

$$S(f(N/2)) = \frac{1}{N^2} \Biggl| \sum_{i=0}^{N-1} U(i) e^{jw_{N/2}} \Biggr|^2,$$
 (17)

where U(i) is the discrete Fourier transform of u(i), and the frequencies are given by

$$f(k) = kf_{\rm s} \tag{18}$$

with $f_s = 1/\Delta$ being the sampling frequency, Δ the sampling time interval, and $w_k = 2\pi i k/N$ for k = 0, 1, 2, ..., N/2. In Fig. 2, the time sequences described by expressions (15) and (16) are shown by solid curve, where the time sequence from discrete Hilbert transform for both inputs are illustrated by dash-dotted curves. It is verified that a cosinoidal input corresponds to a sinusoidal output, but a sinusoidal input for a DHT filter results in a minus cosinoidal output, regardless of the period of the input. This means that a DHT filter produces signals with $\pi/2$ phase delay, i.e. quadrature signal. This property can be proved by direct mathematical derivation from the original definition of Hilbert transform.

Note that, since the pair of input and output signals may consist of an analytic signal, currently, the DHT filter has been used in many systems and devices. In Fig. 2(c), the two energy spectrums for both harmonic input samples mentioned above are shown, where the dash-dotted impulse is for the input of Fig. 2(b) with another impulse relevant to the input of Fig. 2(a) which indicates that the energy spectrums for both input and output time sequences maintain the same spectral distribution, and this is coincided with analytical solution. For harmonic signals with a single intrinsic frequency, the energy spectrum is of course concentrated to the frequency, this is confirmed by Fig. 2(c). In addition, the results shown in Fig. 2 further indicate that the procedure written in FORTRAN language for the wind speed prediction is correct.

The data of wind speed in Hong Kong observed by observatory in June of 1989 (see the solid curve in Fig. 3(c)) were chosen as the input sample, since previous work based on statistical analysis has indicated that the meteorological data in 1989 are the most suitable to be used in building services engineering [12], e.g. for cooling load and heat gain estimations. Subsequently, the wind speed during June in Hong Kong really manifests the dramatic variation stemming from manifold meteorological conditions.

In Fig. 3(a) and (b), the solid curves show the time variation of the amplitude p(i) and phase q(i), which are defined by Eq. (14), respectively. As an evidence of the manner of variation described in the foregoing paragraph, the solid curves show sharp crests and valleys. The mean values for the sample wind speed were $\bar{\alpha} = -0.7088$ and $\bar{\beta} = 1.370$, and the standard deviation of the data sequences p(i) and q(i), whose mean values were removed, were, respectively, $\sigma_p = 0.371$, $\sigma_q = 0.710$. This quantitatively indicated the ripple property of the input samples considered.

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Fig. 3. (a) The time sequence of p(i) and its discrete Hilbert transform; (b) The time sequence of q(i) and its DHT and (c) the sample of wind speed and the estimated wind speed by the DHT.

The application of discrete Hilbert transform to p(i) and q(i) gives rise to the dashdotted curves denoted by Hp(i) and Hq(i). It is readily seen that the output of DHT maintains the primary variation trend given by the relevant solid curve.



Fig. 4. The energy spectra of p(i), Hp(i) and q(i), Hq(i), where the thin solid curve denotes the spectrum of p(i) and Hp(i), and the dashed curve represents the spectrum of q(i) and Hq(i).

The wind speed estimated is given in Fig. 3(c) with dash-dotted curve, which certainly reveals phase shift but has local variation properties due to the convolution of the input samples with $1/\pi t$.

What seems fresh is that over-predictions of the wind speed occurred at several time points. The maximum of the wind speed at the instant t = 150 h was $\hat{u}_{max} = 25.79$ m/s; this value is twice the maximum of the input sample of wind speed $u_{max} = 13.00$ m/s at t = 458. From the viewpoint of wind engineering, the overprediction points for wind speed may be adjusted according to the distribution property of the corresponding input sample without introducing significant influence on the energy spectrum, since these points have merely occurred at several positions.

The energy spectrum of the amplitude variable p(i) is shown in Fig. 4 with a solid curve, with the energy spectrum of the phase variable q(i) denoted by a dash-dotted curve. The energy spectrum for Hp(i) remains the same as that of p(i), and so does the energy spectrum for Hq(i).

4. Conclusion

A knowledge of how to estimate the wind speed is useful for the wind engineering and building services engineering. The discrete Hilbert transform method, which was used to propose a new view in nonlinear waves successfully with a broadening area found in digital data processing, was applied for wind speed estimation. Not only the wind amplitude but also the instant phase angle of the wind speed were treated with the DHT filter, which indicated that the energy spectrum of the output is completely coincided with that of the input, regardless of whether the input sequence is a harmonic or stochastic sequence. The effectiveness of the DHT filter was checked with simple trigonometric functions which have definite theoretical solution. Although the wind speed as sample data was selected from the record in June of 1989, where dramatic variations were encountered, the output from the filter can reflect the locality due to the convolution between the input and the reciprocal of time. The main characteristic of the estimated wind speed is that it has the same energy spectrum as the original input sample.

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