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Estimation of travel time through a composite ring road by a viscoelastic traffic flow model

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Abstract

To estimate travel time through a composite ring road, a viscoelastic traffic flow model is developed by assuming traffic sound speed on empty road is just equal to free flow speed. Based on the viscoelastic model, numerical tests of traffic flows were conducted to provide node traffic speed for estimating travel time. The composite ring road with three ramp intersections has five parts, each part is composed of a tunnel, a horizontal, an uphill and a downhill segment. The length of uphill segment is the same as the length of downhill segment, both are 1 km, while the tunnel length can be 1, 0.5, and 0.1km. To validate the reliability and feasibility of the viscoelastic traffic flow model, the Navier–Stokes like model Zhang (2003) is extended and adopted to provide the counterpart numerical results for comparison. It was found that in case without ramp effects any tunnel inlet becomes a starting point of traffic congestion region when initial density normalized by its jam value is not below 0.2. But in case with ramp effects, even if initial density is 0.15, downstream an on-ramp intersection, any tunnel inlet can also induce traffic shock when the tunnel is positioned upstream another off-ramp intersection. The off ramp flow can shorten mean travel time and increase its root mean square value significantly. The fitted expression of mean travel time has the form

 $\sigma_{\rm tf} = A\rho_0^m + b$

where A = 8.9686, m = 1.6260, b = 0.8424, ρ_0 is the initial density varying from 0.1 to 0.625. © 2020 International Association for Mathematics and Computers in Simulation (IMACS). Published by Elsevier B.V. All rights reserved.

Keywords: Viscoelastic traffic flow model; Travel time; Ramp effects; Traffic sound speed; Tunnel speed limit

1. Introduction

Since traffic flow affects the work and life of human beings significantly, many macroscopic traffic flow models have been developed, what can be enumerated are the well-known model LWR [24,34], the Euler model [31], the

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gas-kinetic-based model [15,16,28], the Navier–Stokes like model [18], the generic model (Lebacque, et al. [22,23]), and the urgent-gentle class traffic model [48], although researchers have to face the well-known violent criticism [5].

For the estimation of travel time, two heuristic rules were examined by Chang and Mahmassani [4], the rules were proposed for describing urban commuters' predictions of travel time as well as the adjustments of departure time in response to unacceptable arrivals in their daily commute under limited information. A relatively complete research background for travel time can be ascertained by tracing some references (Dailey [6]; Lint and der Zijpp [25]; Wu, Mo and Lee [43]; van Lint, Hoogendoorn and Zuylen; [40]; Yildirimoglu and Geroliminis [45]; Hans, Chiabaut and Leclercq [14]; Kumar, et al. [20]; Ladino, et al. [21]; Ma, et al. [26]; Rahmani, et al. [32]).

In the study aimed to provide a step forward from that research by introducing a more sophisticated understanding of the capacity drop phenomenon for the benefit of designers of street environments, Cepolina and Tyler [3] planned some experiments to evaluate the effects on capacity drop of the density at the bottleneck entrance, the pedestrians desired speed and the pedestrian motivation in passing through the bottleneck.

For the influences of road infrastructure conditions on traffic flow dynamics, Patire and Cassidy [30] found that lane changing patterns of bane and benefit and unveiled a mechanism by which congestion forms on a 3-lane, uphill expressway section, and causes reductions in output flow. Vehicular lane-changing is key to the mechanism, particularly lane-changing induced by speed disturbances that periodically arise in the expressways median and center lanes.

As uphill segments have often been identified as capacity bottlenecks in freeway networks, Goñi-Ros et al. [8] explored whether equipping the leader of a platoon with an in-vehicle Gradient Compensation System can improve traffic flow efficiency on uphill segments. Sags are freeway segments along which the gradient changes significantly from downward to upward, being considered as bottlenecks in freeway networks, extensive studies were conducted by Goñi-Ros et al. [9–12]. It was revealed that mainstream traffic flow control strategies that use variable speed limits have the potential to improve substantially the performance of freeway networks containing sags (Goñi-Ros et al. [10]); there exists a potentially highly effective and innovative way to reduce congestion at sags, which could possibly be implemented using cooperative adaptive cruise control systems (Goñi-Ros et al. [12]).

When vehicles accelerate away from the upstream queue, sags and tunnels can cause capacity reduction, capacity drop, and extreme low acceleration rates. This fact promotes the development of a behavioral kinematic wave model [17], which introduces a tunnel time gap that increasing with the distance of the tunnel inlet. The model was calibrated and validated with four trajectories at the Kobotoke tunnel in Japan, and may be helpful to develop better design and control strategies to improve the performance of a sag or tunnel bottleneck.

A class of models with nonlocal point constraints for traffic flow through bottlenecks were introduced and analyzed by Andreianov et al. [1], their numerical results show that these constraints are able to reproduce features in vehicular traffic and crowd dynamics such as the self-organization.

However, for the effect of road segment condition on travel time from the view of macroscopic model, less work has been reported. In this paper, a viscoelastic traffic flow model (VEM) is developed to study the effect on travel time through a composite ring road. The road with three ramp intersections has five parts, each is composed of a tunnel, a horizontal, an uphill, and a downhill segment arranging subsequently, as shown schematically in Fig. 1(a–b). The road segment is labeled by the index \hat{k} , each has its particular fundamental diagram (FD) characterized by the corresponding braking distance, free flow speed of traffic flow. Note that in specific tunnel, the vehicles must follow the tunnel speed limit. In fact, any vehicle running on downhill segment should have a larger free flow speed than on the uphill and horizontal segments. As braking distance is generally relevant to free flow speed (Kiselev, et al. [19]), the composite ring road has four fundamental diagrams relating to the four segments, as shown in Fig. 2.

In particular, in each road part, the tunnel is positioned upstream the uphill that linking directly with the downhill, with the horizontal segment separating the tunnel and uphill [see Fig. 1(b)]. As traffic flow has to subject to road segment condition, how the segment condition affecting the travel time through the composite road is certainly an academic problem of traffic flow theory.

The VEM model describes traffic pressure algebraically, expresses traffic sound speed by using the definition of sound speed under isentropic condition in classical mechanics. This approach of modelling is much easier than that in the gas-kinetic-based models [15,16,28], in which traffic pressure is described by a partial differential equation. Therefore, we planned to build a simulation platform for vehicular traffic on the basis of the VEM, conduct a series of numerical tests to estimate the travel time through a composite ringroad, and then use a linear regression after



Fig. 1. (a) Schematic diagram of ring traffic flow with five initial jams located at X_I , the tunnel starts at X_{si} , the uphill starts at X_I^u and is closely followed by the downhill that ends at X_I so that the uphill or downhill section length can be calculated by $L_h = (X_I - X_I^u)/2$, for I = A, B, C, D, E, and i = 1, 2, ..., 5. (b) Road segments labeled by $\hat{k} = 1, 2, 3, 4$.



Fig. 2. Fundamental diagram for traffic flows on a ring road with downhill (krd = -1), horizontal (krd = 0), uphill (krd = 1), and specific tunnel (krd = 2) segments. Note that ρ is measured by jam density ρ_m , the flow rate $q_e^{(krd)}$ is measured by $\rho_{*2}v_{f2}$, and $q_{es}^{(krd)} = c_{\tau \hat{k}}/e \cdot [\rho_{*\hat{k}}v_{f\hat{k}}/\rho_{*2}v_{f2}]$, $krd = \hat{k} - 2 = -1, 0, 1, 2$ for $\hat{k} = 1, 2, 3, 4$ respectively.

logarithmic processing to obtain a fitted expression for mean travel time. To validate the VEM, the Navier–Stokes like model [46] is extended and adopted to provide the counterpart numerical results for comparison. To show the reliability of the VEM, the observation data from Ref. (McShane, Roess and Prassas [27]) and the field observation data extracted from Ref. (Patire and Cassidy [29]) are used for comparison.

We will introduce the VEM and numerical method before describing the method of travel time prediction and model for validation, then we will discuss the numerical results, and give the conclusions finally.

Table 1 Simulation parameters of traffic flow on the ring road.

1			U				
v_{f1} (km/h)	140	$c_{\tau 1} / v_0$	5.003	ρ_{c21}	0.7191	$X_{\rm A}^{\rm u}$ (km)	10
v_{f2} (km/h)	120	$c_{\tau 2}/v_0$	4.587	ρ_{c22}	0.6979	$X_{\rm B}^{\rm u}$ (km)	34
v_{f3} (km/h)	100	$c_{\tau 3}/v_0$	4.146	ρ_{c23}	0.6716	$X_{\rm C}^{\rm u}$ (km)	58
v_{f4} (km/h)	80	$c_{\tau 4}/v_0$	3.702	ρ_{c24}^{a}	0.6404	$X_{\rm D}^{\rm u}$ (km)	82
$X_{\rm br1}$ (m)	96	$\widehat{\gamma}_1$	2.779×10^{-3}	$X_{\rm R1}~({\rm km})$	20	$X_{\rm E}^{\rm u}$ (km)	106
$X_{\rm br2}$ (m)	80	$\widehat{\gamma}_2$	2.316×10^{-3}	X_{R2} (km)	60	$X_{\rm A}$ (km)	12
$X_{\rm br3}$ (m)	65	$\widehat{\gamma}_3$	1.985×10^{-3}	X_{R3} (km)	100	$X_{\rm B}~({\rm km})$	36
$X_{\rm br4}$ (m)	50	$\widehat{\gamma}_4$	3.474×10^{-3}	$\sigma_{ m lav}$	0.1	$X_{\rm C}~({\rm km})$	60
τ_{01} (s)	6.166	$2\beta_1$	3.769×10^{-3}	$\sigma_{2\mathrm{av}}$	[-0.4, -0.1]	$X_{\rm D}~({\rm km})$	84
τ_{02} (s)	7.194	$2\beta_2$	3.406×10^{-3}	$\sigma_{3\mathrm{av}}$	-0.1	$X_{\rm E}~({\rm km})$	108
τ_{03} (s)	8.632	$2\beta_3$	3.123×10^{-3}	X_{s1} (km)	5	$L_{\rm h}~({\rm km})^{\rm b}$	1
τ_{04} (s)	10.791	$2\beta_4$	4.221×10^{-3}	X_{s2} (km)	29	<i>l</i> (m)	8
Λ_1	3.032	$ ho_{*1}$	0.0769	X_{s3} (km)	53	ρ_m (veh/km)	124
Λ_2	2.780	ρ_{*2}	0.0909	X_{s4} (km)	77	u_{c2} (km/h)	18
Λ_3	2.513	ρ_{*3}	0.1096	X_{s5} (km)	101	l_0 (m)	100
Λ_4	2.244	$ ho_{*4}$	0.1379	$L_{\rm sp}$ (km)	1	<i>v</i> ₀ (m/s)	3.0303

^aThe critical densities $\rho_{*\hat{k}}$, and $\rho_{c2\hat{k}}$ for $\hat{k} = 1, 2, 3, 4$ are measured by ρ_m .

^bThe uphill or downhill length is $L_h = (X_I - X_I^u)/2$, I = A, B, C, D, E.

2. Viscoelastic traffic model

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For the simplicity of mathematical modeling of traffic flow on a composite ring road, we assume: (i) traffic flow satisfies a linear viscoelastic constitutive relation; (ii) on empty road, traffic sound speed is just equal to free flow speed. (iii) the road is composed of tunnel, horizontal, uphill and downhill segments, it is a ring type for the convenience of assigning boundary conditions in numerical tests, and has three ramp intersections. For the first assumption, the main reason is that in many high-order traffic models relaxation time that usually relates the elasticity and viscosity of fluids (Han [13]) has been adopted to define the driven force of vehicles, while vehicular motion may have a memory behavior for driver safety concern. The third assumption implies that we attempt to describe traffic flows by including more road segment conditions, such as tunnel speed limit.

The tunnel, horizontal, uphill and downhill segments are labeled by index \hat{k} , as shown in Fig. 1(a–b).

Defining the traffic density and speed by $\rho(x, t)$ and u(x, t) respectively so that the traffic flow rate can be written as $q(x, t) = \rho u$, assuming that the second critical traffic speed does not depend on the road segment condition, for instance $u_{c2} = 18 \text{ km/h}$ as shown in the 3rd-to-last line of the 4th column in Table 1, using a maximum relaxation order of 2 in viscoelastic traffic flow modeling (Ma, et al. [26]), the governing equations of the viscoelastic traffic model (VEM) have the following form

$$\begin{cases} \rho_t + q_x = \sigma q/l_0, \\ \rho(u_t + uu_x) = R. \end{cases}$$
(1)

where R satisfies the expression (Zhu and Yang [50]; Ma, et al. [26])

$$R + [\rho \gamma (R/\rho)_x]_x = (q_e - q)/\tau - p_x + [(2G\tau + \rho \nu_1)u_x]_x,$$
(2)

where R/ρ represents traffic flow acceleration, $\rho v_1 = 3\rho \gamma u_x$, $\nu (= 2G\tau/\rho)$ is traffic kinematic viscosity, $\gamma (=$ $(0.68\nu\tau)$ is traffic elasticity, here G is the modulus of vehicular fluid elasticity, τ is relaxation time of traffic flow, σ is the ramp parameter assigned by a random generator based on a Gaussian normal distribution, p_x is traffic pressure gradient, q_e is equilibrium traffic flow rate obtained by a fundamental diagram as given by Fig. 2.

The term $[(2G\tau + \rho v_1)u_x]_x$ on the right hand side of Eq. (2) reflects some properties of traffic self-organization. In the reality of traffic flow, ahead of a vehicle, a speed increase tends to produce a positive value of u_{xx} , and thus provides drivers' motivation for acceleration, as the density reduction ahead of the vehicle can be foreseen in congested traffic flows. If we assume traffic elasticity γ is a constant, then the kinematic viscosity ν is inversely proportional to relaxation time τ , indicating that ν approaches to zero for $\tau \to \infty$.

The ramp effects are described by using a ramp variable σ , its instantaneous value is based on random number generator with Gaussian normal distribution, as reported previously (Zhang, et al. [48]). It is noted that the coordinate x is road fitted rather than a horizontal line, as the road has included the uphill and downhill segments [$\hat{k} = 3$, 1, see in Fig. 1(b)].

The traffic equilibrium flow rate (q_e) depends on segmental condition, as shown in Fig. 2. Let the jam density be ρ_m , on the segment $\hat{k}(= \text{krd} + 2)$, the equilibrium flow $[q_e^{(\text{krd})}]$ can be written as

$$q_e^{(\mathrm{krd})} = \begin{cases} \rho v_{f\hat{k}}, & \text{for } \rho \le \rho_{*\hat{k}}, \\ -c_{\tau\hat{k}}\rho \ln(\rho/\rho_m), & \text{for } \rho_{*j} < \rho \le \rho_{c2\hat{k}}, \\ B_{\hat{k}}\rho\{1 - \mathrm{sech}[\Lambda_{\hat{k}}\ln(\rho/\rho_m)]\}, & \text{for } \rho_{c2\hat{k}} < \rho \le \rho_m. \end{cases}$$
(3)

At second critical density $\rho_{c2\hat{k}}$, the equilibrium speed $u_{c2\hat{k}}$ is called second critical speed, it is used to define a ratio $\Lambda_{\hat{k}} = c_{\tau j}/u_{c2\hat{k}}$, the parameter $B_{\hat{k}}$ is defined by

$$B_{\widehat{k}} = u_{c2\widehat{k}}/\{1 - \operatorname{sech}[\Lambda_{\widehat{k}}\ln(\rho_{c2\widehat{k}}/\rho_m)]\}.$$
(4)

It is noted that the fundamental diagrams in Fig. 2 labeled by the index krd(= -1, 0, 1, and 2) are obtained using the free flow speeds $v_{f\hat{k}}$ and braking distances $X_{br\hat{k}}$, as can be seen in Table 1. The subscript \hat{k} is used to distinguish variables that being relevant to the downhill ($\hat{k} = 1$), horizontal ($\hat{k} = 2$), uphill ($\hat{k} = 3$), and tunnel ($\hat{k} = 4$) segments respectively. The equilibrium flow-density relations on the four segments described by Eq. (3) are based on the assumption that the second critical point of traffic flow does exist.

On road segment \hat{k} , for the braking distances X_{brk} , the corresponding maximum permissible density $\rho_{*\hat{k}}$ at free flow speed $v_{f\hat{k}}$ is given by

$$\rho_{\ast\hat{k}} = \rho_m \exp(-v_{f\hat{k}}/c_{\tau\hat{k}}). \tag{5}$$

As maximum permissible density $(\rho_{*\hat{k}})$ itself implies that the distance between vehicles is not shorter than the braking distance $X_{br\hat{k}}$, using average vehicle length l, then $\rho_{*\hat{k}}$ has the form

$$\rho_{*\hat{k}} = \rho_m [1 + X_{\text{br}\hat{k}}/l]^{-1},\tag{6}$$

which is generally referred as the 1st critical density. Combining Eqs. (5) and (6), we have

$$c_{\tau \hat{k}} = v_{f \hat{k}} / \ln[1 + X_{\text{br}\hat{k}}/l]. \tag{7}$$

As the details of how to describe traffic pressure and traffic sound speed mathematically for vehicular flow on a ring road have been reported recently (Zhang, et al. [49]), here we just directly show the relevant expressions by using the road segment conditions distinguished by \hat{k} . By assuming that on empty road traffic sound speed is exactly equal to the free flow speed, using the parameter of traffic pressure obtained by postulating that the sound speed at the second critical point is exactly equal to the speed $c_{\tau\hat{k}}$, further defining

$$\begin{cases} K_{\hat{k}} = \{c_{\tau\hat{k}}[1 - \alpha\rho_{c2\hat{k}}/\rho_m]\}^2, \\ c_{*\hat{k}}^2 = K_{\hat{k}}/(1 - \alpha\rho_{*\hat{k}}/\rho_m)^2, \\ B_{*\hat{k}} = (v_{f\hat{k}}^2 - c_{*\hat{k}}^2)/\rho_{*\hat{k}}^4, \\ B_{0\hat{k}} = c_{*\hat{k}}^2\rho_{*\hat{k}} + B_{*\hat{k}}\rho_{*\hat{k}}^5/5 - K_{\hat{k}} \cdot \{\rho_{*\hat{k}}/[1 - \alpha(\rho_{*\hat{k}}/\rho_m)]\} \\ c_{0\hat{k}} = c_{\tau\hat{k}}^2, \end{cases}$$

the traffic pressure $p_{\widehat{k}}$ is

$$p_{\hat{k}} = \begin{cases} c_{*\hat{k}}^2 \rho + \frac{1}{5} B_{*\hat{k}} \cdot [\rho_{*\hat{k}}^5 + (\rho - \rho_{*\hat{k}})^5], & \text{for } \rho \le \rho_{*\hat{k}}, \\ K_{\hat{k}} \cdot \{\rho/[1 - \alpha(\rho/\rho_m)]\} + B_{0\hat{k}}, & \rho_{*\hat{k}} < \rho < \rho_m, \end{cases}$$
(8)

using the definition $c^2 = \partial p / \partial \rho$ under isentropic condition in classical mechanics, the form of corresponding sound speed $c_{\hat{k}}$ can be expressed by

$$c_{\hat{k}} = \begin{cases} \sqrt{c_{*\hat{k}}^2 + B_{*\hat{k}}(\rho - \rho_{*\hat{k}})^4}, & \text{for } \rho \le \rho_{*\hat{k}}, \\ K_{\hat{k}}^{1/2} / [1 - \alpha(\rho / \rho_m)], & \rho_{*\hat{k}} < \rho < \rho_m. \end{cases}$$
(9)

It is assumed that the length scale l_0 is just equal to the product of sound speed and relaxation time, i.e., $l_0 = c_{0\hat{k}}\tau_{0\hat{k}}$, the speed $c_{\tau\hat{k}}$ in the road segment \hat{k} is just equal to the sound speed $c_{0\hat{k}}$ at the second critical point $\rho_{c2\hat{k}}$ (Zhang, et al. [48]).

3. Numerical method

To solve the governing equations of VEM, essentially non-oscillatory and non-free-parameter dissipation difference scheme (ENN) developed by Zhang et al. [47] is adopted for predicting the numerical flux, and the 3rd order Runge–Kutta scheme that also maintaining the total variation diminishing property [35,36] is used for time marching in numerical tests. The ENN is the third order improvement of the non-oscillatory and non-free-parameter dissipation difference scheme (NND) (Shui [35]) that is well known in Aerodynamics, has been adopted to propose proper weighted difference scheme (Wang and Shen [42]).

Using definition $\partial p/\partial \rho = c^2$, taking $R_1 = R + p_x + \sigma q/l_0 \cdot u$ instead of R, the governing Eqs. (1) and (2) become

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} = \mathbf{S},\tag{10}$$

where $\mathbf{U} = (\rho, q)^T$, $\mathbf{F}(\mathbf{U}) = (q, q^2/\rho + p)^T$, and $\mathbf{S} = [\sigma q, R_1]^T$, with superscript 'T' representing vector transpose. The Jacobian matrix is

$$\mathbf{A} = \begin{pmatrix} \frac{\partial F_1}{\partial U_1} & \frac{\partial F_1}{\partial U_2} \\ \frac{\partial F_2}{\partial U_1} & \frac{\partial F_2}{\partial U_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -u^2 + c^2 & 2u \end{pmatrix}.$$
(11)

where $U_1 = \rho$, $U_2 = q$. It is easy to see that the eigenvalues of Matrix A may be expressed as $a_1 = u - c$, and $a_2 = u + c$.

Let the eigenvalues, the left and right eigenvectors be

$$a_k(\mathbf{U}), \quad \mathbf{I}_k(\mathbf{U}), \quad \mathbf{r}_k(\mathbf{U}), \quad k = 1, 2,$$

$$(12)$$

then the A(U) can be written as

$$\mathbf{A}(\mathbf{U}) = \mathbf{R}\mathbf{a}\mathbf{L}, \quad \mathbf{L} = \mathbf{R}^{-1},\tag{13}$$

where $\mathbf{a} = \text{diag}(a_1(\mathbf{U}), a_2(\mathbf{U}))$ is a diagonal matrix composed of eigenvalues; **R**, **L** are respectively the left and right characteristic matrix being composed of relevant eigenvectors,

$$\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2], \quad \mathbf{L} = \begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \end{bmatrix}.$$
(14)

Hence, the splitting form of F(U) can be written as

$$\mathbf{F}^{\pm}(\mathbf{U}) = \mathbf{A}^{\pm}(\mathbf{U})\mathbf{U}, \quad \mathbf{A}^{\pm}(\mathbf{U}) = \mathbf{R}\mathbf{a}^{\pm}\mathbf{L}, \tag{15}$$

where

$$\mathbf{a}^{\pm} = \operatorname{diag}(a_1^{\pm}(\mathbf{U}), a_2^{\pm}(\mathbf{U})),$$
$$a_k^{\pm}(\mathbf{U}) = \frac{1}{2}(a_k(\mathbf{U}) \pm |a_k(\mathbf{U})|), \quad k = 1, 2.$$

The essentially non-oscillatory and non-free-parameter dissipation difference scheme (ENN) has introduced a function $ms(c_1, c_2)$ in the form

$$ms(c_1, c_2) = \begin{cases} c_1, \text{ for } |c_1| < |c_2|, \\ c_2, \text{ for } |c_1| > |c_2|, \\ c_1, \text{ for } |c_1| = |c_2| \text{ and } |c_1 \cdot c_2 > 0|, \\ 0, \text{ for } |c_1| = |c_2| \text{ and } |c_1 \cdot c_2 \le 0|. \end{cases}$$
(16)

This function is symmetrical to its variables c_1 and c_2 . Then labeling the time step and spatial node size respectively by Δt and Δx , the difference scheme ENN discretizes $\partial \mathbf{F}(\mathbf{U})/\partial x$ by

$$\left. \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} \right|_{i} = \frac{\Delta t}{\Delta x} (\widehat{\mathbf{F}}_{i+1/2} - \widehat{\mathbf{F}}_{i-1/2}), \tag{17}$$

where

$$\widehat{\mathbf{F}}_{i+1/2} = \widehat{\mathbf{F}}_{i+1/2}^+ + \widehat{\mathbf{F}}_{i+1/2}^-.$$

By defining

 $\Delta_{i+1/2}\mathbf{F}^+ = \mathbf{F}^+(\mathbf{U}_{i+1}) - \mathbf{F}^+(\mathbf{U}_i), \quad \Delta_{i+1/2}\mathbf{F}^- = \mathbf{F}^-(\mathbf{U}_{i+1}) - \mathbf{F}^-(\mathbf{U}_i),$

the ENN gives the numerical flux as follows [37]

$$\widehat{\mathbf{F}}_{i+1/2}^{+} = \begin{cases} \widehat{\mathbf{F}}^{+}(\mathbf{U}_{i}) + \frac{1}{2} \operatorname{ms} \left(\Delta_{i+1/2} \mathbf{F}^{+}, \Delta_{i-1/2} \mathbf{F}^{+} \right) + \frac{1}{3} \operatorname{ms} \left(\Delta_{i+1/2} \mathbf{F}^{+} - \Delta_{i-1/2} \mathbf{F}^{+}, \Delta_{i-1/2} \mathbf{F}^{+} \right) \\ \Delta_{i-1/2} \mathbf{F}^{+} - \Delta_{i-3/2} \mathbf{F}^{+} \right), \quad \text{for} \quad |\Delta_{i+1/2} \mathbf{F}^{+}| > |\Delta_{i-1/2} \mathbf{F}^{+}|; \\ \widehat{\mathbf{F}}^{+}(\mathbf{U}_{i}) + \frac{1}{2} \operatorname{ms} \left(\Delta_{i+1/2} \mathbf{F}^{+}, \Delta_{i-1/2} \mathbf{F}^{+} \right) - \frac{1}{6} \operatorname{ms} \left(\Delta_{i+1/2} \mathbf{F}^{+} - \Delta_{i-1/2} \mathbf{F}^{+}, \Delta_{i+3/2} \mathbf{F}^{+} - \Delta_{i+1/2} \mathbf{F}^{+} \right), \quad \text{otherwise} . \end{cases}$$

$$(18)$$

$$\widehat{\mathbf{F}}_{i+1/2}^{-} = \begin{cases} \widehat{\mathbf{F}}^{-}(\mathbf{U}_{i+1}) - \frac{1}{2} \operatorname{ms} \left(\Delta_{i+1/2} \mathbf{F}^{-}, \Delta_{i+3/2} \mathbf{F}^{-} \right) + \frac{1}{3} \operatorname{ms} \left(\Delta_{i+3/2} \mathbf{F}^{-} - \Delta_{i+1/2} \mathbf{F}^{-}, \Delta_{i+1/2} \mathbf{F}^{-} \right) \\ \Delta_{i+5/2} \mathbf{F}^{-} - \Delta_{i+3/2} \mathbf{F}^{-} \right), \quad \text{for} \quad |\Delta_{i+1/2} \mathbf{F}^{-}| > |\Delta_{i-1/2} \mathbf{F}^{-}|; \\ \widehat{\mathbf{F}}^{-}(\mathbf{U}_{i+1}) - \frac{1}{2} \operatorname{ms} \left(\Delta_{i+1/2} \mathbf{F}^{-}, \Delta_{i-1/2} \mathbf{F}^{-} \right) - \frac{1}{6} \operatorname{ms} \left(\Delta_{i+1/2} \mathbf{F}^{-} - \Delta_{i-1/2} \mathbf{F}^{-}, \Delta_{i+3/2} \mathbf{F}^{-} - \Delta_{i+1/2} \mathbf{F}^{-} \right), \quad \text{otherwise} . \end{cases}$$

$$(19)$$

It is necessary to mention the numerical approach for the approximation of several, widely applied, macroscopic traffic flow models (Delis, et al. [7]), where the family of spatial discretizations includes a second-order monotone upstream-centered scheme for conservation laws (MUSCL) scheme and a fifth-order weighted essentially non-oscillatory (WENO) scheme, and a detailed formulation of the scheme, while emphasis is given on the WENO scheme and its performance for solving the different traffic models. However, considering the ENN scheme that having a third-order accuracy in spatial discretization is essentially non oscillatory and non-free-parameter, and easy for building simulation platform, it is adopted preferentially in this study.

Further defining

$$\mathscr{L}(\mathbf{U}) = -\frac{\partial \mathbf{F}(\mathbf{U})}{\partial \mathbf{x}} + \mathbf{S}, \qquad (20)$$

to seek the numerical solution of

$$\frac{\partial \mathbf{U}}{\partial t} = \mathscr{L}(\mathbf{U}), \tag{21}$$

the 3rd order Runge-Kutta scheme [35,36] has the form

$$\begin{cases} \mathbf{U}_{i}^{(1)} = \mathbf{U}_{i}^{n} + \Delta t \mathscr{L}(\mathbf{U}^{n}), \\ \mathbf{U}_{i}^{(2)} = (3\mathbf{U}_{i}^{n} + \mathbf{U}_{i}^{(1)})/4 + \Delta t \mathscr{L}(\mathbf{U}^{(1)})/4, \\ \mathbf{U}_{i}^{n+1} = (\mathbf{U}_{i}^{n} + 2\mathbf{U}_{i}^{(2)})/3 + 2\Delta t \mathscr{L}(\mathbf{U}^{(2)})/3, \end{cases}$$
(22)

where the superscript n denotes the time level.

Labeling the step ratio by $\omega = \Delta t / \Delta x$, the Courant–Friedrichs–Lewy (CFL) condition of TVD is satisfied by

$$\omega = C_{\rm FL}/\max|a_{k,i}|, \quad k = 1, 2;$$

$$i = 0, 1, 2, \dots, I_{\rm max} - 1,$$
(23)

where $a_{k,i}$ represents the *k*th eigenvalue for **A** at x_i , I_{max} is the maximum number of node. In the numerical tests, we assumed that the Courant number $C_{\text{FL}} = 0.7$ [37].

Note that as reported by Aw and Rascle [2], negative speeds can possibly occur in the solutions, if pressure gradient p_x/ρ is used to describe the acceleration R/ρ . This promotes the development of anisotropic models (Rascle [33]; Xu, et al. [44]).

4. Method of travel time prediction

Numerical tests based on the VEM can output the node traffic speed $u(x_i, t^n)$. Following the previous work (Zhang, et al. [48]), using a pre-assigned time period Δ_0 , the local average speed $\overline{u}_i(t)$ at the node x_i is

$$\overline{u}_i(t) = \frac{1}{\Delta_0} \int_{t-\Delta_0}^t u_i(\xi) d\xi,$$
(24)

the node number i depends on the segment index krd. As the road total length L is expressed by

$$L = \sum_{
m krd} L_{
m krd},$$

the instantaneous travel time $T_t(t)$ through the road is

$$T_{t}(t) = \sum_{krd=-1}^{2} T_{t}^{(krd)},$$
(25)

$$T_t^{(\rm krd)}(t) = \sum_i \left[\Delta x_i / \overline{u}_i(t) \right]^{(\rm krd)}.$$
(26)

Note that $\Delta x_i^{(\text{krd})} = l_0$, and Δ_0 is assumed to be 7.5 min. As the propagation and interaction of traffic wave causes evident time dependent properties of traffic speed and density, we have to predict the mean travel time by time averaging of $T_t(t)$, which allows us to calculate the relevant root means square. The mean travel time T_{tav} and $T_{\text{tav}}^{(\text{krd})}$ can be expressed by

$$\begin{cases} T_{\text{tav}} = \frac{1}{t_{\text{end}} - t_{\text{st}}} \int_{t_{\text{st}}}^{t_{\text{end}}} T_{\text{t}}(\xi) d\xi ,\\ T_{\text{tav}}^{(\text{krd})} = \frac{1}{t_{\text{end}} - t_{\text{st}}} \int_{t_{\text{st}}}^{t_{\text{end}}} T_{\text{t}}^{(\text{krd})}(\xi) d\xi . \end{cases}$$
(27)

with the root mean square (rms) values given by

$$\begin{cases} [T_{t}']^{2} = \frac{1}{t_{\text{end}} - t_{\text{st}}} \int_{t_{\text{st}}}^{t_{\text{end}}} [T_{t}(\xi) - T_{\text{tav}}]^{2} d\xi , \\ [T_{t}^{(\text{krd})'}]^{2} = \frac{1}{t_{\text{end}} - t_{\text{st}}} \int_{t_{\text{st}}}^{t_{\text{end}}} [T_{t}^{(\text{krd})}(\xi) - T_{\text{tav}}^{(\text{krd})}]^{2} d\xi . \end{cases}$$
(28)

Where t_{st} is the time to start the simulation, in general $t_{st} = 0$, with t_{end} representing the time to end the simulation. Making dimensionless by time scale $t_2 (= L/v_{f2})$ gives

$$\begin{cases} \sigma_{t} = T_{t}/t_{2}, & \sigma_{t}' = T_{t}'/t_{2}, \\ \sigma_{tav} = T_{tav}/t_{2}, & \sigma_{t}' = T_{t}'/t_{2}, \\ \sigma_{t}^{(krd)} = T_{t}^{(krd)}/t_{2}, & \sigma_{t}^{(krd)'} = T_{t}^{(krd)'}/t_{2}. \end{cases}$$
(29)

where σ_t is the instantaneous travel time through the road, with the mean travel time and rms denoted respectively by σ_{tav} and σ'_t ; while σ'_t is the instantaneous travel time through the segment labeled by superscript (krd), corresponding to the mean travel time $[\sigma'_{tav}]$ and rms $[\sigma'_t]$. The time scale t_2 represents the travel time through the road in case at the free flow speed v_{f_2} .

5. Model for validation

On the other hand, to validate the VEM, the Navier–Stokes like traffic model (Zhang [46]) is extended and named EZM by assuming that traffic pressure and sound speed have the same as adopted in the VEM. Further assuming that there is constant traffic elasticity, then we can describe the EZM in the following form

$$\begin{cases} \rho_t + q_x = \sigma q/l_0, \\ q_t + \{q^2/\rho + p + [(2\beta c_0) \cdot (c/c_0)](q/\rho)\}_x = R, \end{cases}$$
(30)



Fig. 3. Sound speed ratio c/c_0 (a) and traffic pressure p (b) versus density ρ , where ρ is measured by jam density ρ_m , where p is measured by $\rho_m v_{f\hat{k}}^2$, $\hat{k} = 2$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

with

$$R = (q_e - q)/\tau + [(2\beta c_0) \cdot (c/c_0)\rho](q/\rho)_{xx} + (q/\rho)[(2\beta c_0) \cdot (c/c_0)\rho]_x,$$
(31)

where traffic equilibrium flow q_e , traffic pressure p, sound speed c, and relaxation time τ are obtained by the same expressions as used in the VEM. c_0 represents the sound speed at the second critical density. It is also assumed that the product of sound speed and relaxation time is just equal to the length scale l_0 . β is the dimensionless traffic kinematic viscosity

$$\beta = \frac{\nu}{2\tau_0 c_0^2}.\tag{32}$$

In the EZM, the discretization of $[(2\beta c_0) \cdot (c/c_0)\rho]_x$ is implemented by a second order upwind scheme [39]. The extended traffic model EZM is adopted for validating the reliability and feasibility of the VEM.

The traffic density dependence of traffic sound speed ratio c/c_0 and traffic pressure p on the four segments labeled by krd are shown respectively in Fig. 3(a–b). The blue, black, green and purple solid curves represent the sound speed ratio c/c_0 of traffic flows respectively on the downhill, horizontal, uphill and tunnel segments [see Fig. 3(a)]. It is see that the c/c_0 and p have the largest value in tunnel and the lowest on downhill segment for a given density, with the c/c_0 and p curves on the uphill segment just below that in the tunnel, and the c/c_0 and p curves on horizontal segment just above that on the downhill segment.

5.1. Parameters and conditions

To predict travel time of vehicles through the composite ring road [Fig. 1(a–b)], numerical tests of ring road traffic flows are conducted by the VEM and EZM, with the macroscopic fundamental diagrams shown in Fig. 2. In Table 1, it can be seen the simulation parameters of traffic flow on the ring road, such as the free flow speeds $v_{f\hat{k}}$, braking distances $X_{br\hat{k}}$, relaxation times $\tau_{0\hat{k}}$. In particular, the second critical speed u_{c2} is assumed to be 18 km/h, and mean vehicle length l is 8 m, hence the jam density ρ_m is set to 124 veh/km. The five initial jams artificially assumed at X_I , (I = A, B, C, D, E), the uphill and downhill length is identical and denoted by L_h , and tunnel length is labeled by L_{sp} . The normalized elasticity in the VEM $\hat{\gamma} \left[= \frac{0.68 \times 2G(\tau_0)}{l_{0}q_0} \right]$ is 2.178×10⁻³, corresponding to $2\beta = 3.023 \times 10^{-3}$ in the EZM.

The initial density is assumed to be

$$\rho(x,0) = \begin{cases} 1, \text{ for } x = \in [x_I - 1/2, x_I + 1/2], \\ \rho_0, \text{ otherwise.} \end{cases}$$
(33)

with $q(x, 0) = q_e[\rho(x, 0)]$. The initial density ρ_0 plays a significant role in affecting the propagation of the traffic jams artificially assumed to be located at X_I , (I = A, B, C, D, E) [see, in Fig. 1], in addition to the traffic sensitivities to the visco-elasticity γ , as reported (Smirnova, et al. [38]).

For the traffic flows on the composite ring road, the boundary condition is

$$p(x,t) = \rho(L+x,t), \quad q(x,t) = q(L+x,t)$$
(34)

where L is the total length of the ring road.

h

Table 2 ρ_0 – dependence of σ_{tav} and $\sigma_{tav}^{(krd)}$ (krd = 1, -1, 2) in case without ramp effects for $L_{sp} = 1$ km.

$ ho_0$	VEM				EZM			
	$\sigma_{\rm tav}$	$\sigma_{ m tav}^{(1)}$	$\sigma_{\rm tav}^{(-1)}$	$\sigma_{ m tav}^{(2)}$	$\sigma_{\rm tav}$	$\sigma_{ m tav}^{(1)}$	$\sigma_{\rm tav}^{(-1)}$	$\sigma^{(2)}_{ m tav}$
0.1	1.1028	0.0547	0.0453	0.0569	1.1061	0.0547	0.0452	0.0574
0.15	1.3244	0.0679	0.0537	0.0759	1.3256	0.0683	0.0539	0.0760
0.17	1.4196	0.0730	0.0571	0.0892	1.4211	0.0734	0.0573	0.0893
0.185	1.4889	0.0730	0.0588	0.1103	1.4903	0.0736	0.0590	0.1097
0.2	1.5934	0.0672	0.0570	0.1724	1.5906	0.0681	0.0575	0.1700
0.368	2.8891	0.0814	0.0614	0.1755	2.9052	0.0778	0.0609	0.1829
0.625	5.1753	0.2299	0.2151	0.2034	5.1752	0.2299	0.2151	0.2036

Table 3

 ρ_0 – dependence of σ_{tav} and $\sigma_{tav}^{(krd)}$ (krd = 1, -1, 2) in case without ramp effects for $L_{sp} = 0.5$ km.

$ ho_0$	VEM				EZM			
	$\sigma_{ m tav}$	$\sigma_{ m tav}^{(1)}$	$\sigma_{\rm tav}^{(-1)}$	$\sigma^{(2)}_{ m tav}$	$\sigma_{\rm tav}$	$\sigma_{ m tav}^{(1)}$	$\sigma_{ m tav}^{(-1)}$	$\sigma^{(2)}_{ m tav}$
0.1	1.0987	0.0552	0.0455	0.0267	1.0999	0.0552	0.0454	0.0269
0.15	1.3171	0.0681	0.0541	0.0355	1.3172	0.0681	0.0540	0.0355
0.17	1.4088	0.0732	0.0576	0.0433	1.4106	0.0735	0.0576	0.0432
0.185	1.4853	0.0684	0.0578	0.0755	1.4853	0.0698	0.0582	0.0727
0.2	1.5804	0.0682	0.0579	0.0925	1.5783	0.0681	0.0579	0.0925
0.368	2.8636	0.0860	0.0624	0.0938	2.8642	0.0828	0.0624	0.0939
0.625	5.1523	0.2287	0.2135	0.1057	5.1514	0.2287	0.2134	0.1058

5.2. Comparison of results

For $L_{sp} = 1$ km, the initial density ρ_0 -dependence of travel times σ_{tav} , and $\sigma_{tav}^{(krd)}$ (krd = 1, -1, 2) in case without ramp effects is shown in Table 2, where the left part including the 2 to 5 columns is predicted on the basis of VEM, with the right part including the 6 to 9 columns based on the EZM. It is seen that in general the mean travel time (σ_{tav}) grows monotonically with the increase of initial density ρ_o , and so are the mean travel time through the five separated uphills, thereafter called the uphill mean travel time [$\sigma_{tav}^{(1)}$] and the mean travel time through the five separated downhills, thereafter called the downhill mean travel time [$\sigma_{tav}^{(-1)}$]. When ρ_0 is below 0.2, the mean travel time through the five separated tunnels, thereafter called the tunnel mean travel time [$\sigma_{tav}^{(2)}$] holds an increasing trend with initial density ρ_0 , but for $\rho_0 \ge 0.2$, $\sigma_{tav}^{(2)}$ keeps a value around 0.1868 with a maximum deviation of 0.0168, indicating that the tunnel inlet has originated traffic shocks propagating backward rather than in the tunnel and thus there exists a traffic congestion region generated by the tunnel bottleneck effect. Comparing the left and right parts, it can be seen that the mean travel time σ_{tav} by the VEM is much the same as that predicted by the EZM. For instance, when ρ_0 increases from 0.1 to 0.625, the maximum deviation is less than 0.0065 that occurring at $\rho_0 = 0.625$.

In Table 3, the ρ_0 - dependence of travel times σ_{tav} , and $\sigma_{tav}^{(krd)}$ (krd = 1, -1, 2) for $L_{sp} = 0.5$ km are shown, again for model validation. For $\rho_0 \in [0.1, 0.625]$, in comparison with the mean travel time σ_{tav} predicted by the EZM, the mean travel time σ_{tav} by the VEM shows a maximum deviation of 0.0023 that occurring at $\rho_0 = 0.125$. As the tunnel length is halved, for $\rho_0 \ge 0.2$, $\sigma_{tav}^{(2)}$ holds a value around 0.0934 with a maximum deviation of 0.0114 that occurring at $\rho_0 = 0.625$. The uphill mean travel time $\sigma_{tav}^{(1)}$ is slightly larger than the downhill mean travel time $\sigma_{tav}^{(-1)}$.

However, as shown in Table 4, for $L_{\rm sp} = 0.1$ km, when ρ_0 is in the range from 0.25 to 0.5, $\sigma_{\rm tav}^{(1)}$ can have above doubled value than $\sigma_{\rm tav}^{(-1)}$, indicating that for the case of $L_{\rm sp} = 0.1$ km, the node traffic speed on uphill segment must be much lower than on the downhill segment. For $\rho_0 = 0.625$, comparing the values of $\sigma_{\rm tav}$ in the second and sixth columns of Table 4, it is seen that the maximum deviation is 0.0297. Furthermore, the tunnel mean travel time $\sigma_{\rm tav}^{(2)}$ has a value of about 0.0186 for $\rho_0 > 0.2$ with a maximum deviation of 0.0076.

The main reason of why there is the ρ_0 -dependence of travel times in Table 2 can be sought from traffic flow patterns given by spatiotemporal evolution of density, as shown in Fig. 4(a-c), where patterns in the left part

Table 4

ρ_0 – dependence of σ_{tay} and $\sigma_{tay}^{(Ray)}$ (krd = 1, -1, 2) in case without ramp effects for L_{sp} =

$ ho_0$	VEM				EZM			
	$\sigma_{\rm tav}$	$\sigma_{ m tav}^{(1)}$	$\sigma_{\rm tav}^{(-1)}$	$\sigma^{(2)}_{ m tav}$	$\sigma_{\rm tav}$	$\sigma_{ m tav}^{(1)}$	$\sigma_{\rm tav}^{(-1)}$	$\sigma^{(2)}_{ m tav}$
0.1	1.0923	0.0552	0.0454	0.0050	1.0925	0.0549	0.0454	0.0049
0.15	1.3056	0.0673	0.0540	0.0062	1.3060	0.0671	0.0540	0.0062
0.17	1.4022	0.0742	0.0580	0.0073	1.4040	0.0744	0.0580	0.0073
0.185	1.4758	0.0802	0.0610	0.0089	1.4745	0.0801	0.0611	0.0089
0.2	1.5484	0.0849	0.0636	0.0110	1.5488	0.0850	0.0636	0.0112
0.368	2.7244	0.1813	0.0706	0.0184	2.7114	0.1659	0.0703	0.0183
0.625	5.1379	0.2282	0.2122	0.0223	5.1367	0.2280	0.2122	0.0222



Fig. 4. Spatiotemporal evolutions of traffic density on the ring road in the case of no ramp effects for $L_{sp} = 1 \text{ km}$, (a) $\rho_0 = 0.15$; (b) $\rho_0 = 0.2$; (c) $\rho_0 = 0.25$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

obtained by the VEM are much the same as those in the right obtained by the EZM. The flow patterns on the two parts are extremely dependent on the initial density ρ_0 . In Fig. 4(a), for $\rho_0 = 0.15$, showing by the cyan-colored oblique line, the traffic trajectories of the initial jams at X_I , (I = A, B, C, D, E) have positive slope in initial period, these jams propagate forward and arrive at the downstream tunnel inlets x_{sk} (k = 1, 2, ..., 5) at $t \approx 0.55$ h. In the initial period, tunnel inlet has generated a traffic jam also propagating forward, its trajectory has to change slope due to the interaction with traffic rarefaction waves. The tunnel inlets become the slope change points of trajectories of all forward moving traffic jams.

For $\rho_0 = 0.2$, as shown in Fig. 4(b), any tunnel inlet becomes an origin of traffic congestion region, where traffic density is above 0.6. There exists the interaction between the forward propagating initial jam and the tunnel



Fig. 5. Spatiotemporal evolutions of traffic density on the ring road in the case of no ramp effects for $L_{sp} = 0.1$ km, (a) $\rho_0 = 0.15$; (b) $\rho_0 = 0.2$; (c) $\rho_0 = 0.25$.

inlet induced backward propagating traffic shock wave, leading to self-generated traffic jams downstream the tunnel outlet. In Fig. 4(c), the traffic flow pattern for $\rho_0 = 0.25$ is shown, it is seen that the traffic congestion region originated from the tunnel inlet has a larger width, implying that the propagation speed of the tunnel-generated traffic shock is larger.

For the tunnel length $L_{sp} = 0.1$ km, the traffic flow patterns obtained by the VEM and EZM can be seen in Fig. 5(a)(b)(c). The highly pattern similarity observed by comparing the left and right parts do provide evidence to explain why the mean travel times predicted by the VEM is almost identical to that predicted by the EZM. Particularly, Fig. 5(c) transfers information to explain why the uphill mean travel time for $L_{sp} = 0.1$ km is above doubled than the downhill mean travel time for $\rho_0 \in [0.2, 0.5]$, as on the uphill segments at x = 11, 35, 59, 83, and 107 km, the traffic density is generally higher than that on the downhill segments at x = 12, 36, 60, 84, and 108 km, implying that the node traffic speeds on uphill segments are lower, according to Eqs. (26)–(27), the uphill mean travel time $\sigma_{tav}^{(1)}$ should be larger.

For $L_{sp} = 1$ km, the temporal evolutions of traffic density and acceleration at x = 60 km in case without ramp effects for $\rho_0 = 0.15$ and 0.25 are shown in Fig. 6(a–b), where R/ρ has the unit of 1.5 m/s^2 (the permitted maximum acceleration). The green solid curve obtained by the VEM shows that the evolution at x = 60 km has an overall consistency with the black-dashed curve obtained by the EZM. Any synchronous negative drop of acceleration (R/ρ) corresponds to a positive peak of density (ρ). At x = 60 km, the evolutions of ρ and R/ρ depend on initial density ρ_0 . The absolute value of traffic flow acceleration in the time period $t \in (0, 4$ h) is generally less than 0.5.

Temporal evolutions of traffic density and acceleration at x = 60 km for $L_{sp} = 0.1 \text{ km}$ in the cases of $\rho_0 = 0.15$ and 0.368 are given by Fig. 7(a–b), where R/ρ is normalized by the permitted maximum acceleration 1.5 m/s². Overally, the green solid curves predicted by the VEM are consistent with the black solid curves by the EZM. Corresponding to a positive peak of density (ρ), there is a synchronous negative drop of R/ρ , and vice versa.



Fig. 6. Temporal evolutions of traffic speed and density at x = 60 km for $L_{sp} = 1$ km, (a) $\rho_0 = 0.15$; (b) $\rho_0 = 0.25$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

At x = 60 km, the evolutions of ρ and R/ρ depend on initial density ρ_0 . The absolute value of traffic flow acceleration in the time period $t \in (0, 4 \text{ h})$ is generally less than 0.5, suggesting that the predicted acceleration is reasonable.

Consequently, we can conclude that the numerical results based on the VEM certainly have some reliability and reasonability, suggesting that traffic pressure and sound speed expressions have application potentials.

6. Results and discussion

6.1. Traffic flow patterns

To reflect ramp effects on traffic flow pattern, in the numerical simulation, the ramp parameter σ is generated by a random number generator with Gaussian normal distribution, for which the relevant root mean square σ'_k is assumed to be 0.005, for k = 1, 2, 3, while the relevant mean value of σ_{kav} is assigned as shown in Table 1. In the case with ramp effects, σ_{1av} and σ_{3av} are fixed respectively at 0.1 and -0.1, with σ_{2av} chosen as -0.1, -0.2, -0.3, or -0.4. Numerical tests based on the VEM permit obtaining the ring road traffic flow patterns for $\rho_0 = 0.15$ and 0.2, that are shown respectively in the left and right parts of Fig. 7(a-d).

In the left part of Fig. 7(a), due to the effect of on ramp flow at intersection $X_{R1} = 20$ km, it can be seen that traffic shocks occur at the downstream tunnel inlets X_{s2} and X_{s3} with certain time period of existence about 1 h, then disappear as a result of the influences of the off ramp flows at intersections $X_{R2} = 60$ km and $X_{R3} = 100$ km as well as the use of periodic boundary condition. As seen in the right part of Fig. 7, traffic shocks appear at the tunnel inlets X_{s1} , X_{s2} , X_{s3} and X_{s4} . The time period of existence of traffic shocks originated at the tunnel inlet X_{s3} is longer than that in the left part, suggesting that the traffic flow pattern is not only affected by the ramp flow significantly but also extremely dependent on the initial traffic density.

6.2. Instantaneous distributions

To show the effect of road segment condition on the traffic flow pattern more elaborately, the instantaneous distributions of density and speed for $L_{sp} = 1$ km in the case of $\rho_0 = 0.2$ are given respectively by Figs. 8(a–b) and 9(a–b), where coarse blue-solid curves are labeled by $\sigma_{1av} = \sigma_{2av} = \sigma_{3av} = 0$, green-dashed curves are labeled by $\sigma_{2av} = -0.1$, $\sigma_{1av} = -\sigma_{3av} = 0.1$, dashdotted purple curves are labeled by $\sigma_{2av} = -0.4$, $\sigma_{1av} = -\sigma_{3av} = 0.1$, the dashdotted black vertical lines are used to show the downhill end positions by X_I , (I = A, B, C, D, E). On and off ramp flows can certainly change the distributions of density and speed. In Fig. 8(a–b), it can be seen that each tunnel inlet becomes a starting point of road congestion region.



Fig. 7. Spatiotemporal evolutions of traffic density on the ring road in the case of no ramp effects (a) $\sigma_{2av} = -0.1$; (b) $\sigma_{2av} = -0.2$, (c) $\sigma_{2av} = -0.3$, (d) $\sigma_{2av} = -0.4$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

6.3. Comparison with measured data

To show the reliability of the VEM, the predicted traffic speed (*u*) and equilibrium speed (u_e) at x = 60 km for $\rho_0 = 0.15$, 0.368 are plotted as a function of traffic density as shown in Fig. 10(a–b), where the observation data labeled by symbol '+' are obtained from McShane, Roess and Prassas [27]. The instantaneous speed *u* at the downhill midpoint x = 59.5 km for $\rho_0 = 0.15$, 0.3 is plotted as a function of the local flow *q* in Fig. 10(c), where the field observation data extracted from Ref. [29] are also labeled by symbol '+'.

In Fig. 11(a), it is seen that for $\rho_0 = 0.15$, the initial traffic flow is generally unsaturated, as there are the five initial jams, there are over-saturated points (ρ, u) located in the saturated and over saturated flow range $\rho > 0.368$, but the point number are far less than that in the unsaturated range $\rho < 0.368$. Almost all points of traffic flow state (ρ, u) fall within the range having scattered points of observation data, the predicted speed has taken some value completely in the observation data range, indicating that the speed predicted by the VEM is reliable. From Fig. 10(b), by comparing the left and right parts for $\rho_0 = 0.368$, it can be seen that off-ramp flow at intersection X_{R2} can affect the instantaneous $u - \rho$ relationship at x = 60 km significantly.



Fig. 8. Distribution of traffic density ρ on the ring road for $L_{sp} = 1 \text{ km}$, $\rho_0 = 0.2 \text{ at}$ (a) t = 0.25 h, (b) 0.5 h.



Fig. 9. Distribution of traffic speed u on the ring road for $L_{sp} = 1 \text{ km}$, $\rho_0 = 0.2$ at (a) t = 0.25 h, (b) 0.5 h.

In Fig. 11(c), the data for median lane were recorded on December 23 of 2005, time-of-day tags from 6:30 to 7:10 h, at the site of the Tomei expressway (near Tokyo) instrumented with a series of eleven video cameras and two sets of loop detectors at kilo-post (KP) 21.5 [29]. It can be seen that for the predicted states (q, u) for $\rho_0 = 0.15$, and 0.3 by the VEM at the downhill midpoint x = 59.5 km, many traffic state points (q, u) are located in the zone having scattered field observation points.

Fig. 11 shows that the traffic points (ρ, u) at x = 60 km and the traffic states (q, u) at the downhill midpoint x = 59.5 km predicted by the VEM usually agree well with the observation data. The comparison shows that the VEM is reliable and has some application potential.

6.4. Travel time

Being different from the work reported by Chang and Mahmassani [4], who have examined two heuristic rules proposed for describing urban commuters' predictions of travel time as well as the adjustments of departure time in response to unacceptable arrivals in their daily commute under limited information, here we discuss the road segment condition effects on the travel time through the composite ring road that are estimated with the method given by Section 4.



Fig. 10. Comparison of traffic speed with existing measured data for $\sigma_{2av} = -0.1$, -0.4 in the case of $L_h = L_{sp} = 1$ km, (a) $\rho_0 = 0.15$; (b) $\rho_0 = 0.368$ at x = 60 km; and (c) Comparison of the instantaneous *u-q* relationship at the downhill midpoint x = 59.5 km with field observation data extracted from Ref. (Patire and Cassidy [29]). It is noted that $\sigma_{1av} = -\sigma_{3av} = 0.1$, the observation data used in parts (a) and (b) are obtained from Ref. (McShane, Roess and Prassas [27]), and the jam density for normalization is assumed to be 200 veh/mile; For the convenience of showing the field data from Ref. [29], the speed and flow rate for normalization are respectively assumed to be $130 \text{ km/h} [v_{fe} = (v_{f1} + v_{f2})/2]$, and $1465 \text{ vph} [\approx v_{fe} \times (\rho_m \rho_{*2})]$.



Fig. 11. Travel time σ_t versus time on the ring road in the case of no ramp effects for (a) $L_{sp} = 1 \text{ km}$, (b) $L_{sp} = 0.5 \text{ km}$, (c) $L_{sp} = 0.1 \text{ km}$.



Fig. 12. Density dependence of travel time σ_{tav} (a), its rms value σ'_t (b), travel time $\sigma^{(1)}_{av}$ & $\sigma^{(-1)}_{av}$ (c), and $\sigma^{(2)}_{av}$ (d) in the case of no ramp effects at different tunnel length.

6.4.1. Without ramp effects, $\sigma_{kav} = 0$

Affected only by road segment condition in case without ramp effects, when initial density varies from 0.1 to 0.625, the mean travel time σ_{tav} , $\sigma_{tav}^{(krd)}$ (krd = 1, -1, 2) predicted by Eqs. (27) and (29) for the specific tunnel length $L_{sp} = 1$ km and 0.1 km are predicted and shown in Tables 2 and 3 respectively. As discussed in Section 5, the mean travel time predicted by the VEM is much the same as that predicted by the EZM.

For the four cases of $\rho_0 = 0.2$, 025, 0.368, 0.5, and 0.6, the temporal evolutions of travel time σ_t are shown in Fig. 11(a–c). There is a similar evolution trend: the travel time predicted by Eqs. (25)–(26) increases with time in an initial period of about 1 h, after which the travel time gradually arrives at a steady value. The initial growing magnitude of travel time depends on the initial traffic density ρ_0 , with the maximum occurs near $\rho_0 = 0.368$.

The mean travel time σ_{tav} estimated by Eq. (d.7a) is shown in Fig. 12(a). It increases with initial density ρ_0 monotonically. Shortening the tunnel length can slightly increase the mean travel time that occurring when initial density ρ_0 is above 0.25, otherwise for traffic flow far from saturation $\rho_0 < 0.25$, the effect of decreasing the tunnel length is small and almost negligible.

The variation of the root mean square (rms) values of travel time σ'_t with initial density is shown in Fig. 12(b). It is seen that for the case of $L_{sp} = 0.5$ km, for unsaturated traffic flows $\rho_0 < -.368$, the σ'_t is almost the same as that estimated for the $L_{sp} = 1$ km, when ρ_0 ranges from 0.368 to 0.6, there is a smaller value of σ'_t that existing when the tunnel length L_{sp} is shortened. But for the case of $L_{sp} = 0.1$ km, the σ'_t holds the lowest in the initial density range $\rho_0 \in [0.2, 0.6]$.

In Fig. 12(c), the ρ_0 -dependence of $\sigma_{tav}^{(1)}$ and $\sigma_{tav}^{(-1)}$ can be seen, where for $L_{sp} = 0.1$ km, the solid red line with delta symbols denotes the uphill mean travel time ($\sigma_{tav}^{(1)}$), with the solid black line with gradient symbols representing the downhill mean travel time ($\sigma_{tav}^{(-1)}$); otherwise for $L_{sp} = 1$, 0.5 km, the delta and gradient symbols not linked by solid line are the relevant values of $\sigma_{tav}^{(1)}$ and $\sigma_{tav}^{(1)}$ respectively. Particularly, for $\rho_0 \in [0.2, 0.5]$, the uphill mean travel time $\sigma_{tav}^{(1)}$ for $L_{sp} = 0.1$ km is much larger than that for other cases with larger values of L_{sp} , implying that there is a quite different flow pattern on uphill segment that occurring when the tunnel length L_{sp} changes from 0.5 km to 0.1 km. Indeed, traffic flow pattern in Fig. 5(c) is quite different from that demonstrated in Fig. 4(c). In general, the



Fig. 13. Travel time T_t versus time on the ring road in case with ramp effects for $L_{sp} = 1$ km, (a) $\rho_0 = 0.2$, 0.25, and 0.368, (b) $\rho_0 = 0.45$, and 0.6.

downhill mean travel time $\sigma_{tav}^{(-1)}$ is smaller than the uphill mean travel time $\sigma_{tav}^{(1)}$, with the time difference depending

on the initial density of traffic flow and the tunnel length. The ρ_0 -dependence of the tunnel mean travel time $\sigma_{tav}^{(2)}$ is given by Fig. 12(d). $\sigma_{tav}^{(2)}$ grows with initial density ρ_0 if $\rho_0 < 0.2$. Otherwise, the $\sigma_{tav}^{(2)}$ approaches to a constant value and shows a time plateau that depending on the tunnel length L_{sp} . The longer the tunnel length, the higher is the height of time plateau.

6.4.2. With ramp effects, $\sigma_{kav} \neq 0$

To show the effects of road segment condition on travel time in case with ramp effects $\sigma_{kav} \neq 0$ for $L_{sp} = 1$ km, it was assumed that $\sigma_{1av} = 0.1$ and $\sigma_{3av} = -0.1$, $\sigma_{2av} = -0.1$, -0.2, -0.3, and -0.4, the temporal evolutions of σ_t predicted by the VEM for $\rho_0 = 0.2$, 0.368 and 0.5, 0.6 are shown respectively in Fig. 13(a) and (b). Different from Fig. 11(a), as can be seen for the case of $\rho_0 = 0.368$, the travel time σ_t grows with the increase of time initially, but starts to drop almost linearly after arriving at a peak, with the dropping rate being dependent on σ_{2av} . The smaller the value of σ_{2av} , the larger is the dropping rate of travel time.

When σ_{2av} is used to distinguish the numerical test for the prediction of the mean travel time σ_{tav} , the mean values of σ_{tav} through averaging by case number (= 5) can be obtained for a series values of ρ_0 , which have been adopted in the regression analysis after logarithmic processing (Wang, et al. [41]) to obtain the fitted mean travel time $\sigma_{\rm tf}$ in the form

$$\sigma_{\rm tf} = A\rho_0^m + b,\tag{35}$$

where A = 8.9686, m = 1.6260, b = 0.8424, ρ_0 varies from 0.1 to 0.625. Note that the regression of $\log_{10}A$ and m has a linear correlation coefficient of 0.9985, with a residual standard deviation of about 2.44%. The expression is shown by the fuchsia-colored solid curve in Fig. 14(a), which shows that the mean travel time decreases with the decreasing σ_{2av} , indicating that the ramp diversion at X_{R2} can shorten the mean travel time. For over-saturated traffic flow on the ring road, it has a value around 0.1 for the rms of travel time that depending on the off-ramp flow influence.

The ramp effects on the rms of travel time σ'_t can be seen in Fig. 14(b). If $\sigma_{2av} = -0.1$, σ'_t is usually below 0.1, as shown by the red solid curve with circles filled by yellow color. For $\rho_0 \in [0.2, 0.5]$, σ'_t takes a value in the range from 0.1 to 0.3. Otherwise, σ'_t has a value in the range from 0.17 to 0.45, depending on the choice of σ_{2av} .

Undoubtedly, σ'_t is determined by the traffic flow pattern on the composite ring road. Comparing the ρ_0 - dependence of $\sigma^{(1)}_{tav}$ and $\sigma^{(-1)}_{tav}$ shown in Fig. 14(c) with that shown in Fig. 12(c), it is seen that for unsaturated traffic flow $\rho_0 < 0.368$, the ramp effects on the uphill and downhill mean travel times [$\sigma^{(1)}_{tav}$ and $\sigma_{tav}^{(-1)}$] are rather small; otherwise it becomes more evident. In general, $\sigma_{tav}^{(1)}$ is the larger between the two.

Comparing Fig. 14(d) with Fig. 12(d), it is seen that the ρ_0 -dependence of the tunnel mean travel time $\sigma_{tav}^{(2)}$ is different from that in case without ramp effects. Usually, the $\sigma_{tav}^{(2)}$ is below 0.21 in case with ramp effects, takes a value above $\sigma_{tav}^{(1)}$ and $\sigma_{tav}^{(-1)}$.



Fig. 14. Density dependence of travel time σ_{tav} (a), its rms value σ'_t (b), travel time $\sigma^{(1)}_{tav}$ & $\sigma^{(-1)}_{tav}$ (c), and $\sigma^{(2)}_{av}$ (d) for tunnel length $L_{sp} = 1 \text{ km}$ in case with ramp effects. Note that in the fitted expression of σ_{tav} in part (a), A = 8.9686, m = 1.6260, and b = 0.8424. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

7. Conclusions

In this last section we list the conclusions of our analysis as follows:

- The traffic pressure expression proposed in the viscoelastic traffic flow model and the ENN scheme proposed in aerodynamics have considerable potentialities in applications, as the mean travel time predicted by the viscoelastic model coincides well with that estimated by the Navier–Stokes like model that is extended by using the same traffic pressure, and the ENN scheme for predicting the numerical flux has no free-parameter and third order accuracy in numerical flux approximation.
- 2. For traffic flow on a composite ring road in case without ramp effects, when the initial density normalized by its jam value is not lower than 0.2, any tunnel inlet becomes a starting point of traffic congestion region, causing traffic capacity reduction. The traffic shock propagates backward, interacts with rarefaction waves and other traffic shocks. When the initial density is 0.15, the tunnel generates a forward moving traffic bottleneck, whose trajectory is almost parallel to the trajectory of an initial jam. When the tunnel length is as short as 0.1 km, the uphill mean travel time is above the doubled, compared with the downhill mean travel time if the initial density of the road permits ranging from 0.2 to 0.5. The tunnel mean travel time grows with initial density if the density is lower than 0.2, otherwise, it reaches to a constant value and shows a time plateau that depending on the tunnel length. The longer the tunnel length, the higher is the plateau height.
- 3. For the traffic flow in case with ramp effects, even if the initial density is as low as 0.15, as an on ramp flow allows vehicles entering into the ring road, downstream the on-ramp intersection traffic shock can be generated at any tunnel inlet if the tunnel is upstream another off-ramp intersection. For the ramp conditions that are considered in the numerical tests, it was found that the tunnel induced traffic shock has a time period of existence about 1 h, then disappears as a result of the influences of off ramp flows and the use of periodic boundary condition.
- 4. The off ramp flow can shorten the mean travel time and increase its root mean square significantly. Generally, the tunnel mean travel time is larger than the mean travel time through the uphill or downhill segment.

5. According to the extensive numerical tests for the traffic flow on the composite ring road, a linear regression after logarithmic processing has obtained the fitted mean travel time

 $\sigma_{\rm tf} = A\rho_0^m + b$

where A = 8.9686, m = 1.6260, b = 0.8424, ρ_0 varies from 0.1 to 0.625. When the road is initially over-saturated, it is expected to be about 0.1 for the root mean square value of travel time that depending on the off-ramp flow effect.

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