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# Numerical simulation of turbulent Rayleigh–Benard convection $\stackrel{\mathackar}{\rightarrow}$

Hongxing Yang<sup>a,\*</sup>, Zuojin Zhu<sup>b</sup>

<sup>a</sup> Department of Building Services Engineering, The Hong Kong Polytechnic University, Kowloon, Hong Kong, PR China <sup>b</sup> School of Engineering Science, University of Science and Technology of China, Hefei, 230026, PR China

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## Abstract

This paper presents the numerical results of the turbulent Rayleigh–Benard convection at three Rayleigh numbers:  $Ra=3.80 \times 10^7$ ,  $3.08 \times 10^8$  and  $1.58 \times 10^9$ . The fourth order upwind scheme and coarse staggered grid system are used for the numerical calculation. The results are well agreed with experimental data. The successful computation for the problem implies that there might have simpler fundamentals for turbulent convection modeling, which will be explored in the subsequent theoretical and numerical studies.

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# 1. Introduction

Rayleigh–Benard convection is the thermal induced flow in a thin fluid layer heated from below. It has been the benchmark problem in hydrodynamic instability and has been long-time investigated [1]. At the critical Rayleigh number of about 1708, the flow transition occurs from the sub-critical to super-critical regimes. The turbulent convection regimes appear at even larger Rayleigh numbers, at which the effect of numerical viscosity becomes weak for numerical calculation. This effect is closely involved with numerical schemes used in discretizing the governing equations. Because turbulence is still an unsolved problem in classical physics [2], the turbulent Rayleigh–Benard convection is one of the problems in this category and is an important topic in thermal science.

As the one-point closure turbulence models [3] adopted many artificial coefficients but lacking the verification of their universality, and so does the large eddy simulation models [4]. In this study, we therefore prefer to employ direct numerical simulation (DNS) in terms of coarse grid, because the strictly saying DNS requires the grid number should be at the level of about  $Ra^{2.25}$  [5], leading to the computational demand far beyond the ability of a personal computer.

The wind (i.e. large scale circulation appeared autonomously) in the Rayleigh–Benard convection has been identified recently [6]. It was found that, for the case of  $Ra=10^7$  and Pr=1, the wind boundary layer scales linearly close to the wall and has a logarithmic region further away, indicating that the boundary layer is turbulent. Our previous works [7,8] show that the numerical viscous effect becomes evident when equivalent Reynolds number is large. Hence,

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<sup>\*</sup> Corresponding author. E-mail addresses: Behxyang@polyu.edu.hk (H. Yang), zuojin@ustc.edu.cn (Z. Zhu).

in this study, we use a fourth order upwind scheme in the discretization of the convective terms of the governing equations, so that, for the simulation of the turbulent Rayleigh–Benard convection, the numerical viscous influence can be maintained in a comparatively low level.

The main aim of this study is to prove that the direct numerical simulation (DNS) in terms of coarse grid is effective in the study of turbulent natural convections for further theoretical and numerical study of the turbulent natural convection.

## 2. Governing equations and numerical method

## 2.1. Governing equations

The turbulent Rayleigh–Benard convection is schematically illustrated in Fig. 1. The origin of the Cartesian coordinate system is allocated at the center of the rectangular cavity whose width–length–height is 6:12:1, where x is the horizontal coordinate, with y and z denoting the vertical and spanwise direction. The cavity is filled with a fluid with kinematic viscosity i and thermal diffusivity  $\kappa$ , with all vertical wall insulated. The bottom and top walls are assumed as isothermal, and the heat flux flows through the thin fluid layer from below. Following the approach of Wakitani [9], we select  $v_0 = \sqrt{g\beta_T H \Delta T}$  as the velocity scale, the height H as the length scale and the time scale should be  $t_0 = H/v_0$ , with  $\rho v_0^2$  being the measure of pressure. When we further define  $\Theta = [T - T_w]/\Delta T$ , the dimensionless governing equations for the turbulent natural convection problem can be written as follows:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\mathbf{u}_t + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \Theta \lambda + (Ra/Pr)^{-1/2} \nabla^2 \mathbf{u}$$
<sup>(2)</sup>

$$\Theta_t + (\mathbf{u} \cdot \nabla)\Theta = (RaPr)^{-1/2} \nabla^2 \Theta \tag{3}$$

where  $\lambda = (0,1,0)$  denotes the unit vector in the vertical direction and *Pr* is the Prandtl number, with the Rayleigh number  $Ra = g\beta_T(T_{w1} - T_{w2})H^3/(v\kappa)$ . It is noted that the variables in the governing equations are point averaged, i.e. averaged in an arbitrary standard monad in the turbulent field, with effect from the nonstandard monads in the standard one being totally neglected. Further work will be done to include these nonstandard monad influences on the turbulent natural convection in our subsequent papers.

The boundary conditions on the two vertical walls can be written as:

$$u = 0, v = 0, w = 0, \partial \Theta / \partial x = 0, \text{ for } x = -6, y \in (-0.5, 0.5), z \in (-3, 3)$$
 (4)

and

$$u = 0, v = 0, w = 0, \partial \Theta / \partial x = 0, \text{ for } x = 6, y \in (-0.5, 0.5), z \in (-3, 3)$$
 (5)

For the bottom wall, the boundary conditions are:

$$u = 0, v = 0, w = 0, \Theta = 1, \text{ for } y = -0.5, x \in (-6, 6), z \in (-3, 3)$$
 (6)

and for the top wall,

$$u = 0, v = 0, w = 0, \Theta = 0, \text{ for } y = +0.5, x \in (-6, 6), z \in (-3, 3)$$
 (7)



Fig. 1. Schematic diagram of the turbulent Benard convection in the case of  $T_{w1} > T_{w2}$  (the width–length–height ratio is 6:12:1 in this study).



Fig. 2. Evolution of overall Nusselt numbers on the bottom wall at three Rayleigh numbers.

where

$$u = 0, v = 0, w = 0, \partial \Theta / \partial z = 0, \text{ for } z = \pm 3, x \in (-6, 6), y \in (-0.5, 0.5)$$
 (8)

for the boundary conditions on the front and rear vertical walls.

## 2.2. Numerical method

The solution method is characterized by the use of the projection method (PmIII) developed by Brown et al. [10]. The staggered grid system is used in the numerical simulation. The convective terms are differenced by a fourth order upwind scheme instead of the second-order one used in our previous work [8]. The equation for the pressure potential is solved by the approximate factorization method developed by Baker [11] at first and then followed with solution accuracy improvement by using the method Bi-CGSTAB of Von der Vorst [12]. The accuracy of the numerical method is verified by the measured Nusselt number at  $Ra=3.80 \times 10^7$  in Refs. [1,13]. Our computation gives an overall Nusselt number of 22.95, as shown by the solid curve in Fig. 2. This value is excellently consisted with the measured data 22.72. Table 1 shows that at other two larger Rayleigh numbers, agreement between the numerical calculation and the measured results is also well, indicating that the numerical viscous effect has been certainly suppressed to a lower level. For turbulent flow simulation, higher order discretization for non-linear terms is useful. The detail of the numerical scheme will be presented in our subsequent works.

Table 1

Comparison of the overall Nusselt numbers between calculation results and experimental results (extrapolated from Refs. [1,13] with respect to  $Nu_{exp}=0.125Ra^{0.303}Pr^{0.25}$ )

Ra	$3.80 \times 10^{7}$	$3.08 \times 10^{8}$	$1.58 \times 10^{9}$
$\overline{Nu}_{av}$ (bottom)	22.95	42.74	72.54
$\overline{Nu}_{av}$ (top)	22.09	41.64	71.61
$\overline{Nu}_{av}$ (measured*)	22.72	42.83	70.29

### 3. Results and discussion

The fluid in the rectangular cavity with width–legnth–height ratio 6:12:1 is air whose Prandtl number is 0.71. The numerical simulation was terminated at the t=240 on a personal computer with an internal memory of 1 G bytes. Three Rayleigh numbers are used for the numerical computation, i.e.  $3.80 \times 10^7$ ,  $3.08 \times 10^8$  and  $1.58 \times 10^9$ . The grid number used in the present simulations is chosen as:  $161 \times 41 \times 51$  ( $\sim 3.36 \times 10^5$ ). Clearly, it is much less than the required grid number in the DNS with respect to the traditional meaning, which should be at the level of  $(Ra/Pr)^{9/8} (\approx 5.2 \times 10^9)$  for  $Ra=3.08 \times 10^8$ .

#### 3.1. Overall Nusselt number

Table 1 summarizes the computation results, which show that the overall Nusselt numbers on the bottom and top walls are very close to each other. The measurement results were obtained by the extrapolation of the experimental curve given by Silveston [13] and also from Ref. [1], which can be expressed in a power-law form as  $Nu_{exp}=0.125Ra^{0.303}Pr^{0.25}$ . The excellent agreement between the calculated Nusselt numbers and the measured ones shows that the coarse-grid DNS is able to present satisfactory results and the method with forth order upwind scheme is favorable.

Corresponding to the calculated Nusselt numbers shown in Table 1, Fig. 2 further illustrates the evolution of the overall Nusselt numbers on the bottom wall. It was found that there exist significant oscillations of the Nusselt number curves. Evidently, the time-averaged Nusselt number increases with the increase of the Rayleigh number, and so does the corresponding oscillating magnitude. This means that the chaotic degree of Rayleigh–Benard convection grows as the Rayleigh number is increased.

#### 3.2. Velocity history

The historical variations of the velocity components at four particular points in the rectangular cavity have been shown in Fig. 3. It is seen that, even though the time averaged velocity components is very close to zero, at the given Rayleigh number of  $3.08 \times 10^8$ , the instantaneous velocity can arrive at a value of about 0.5  $v_0$ , where  $v_0$  is the velocity scale. This finding is useful for recognizing the wind intensity in the Rayleigh–Benard convection.



Fig. 3. Historical variations of velocity components at four different points near the cavity walls at  $Ra=3.08\times10^8$ , (a) at points (0, -0.47, 0) and (0, +0.47, 0); (b) at points (-5.97, 0, 0) and (+5.97, 0, 0).



Fig. 4. The diagram of power spectra of the historical velocity variations given in Fig. 3(a).

To show whether there exists a turbulent inertial range for the turbulent convection, we further present the discrete Hilbert transform [14] to evaluate the diagram of power spectrum of the velocity evolutions curves given in Fig. 3(a). From Fig. 4, there does have an inertial range at the frequency of about 0.1, but the inertial range is comparative narrow, in which the power spectrum decays with the frequency with respect to the well-known negative 5/3 law.

## 3.3. Flow structures

To depict the turbulent Rayleigh–Benard convection patterns, the flood-type contours of vortices in the y-direction in the midplane (y=0) has been given in Fig. 5. Comparing part (a) and part (b) in Fig. 5, it can be seen that, at the instant of t=240, the turbulent vortical structures at  $Ra=3.08 \times 10^8$  is finer than that obtained at  $Ra=3.80 \times 10^7$ . This is qualitatively coincident with what



Fig. 5. The vortical fields in the horizontal mid-plane (y=0) at the instant of t=240, at two Rayleigh numbers: (a)  $Ra=3.80 \times 10^7$ , (b)  $Ra=3.08 \times 10^8$ .



Fig. 6. The vortical fields in the vertical mid plane (z=0) at three Rayleigh numbers: (a)  $Ra=3.80\times10^7$ , (b)  $Ra=3.08\times10^8$ , (c)  $Ra=1.58\times10^9$ .

was found by Azevedo and Sparrow [15], i.e. the larger the Rayleigh number, the smaller the spatial wavelength. On the other hand, it indicates that the numerical results in our previous study might have been deteriorated by the possible numerical viscosity, since larger deviations of spatial wavelength were found. This dependence of turbulent structure scale on the Rayleigh number can also be observed in Fig. 6. It is indicated that, for Rayleigh–Benard convection, the 3-D simulation can obtain more reliable numerical results from the points of fluid physics.

# 4. Conclusions

The normalized oscillating velocity magnitude of the wind in the Rayleigh–Benard convection is about 0.5 at the Rayleigh number of  $3.08 \times 10^8$ . The overall Nusselt numbers at the Rayleigh numbers of  $3.80 \times 10^7$ ,  $3.08 \times 10^8$  and  $1.58 \times 10^9$  from the coarse grid direct numerical simulation (DNS) are in excellent agreement with the measured results. The turbulent vortical structures become finer with the increase of the Rayleigh number. It implies that the fourth order upwind scheme can efficiently suppress the numerical viscous effect and make it at a lower level. The underlying reasons that the coarse grid DNS can present favorable numerical results will be studied in our subsequent works.

# Nomenclatures

- *Cp* Specific heat under constant pressure
- **G** Gravitational acceleration  $(m^2/s)$
- *H* Height of the cavity m
- L(=H) Length of the cavity m
- W(=2H) Width of the cavity m
- Nu<sub>av</sub> Overall Nusselt number
- *p* Pressure
- Pr Prandtl number of fluid
- *Ra* Rayleigh number
- $t_0$  Time scale (s)
- t Time
- $T_{w1}$  Absolute temperature of the hot vertical wall (K)
- $T_{\rm w2}$  Absolute temperature of the cold vertical wall (k)

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- u Velocity vector
- $v_0$  Velocity scale (m/s)
- *u* Velocity component in *x*-direction
- v Velocity component in y-direction
- w Velocity component in z-direction

Greek symbols

 $\beta_T$  Coefficient of volumetric expansion

 $\Delta T = T_{\rm h} - T_{\rm c}$  Temperature difference between the two vertical walls

- $\rho$  Mean density of the fluid (kg/m<sup>3</sup>)
- $\Theta$  Normalized temperature
- $\nu$  Kinematic viscosity (m<sup>2</sup>/s)

**Subscripts** 

- w1 Bottom wall
- W2 Top wall

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