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Numerical exploration of 1 + 2 type laminar natural convection in a differentially heated square cavity using Chebyshev spectral collocation method ☆

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ABSTRACT

In this paper, the Chebyshev spectral collocation method is applied to explore the unsteady two dimensional (1 + 2 type) laminar natural convection in a differentially heated square cavity at a Rayleigh number (Ra) of 10^7. The method has embedded the traditional Chorin's algorithm so as to avoid the trouble of seeking the pressure field in the buoyancy driven wall-jet flow. The sensitivity of the $\delta-$ parameter has been numerically investigated. It is found that when the δ value is over 11.6173, numerical instability occurs. Comparing the maximum horizontal velocity component with the existing numerical data obtained by solving the Poisson's equation of pressure field reveals that the Chorin's algorithm should be inapplicable for the solution of the benchmark problem of natural convection at $Ra = 10^7$ in thermal science.

1. Introduction

Spectral method is prevailing in computational fluid dynamics(CFD) [1–4]. The potentiality of the spectral method is that it has a very large converging speed. The converging speed of the approximated numerical solution to the primitive problem is faster than any one expressed by any power-index of N^{-1}. From the view of approximation to the original equation, the spectral method can be classified as the *collocation method* which presents discretization in physical space, the *Galerkin method* which seeks solution in spectral space, and the *pseudo-spectral method* which provides discrete integration in physical space at first and then presents transformation into spectral space for seeking the solution. Among the three methods, the collocation method is much more suitable for treating with non-linear problems. Evidences can be found in the previous numerical work of Huang et al. [5].

The so-called Chebyshev collocation spectral method is based on the expansion by virtue of the Chebyshev polynomials. At first, it expands the variable at collocation points and seeks the variable derivatives at these points, then substitutes the expansions into the partial differential equations, transfers the equation to an ordinary differential equation, and finally seeks the approximated solution in physical space. Recent numerical work concerned with the solution of non-linear differential equations has also provided more and more evidence of the applicability and accuracy of the Chebyshev collocation method [6,7]. However, less exploration is reported in CFD.

On the other hand, natural convection in rectangular enclosures has been extensively studied, since its potentiality in engineering applications such as building air distribution [9,10] is relatively evident. A brief literature view about the eddy viscosity type k-ϵ model simulation of turbulent natural convection in a rectangular cavity were reported by Niu and Zhu [11], and it was concluded that the k-ϵ model cannot satisfactorily predict the turbulent quantities, such as the root mean square of velocity components and temperature.

The laminar air natural convection in a square cavity with a heating strip was investigated experimentally by virtue of a two-dimensional particle image velocimetry (PIV) system [12], and the turbulent air natural convection in a differentially heated square cavity was studied using micro-thermocouples and a Laser Doppler Anemometer(LDA) [13–15].

Current numerical simulations have studied the second largest Lyapunov exponent and transition to chaos of natural convection in a rectangular cavity [16], the laminar natural convection in a closed square cavity with special heating modes [17–19] and with special numerical algorithm [20,21], the turbulent natural convection in an inclined cavity with a wavy heating side wall [22], and the conjugate turbulent air natural convection and surface radiation in rectangular enclosures heated from below and cooled from other walls [23].

The primary objective of this paper is to apply the Chebyshev collocation spectral method to explore the practical applicability of such a spectral method by seeking the numerical solutions of the 1 + 2 type laminar natural convection in a differentially heated square cavity at a Rayleigh number of 10^7. Even though Huang et al. [5] have confirmed that that the method can obtain especially satisfactory numerical results when the Chorin's algorithm is used together, the solution sensitivity to

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0735-1933/$ – see front matter
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the $\delta-$ parameter has not been explored. This paper studies this issue, which would reveal that the $\delta-$ value has a significant impact on numerical stability, and the Chorin's algorithm may be unsuitable to the numerical simulation at this Rayleigh number.

2. Governing equations

Consider the turbulent natural convection of air in a rectangular enclosure in a Cartesian coordinate system, in which x is the horizontal coordinate, with z denoting the vertical direction. The origin is allocated at the left bottom corner of the square cavity. For the benchmark problem considered, a schematic is depicted in Fig. 1, where the enclosure is filled with a fluid with kinematic viscosity v and thermal diffusivity κ. Buoyancy results in a clockwise flow in the mid plane ($z = 0$) because of the heating on the left hot wall with temperature T_h and cooling on the right cold wall with temperature T_c. The horizontal walls are assumed as adiabatic. Also, we assumed that the Boussinesq approximation of buoyancy is valid and can be used to simplify the momentum equations.

Let $\Delta T = T_h - T_c$, ρ_0 be the air density at temperature $(T_h + T_c)/2$, and β_T be the coefficient of volumetric expansion of fluid, following the approach of Le Quéré et al. [8], we select $W_0 = \kappa/H$ and the cavity height H as the velocity and length scales, hence the time and pressure scales should be $t_0 = H/W_0$ and $\rho_0 W_0^2$. Let $\Theta = [T - T_c]/\Delta T$, and the dimensionless governing equations for the turbulent natural convection problem can be written as follows:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \lambda(\Theta - 0.5)\mathrm{Pr}Ra + \mathrm{Pr}\nabla^2 \mathbf{u} \tag{2}$$

$$\Theta_t + \mathbf{u} \cdot \nabla \Theta = \nabla^2 \Theta \tag{3}$$

where $\lambda = (0, 1)$ denotes the unit vector in the vertical direction, and $\mathrm{Pr}(= \nu/\kappa)$ is the Prandtl number of fluid, with the Rayleigh number $Ra = g\beta_T H^3 \Delta T/(\nu\kappa)$.

The non-slip boundary condition for $\mathbf{u}$ is used on the cavity walls with the adiabatic condition for Θ used on the front and rear walls. As shown in Fig. 2, the boundary condition for Θ on the left hot wall is

$$\Theta = 1, \text{ for } x = 0, z \in (0, 1) \tag{4}$$

On the right cold wall, it becomes

$$\Theta = 0, \text{ for } x = 1, z \in (0, 1) \tag{5}$$

On the top and bottom walls, the value of Θ should be carefully described according to the adiabatic condition, which will be detailed in the following section.

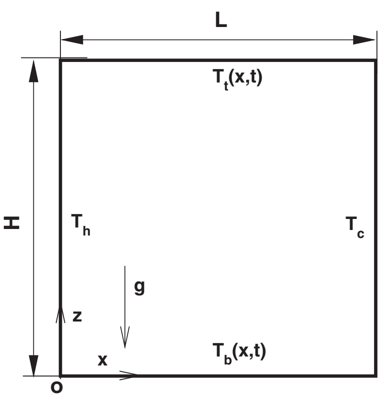

Fig. 1. Schematic of turbulent natural convection in an air-filled square cavity.

3. Numerical method

3.1. Chebyshev collocation method

The governing Eqs. (1)–(3) of laminar natural convection were solved by the Chebyshev collocation spectral method, which, has been detailed by Canuto and Hussaini et al. [24]. Here, we describe the spectral method briefly. The Chebyshev polynomial can be written as

$$T_k(x) = \cos(k\theta), \quad \theta = \arccos(x) \tag{6}$$

where $x \in [-1, 1]$. The Chebyshev function $T_k(x)$ is characterized by the relationship to its first order derivative as given below

$$T_k(x) = \begin{cases} T'_{k+1}(x)/(k+1), & \text{for } k = 0; \\ T'_{k+1}(x)/[2(k+1)], & \text{for } k = 1; \\ [T'_{k+1}(x) - T'_{k-1}(x)]/[2(k+1)], & \text{for } k > 1. \end{cases} \tag{7}$$

Based on which, the expression for $T'_k(x)$ can be written as

$$T'_k(x) = \sum_{n=0}^{k-1} \frac{2k}{C_n} T_n, \quad \text{merely for odd } (n+k) \tag{8}$$

where

$$C_n = \begin{cases} 1/2, & \text{for } n = 0, N \\ 1, & \text{for } n = 1, \cdots, N-1 \end{cases} \tag{9}$$

with N being the collocation point number. For any function $f(x)$, selecting the Gauss–Lobatto type collocation points $x_j = \cos(j\pi/N)$, the function has a Chebyshev polynomial as follows

$$f(x) = \frac{1}{2}[a_0 T_0(x) + a_N T_N(x)] + \sum_{j=1}^{N-1} a_j T_j(x) \tag{10}$$

in which the coefficient of expansion has the form of $a_j = \frac{2}{N}\sum'^{N}_{n=0} T_j(x_n) f(x_n)$. Note that the summation with *a prime* implies that the first and the end term must have a factor of 1/2.

Taking the first derivative of Eq. (10), it is convenient to yield

$$\begin{cases} f'(x_i) = \sum'^{N}_{j=0} a_j T'_j(x_i) = \sum'^{N}_{n=0} \left(\frac{2}{N} \sum'^{N}_{j=0} T'_j(x_i) T_j(x_n) \right) f(x_n) \\ \quad = \sum'^{N}_{n=0} [A_x]_{in} f(x_n) \end{cases} \tag{11}$$

where $[A_x]_i n = \frac{2C_n}{N} \sum'^{N}_{n=0} T'_j(x_i) T_j(x_n)$, for $n = 0, 1, \cdots, N$. Similarly, taking the derivative of Eq. (11), we have

$$\begin{cases} f''(x_i) = \sum_{n=0}^{N} [A_x]_{in} f'(x_n) = \sum_{j=0}^{N} \left(\sum_{n=0}^{N} [A_x]_{in} [A_x]_{nj} \right) f\left(x_j\right) \\ \quad = \sum_{j=0}^{N} [B_x]_{ij} f\left(x_j\right) \end{cases} \tag{12}$$

where $[B_x]_{ij} = \sum_{n=0}^{N} [A_x]_{in} [A_x]_{nj}$, for $n = 0, 1, \cdots, N$. Similarly repeating again, we can derive the expressions for higher order derivatives based on the Chebyshev expansion given by Eq. (10).

More recent fresh result has been obtained from a preliminary investigation, by Huang et al. [5], where the foregoing briefly described Chebyshev collocation spectral method has been used to seek the solution of the 1 + 2 type laminar natural convection in a differentially heated square cavity. Assume the solution region be $D = \{(x, y) : x \in [0, 1], y \in [0, 1]\}$, to maintain the cross-sectional property of function, the Chebyshev function must be transformed into $\tilde{T}_j(x_n) = T_j(2x_n - 1)$ so that

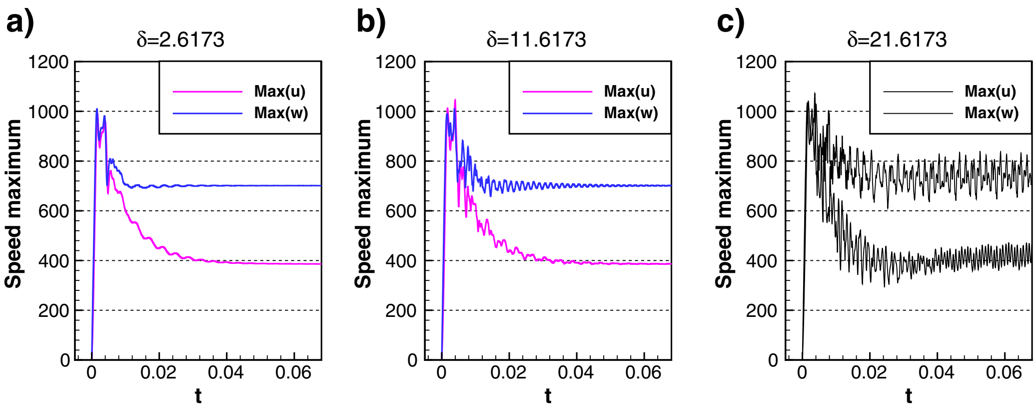

Fig. 2. Evolution of maximum values of velocity components in the square cavity for three cases: (a) $\delta = 2.6173$; (b) $\delta = 11.6173$; (c) $\delta = 21.6173$.

the collocation point (x_n, z_m) for the integer ranges $n \in (0, N)$ and $m \in (0, M)$ can be written as

$$\begin{cases} x_n = \dfrac{1}{2}[1-\cos(n\pi/N)] \\ z_m = \dfrac{1}{2}[1-\cos(m\pi/M)] \end{cases} \tag{13}$$

Using Eqs. (11) and (12), we can yield the following derivatives of the general function $\phi(x_i, z_j, t)$

$$\begin{cases} \phi_x\left(x_i, z_j, t\right) = \sum_{n=0}^{N} [A_x]_{in} \phi_{nj}(t) \\ \phi_{xx}\left(x_i, z_j, t\right) = \sum_{n=0}^{N} [B_x]_{in} \phi_{nj}(t) \\ \phi_z\left(x_i, z_j, t\right) = \sum_{n=0}^{N} [A_z]_{in} \phi_{nj}(t) \\ \phi_{zz}\left(x_i, z_j, t\right) = \sum_{n=0}^{N} [B_z]_{in} \phi_{nj}(t) \end{cases} \tag{14}$$

The general function ϕ is just a representative of flow variables **u**, p and Θ. Using Eq. (14), we can transfer the Boussinesq type governing equations into ordinary differential ones. Different from the previous work of Chorin [25] in which the non-linear convective terms are treated by the Adams–Bashforth scheme, in the present numerical work, we prefer to use the fourth order Runge–Kutta scheme. Following the work of Chorin [25], we also use Chorin's algorithm to avoid the solution of pressure field. Otherwise, the solution of Poisson's equation with non-linear right-hand term certainly requires a larger amount of computational resource. We will focus on the numerical sensitivity of $\delta-$ parameter used in the expression of artificial compressibility.

3.2. Chorin's algorithm

In Chorin's algorithm, an artificial density is introduced to link flow pressure algebraically in the following form

$$p = \left[\frac{1}{\delta}\mathrm{Pr}Ra\right] \cdot \rho \tag{15}$$

where ρ is the artificial density, and $\delta-$ is the parameter usually called as artificial compressibility. The density ρ is assumed to satisfy the revised continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \mathbf{u} = 0 \tag{16}$$

As the solid wall boundary condition should be applied, from the Boussinesq type governing Eqs. (2) and (3), conveniently, we can obtain the boundary conditions as follows

$$\begin{cases} \dfrac{\partial \rho}{\partial x} = \delta \cdot \left[Ra^{-1}\nabla^2 u\right] \\ \dfrac{\partial \rho}{\partial z} = \delta \cdot \left[Ra^{-1}\nabla^2 w + (\Theta - 0.5)\right] \end{cases} \tag{17}$$

where u and w are respectively the horizontal and vertical velocity components of **u**. What should be addressed is: the Chorin's algorithm has a potential of avoiding the further solution of pressure field, instead of time-step marching solution of Eq. (16).

3.3. The second type boundary condition of Θ

On the top and bottom walls of the cavity, we assume that the walls are adiabatic, leading to the second type homogeneous boundary condition. To accurately implement this condition, special care is required. Using the Chebyshev expansion as reported previously [7], we obtain

$$\Theta_z\left(x_i, z_j, t\right) = \left\{[A_z]_{j0}\Theta_{i0}(t) + [A_z]_{jM}\Theta_{iM}(t)\right\} + \sum_{m=1}^{M-1} [A_z]_{jm}\Theta_{im}(t) \tag{18}$$

Setting the subscript j as 0, or M, using the adiabatic condition, we have

$$\begin{cases} [A_z]_{00}\Theta_{i0}(t) \quad +[A_z]_{0M}\Theta_{iM}(t) = -\sum_{M=1}^{M-1} [A_z]_{0m}\Theta_{im}(t) \\ [A_z]_{M0}\Theta_{i0}(t) \quad +[A_z]_{MM}\Theta_{iM}(t) = -\sum_{M=1}^{M-1} [A_z]_{Mm}\Theta_{im}(t) \end{cases} \tag{19}$$

Based on which, we can yield the temperatures on the top and bottom walls, instantaneously. Similarly, we can yield the artificial densities ρ on relevant walls.

4. Results and discussion

Using the numerical method described above, computer simulation of air laminar natural convection schematically shown in Fig. 2 was carried out in a domestic personal computer with a memory of 1Gb. The time step is set as 4.125×10^{-7}, with the time unit equal to κ/H. The grid number is set as 65×65, identical to the previously published spectral work of Le Quéré et al. [8], where the pressure field is obtained by solving the Poisson's equation. Using the present simulator, a single case requires a computational time of around 23 h 39 min 23 s. Classified by the $\delta-$ parameter, three cases are

Table 1
The values of δ parameter used in the computer simulation.

Case	Δ
I	2.6173
II	11.6173
III	21.6173

labeled by I, II, and III, with the δ− values shown in Table 1. Note that when the same time step is used, the computer simulator diverges for the case of δ = 1.6173. When the δ = 3.6173 and 7.6173, the simulator can normally complete the calculating process. This indicates that the Chorin's algorithm must be sensitive to the parameter δ.

Furthermore evidence of the solution to the δ− parameter's sensitivity can be seen in Fig. 2(a–c). This figure indicates that when δ− value is identical to 11.6173, as seen in Fig. 2(b), the calculating process illustrated by both the evolution curves of velocity components' maximums decaying oscillation, implies that δ = 11.6173 is close to the critical point value for numerical stability. When δ is assigned as 21.6173, as seen in Fig. 2(c), the calculating process must be in the regime of numerical instability. The sensitivity to δ can also be found in the distributions of left wall (or heating wall) Nusselt number at the computational terminative moment, as shown in Fig. 3. It is seen that the temperature at the near wall collocation point has unpractical fluctuation which does not appear for the cases of I(δ = 2.6173) and II(δ = 11.6173).

It should be noted that: even though the calculating process is stable, the maximum horizontal velocity component is evidently over predicted, as can be seen in the third line of Table 2. Referring to the experimental data given by Tian and Karayiannis [14], the calculated value of Le Quéré et al. [8] is more accurate. This suggests that the Chorin's algorithm used for the avoidance of solving Poisson's equation for pressure must be given up for the numerical simulation of the 1 + 2 type benchmark problem in thermal science.

The temperature and vorticity contours at the calculating terminative moment are shown in Fig. 4(a–b). The highly symmetrical characteristics of thermal and flow patterns to the square center do reveal that Chebyshev collocation spectral method is potential.

5. Conclusions

The Chebyshev spectral collocation method has been implemented to explore the unsteady two dimensional (1 + 2 type) laminar natural convection in a differentially heated square cavity at a Rayleigh number(Ra) of 10^7. This method is used together with the well known

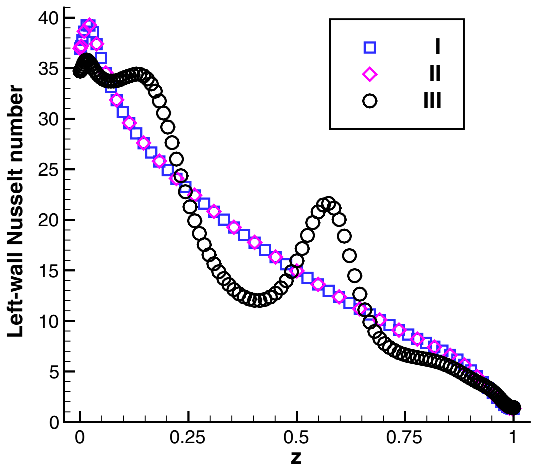

Fig. 3. Left-wall Nusselt number distribution at the terminative moment for three cases.

Table 2
The measured Θ on the top and bottom walls as a function of *x*.

Ra	This work	Le Quéré et al.
$\frac{x}{z}$	$\frac{0.1214}{0.9665}$	$\frac{X_1}{0.879}$ [a]
max(u)	**389.6**	**148.8**
$\frac{x}{z}$	$\frac{0.0215}{0.4655}$	$\frac{0.0213}{Z_2}$ [a]
max(w)	701.2	699.3
$Nu_{1/2}$ [b]	16.571	16.51
Nu_{av} [c]	16.536	16.52
Z	0.0181	0.018
Nu_{max} [d]	39.371	39.37
Z	1	1
Nu_{min} [d]	1.365	1.376

[a] The X_1 and Z_2 values were not given by Le Quéré et al.

[b] $Nu_{1/2}$ is defined by $Nu_{1/2} = \int_0^1 \left[u\Theta - \frac{\partial \Theta}{\partial x}\right]_{x=1/2} dz$.

[c] Nu_{av} is defined by $Nu_{av} = \int_0^1 \left[-\frac{\partial \Theta}{\partial x}\right]_{x=0} dz$.

[d] Nu_{max} and Nu_{min} are respectively the maximum and minimum of left-wall Nusselt number.

Chorin's algorithm to avoid the trouble of pressure finding. When the value of the δ parameter in the Chorin's algorithm is over 11.6173, there occurs numerical instability. It is found that the obtained maximum horizontal velocity component in the current work has been largely over predicted than the existing data. This indicates that for

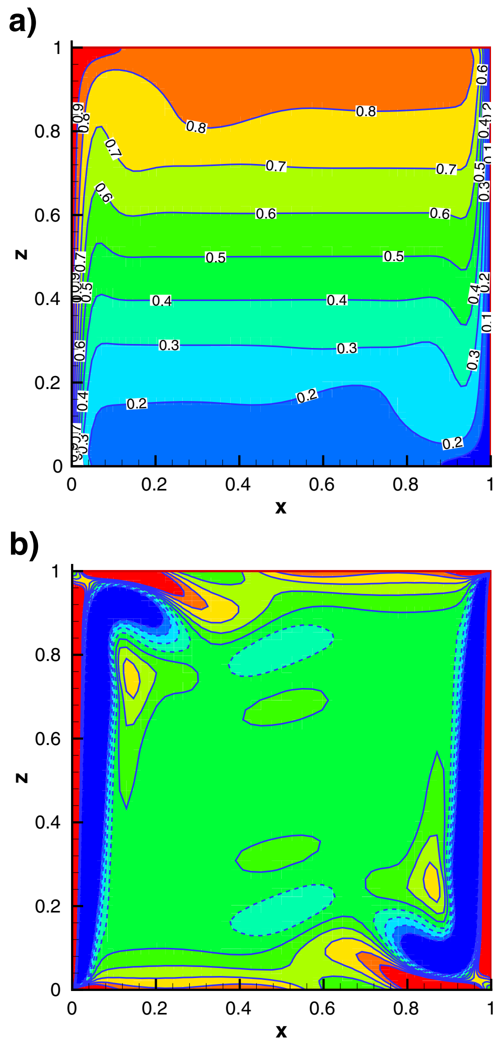

Fig. 4. (a) Contours of temperature field at the moment of terminative simulation; (b) Contours of the vorticity at the terminative moment. Note that, in part (b), the vorticity contours are labeled by values from −4000 to 4000, with an increment of 1000.

Rayleigh number beyond the value of 10^7, the well known Chorin's algorithm cannot be embedded into the Chebyshev collocation method.

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