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NUMERICAL STUDY ON TRANSIENT LAMINAR NATURAL CONVECTION IN AN INCLINED PARALLEL-WALLED CHANNEL

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ABSTRACT

The transient laminar natural convection in an inclined parallel-walled channel when the bottom wall is heated was investigated by direct numerical solution of the governing equations using the accurate projection method, PmIII. Comparison of the investigation results with other published experimental results for the overall Nusselt numbers was made. It was found that the numerical simulation results represented by the overall Nusselt number are in good agreement with the experimental data. The numerical study confirmed that, for the laminar natural convection of water whose Pr is close to 5 in an inclined parallel-walled channel and for a spacing ratio of less than 1/20 when the incline ranges from 45 to 90 degrees, the overall Nusselt number can be considered as a monotonic function of a new parameter, i.e. the product of the Rayleigh number and the ratio of the channel width to the length times the sine of the inclination angle of the parallel channel.

Introduction

Laminar natural convection in parallel-walled channels, as a fundamental problem in thermal science, has been extensively studied due to its importance in applications such as the heat transfer in double-glazing units, the cooling systems of electronic devices, and solar thermal engineering. Earlier work on this problem was reported by Elenbaas [1], who considered heat transfer by natural convection in parallel plates. Twenty years later, in 1962, Bodoia and Osterle [2] studied the development of natural convection. Significant research was conducted during the 1970s and 1980s, when solar energy technology was intensively investigated. Among these, the contributions to natural convection by Aung *et*

al. [3-5], Sparrow and coworkers [6] made significant achievements towards understanding the mechanism of natural convection, including the corresponding experimental results. However, most of the numerical studies were conducted for horizontal and vertical channels. A study on laminar natural convection in parallel-walled inclined channels heated by the bottom plate was needed when the performance of a solar hot-water roof was investigated. This report presents the investigation results.

On the other hand, the onset of instability of natural convection in parallel-walled channels has also been investigated. Chen and Chung [7,8] reported the results of a study on the linear stability of mixed convection in a vertical channel obtained by a numerical method, where the disturbance equations were treated by Galerkin's scheme. It was concluded that, in general, the instability is dominated by the manner of heating. Most reported studies are based on the linear theory of stability, since this is closely involved with the problem under various conditions. In fact, the evolution of cellular flow patterns under over-critical conditions is dominated by the equilibrium of energy released by the buoyancy force and the kinetic energy dissipated by the viscosity and nonlinear mechanism. For this situation, the theory of non-linear stability is more appropriate.

However, most of the aforementioned research mainly concerned the convection flow in vertical parallel-walled channels. There has been less study on the flow in inclined parallel plate channels, notwithstanding the experimental work of Azevedo and Sparrow [9], and the theoretical work of Lavine [10]. Azevedo and Sparrow reported experimental results for natural convection in open-ended inclined channels in which the fluid was water, with the Prandtl number of the water close to 5. When the top wall heating mode was used, a recirculation zone depending on the Rayleigh number was observed in the top portion of the unheated wall. Lavine [10] found theoretically that when the constant heat flux from the boundaries was fixed, for any value of the Rayleigh number corresponding to an adverse temperature gradient, the convection flow was unstable, and was not dependent on the base velocity, temperature distribution and inclination. The purpose of this paper is to report the numerical calculation results and their comparison with the above experimental results.

The authors' previous study [11] on the thermal-induced flow instability in a horizontal parallel plate channel was carried out numerically using a fractional algorithm. The pressure field was obtained by the Bi-CGSTAB scheme of Von de Vorst [12]. The focus of attention is now on the natural convection in inclined parallel-walled channels. The overall Nusselt number is presented by direct numerical solution of Navier-Stokes equations by using an accurate projection method, the PmIII, developed by Brown *et al.* [13]. The problem under examination is shown schematically in Figure 1.

Mathematical Model

Assuming the fluid properties are constant, using the Boussinesq assumption, the channel height,

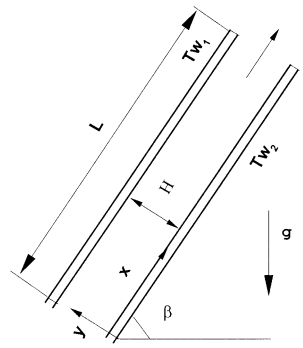

FIG. 1

Schematic diagram of an inclined parallel-walled channel

H, the flow velocity, $u_0 = [g\beta_T(T_w - T_\infty)H]^{0.5}$ and the temperature difference between the air inside the channel and the ambient air, $(T_w - T_\infty)$, as the main measures of the flow in the channel, the two-dimensional governing equations for natural convection in the channel can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial vu}{\partial y} = -\frac{\partial p}{\partial x} + \Theta \sin\beta + \left(\frac{\Pr}{Ra}\right)^{\frac{1}{2}} \nabla^2 u \tag{2}$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial vv}{\partial y} = -\frac{\partial p}{\partial y} + \Theta \cos\beta + \left(\frac{\Pr}{Ra}\right)^{\frac{1}{2}} \nabla^2 v \tag{3}$$

$$\frac{\partial \Theta}{\partial t} + \frac{\partial u\Theta}{\partial x} + \frac{\partial v\Theta}{\partial y} = \frac{1}{\sqrt{Ra \Pr}} \nabla^2 \Theta \tag{4}$$

where Ra is the Rayleigh number, $Ra = g\beta_T(T_w - T_\infty)H^3 / \nu\alpha$;

Θ is the dimensionless temperature, $\Theta = \dfrac{T - T_\infty}{T_w - T_\infty}$;

β_T is the coefficient of the thermal expansion of fluid;

T_∞ and T_w are the ambient and wall temperatures respectively.

The solutions of the governing equations (1)-(4) should be sought under appropriate conditions. As reported by Azevedo and Sparrow [9], the top-heating mode may induce a recirculation zone in the top portion of the unheated wall. Thus it is assumed that the heat flux comes from the bottom wall, whose

temperature difference over the unheated wall temperature is constant. The boundary conditions on the two walls can then be described by:

$$u = 0,\ v = 0,\ \Theta = 0\text{, for the top wall} \tag{5}$$

and

$$u = 0,\ v = 0,\ \Theta = 1,\ \text{for the bottom wall} \tag{6}$$

The boundary conditions on the inlet and outlet boundaries can be assumed as:

$$(\Theta)_{\text{in}} = 0,\ \left[\frac{\partial \Theta}{\partial x}\right]_{\text{out}} = 0,\ v = \frac{\partial \omega}{\partial x} = 0, \quad \text{for } x = 0,\ \text{or L/H} \tag{7}$$

where $\omega = \partial v / \partial x - \partial u / \partial y$ is the viscosity. On the other hand, the initial conditions are simply assigned as:

$$u = 0,\ v = 0,\ \Theta = 0,\ \text{in } \Omega \text{ for } t = 0 \tag{8}$$

Numerical Solution Method

The projection method, PmIII, proposed by Brown *et al.* [13], was used to find the solutions to the above governing equations. The numerical calculation was characterized by alternating the pressure solver with an iteration procedure based on the approximate factorization one (AFI). The convective terms were approximated by the second-order Adams-Bashforth formula. The AFI is developed by Baker [14], whose assessment was made for a benchmark problem.

Results and Discussions

The numerical solutions of the governing equations for the natural convection of water in the inclined parallel channel were sought in terms of the boundary conditions above for the values of $Ra(H/L)$ ranging from 1,000 to 25,000. The channel walls were assumed to have constant temperatures, with the bottom one heated. The space ratio for the inclined parallel channel was either 1/30 or 1/20, and the Prandtl number of the water was 5.03.

A uniform-staggered grid system (150×80) was used together with time interval, $\Delta t = 0.005$. The independence assessment of the spatial and temporal steps is shown in Table 1. For the case $Ra(H/L) = 1000$, the overall Nusselt number when $t = 50$ decreases with the grid number. For instance, from column three in Table 1, for a given time interval of 0.005, the Nusselt number Nu_{av} takes the value of 3.683 for the grid of 80×40, but it is 3.281 for a finer grid of 150×60, and 3.303 for a grid of 180×70. For a given grid of 150×60, the dependence of the overall Nusselt number on the

temporal step can be seen from column six in Table 1. It is evident that the calculated value for $Nu_{av}(t=50)=\frac{H}{L}\int_0^{L/H}\frac{\partial\Theta}{\partial y}|_{t=50}\,dx$ seems to be less correlated with the triple temporal steps used. Accordingly, it seems to be appropriate to use the grid of 150×80 and the temporal step of 0.005 for the solution to the natural convection problem.

TABLE 1
Assessment of Grid-Independence when $Ra(H/L)=1000$

Grid	Δt	$Nu_{t=50}$	Grid	Δt	$Nu_{t=50}$
80×40	5×10^{-3}	3.683	150×60	2×10^{-3}	3.281
100×50	5×10^{-3}	3.419	150×60	5×10^{-3}	3.303
180×70	5×10^{-3}	3.303	150×60	8×10^{-3}	3.278

Table 2 shows the results of the calculated overall Nusselt number for two spacing ratios of the inclined parallel channel and two inclined angles for eight values of $Ra(H/L)$. It is found that, for $\beta=60$ deg, the change of the channel spacing has no evident effect on the calculated overall Nusselt number for t=50, since the peak deviation is less than 4%. Furthermore, for $\beta=45$ deg, the peak deviation is about 9% for $Ra(H/L)=15{,}000$. Thus, for a spacing ratio of less than 1/20, the parameter $Ra(H/L)\sin\beta$ can be used as a suitable monotonic parameter of the overall Nusselt number $Nu_{av}(t=50)$, which is comparable with the experimental correlation proposed by Azevedo and Sparrow [6], i.e.

$$Nu=0.644[Ra(H/L)\sin\beta]^{0.25} \tag{9}$$

TABLE 2
The Overall Nusselt Number for Different β and Ra when $t=50$

H/L	1/30	1/20	1/30	1/20
$Ra(H/L)$	$\beta=45$ deg		$\beta=60$ deg	
1000	3.439	3.280	3.613	3.458
1500	3.792	3.606	3.958	3.811
2500	4.305	4.087	4.468	4.335
5000	5.047	4.928	5.276	5.178
10000	5.934	5.944	6.282	6.170
10000	5.934	5.944	6.282	6.170
15000	6.011	6.552	6.924	6.830
25000	7.398	7.383	7.769	7.738

The comparison between the calculation results and the experimental results is illustrated in

Figure 2. It is found that the numerical calculation results of the overall Nusselt numbers when t=50 are in good agreement with the experimental results.

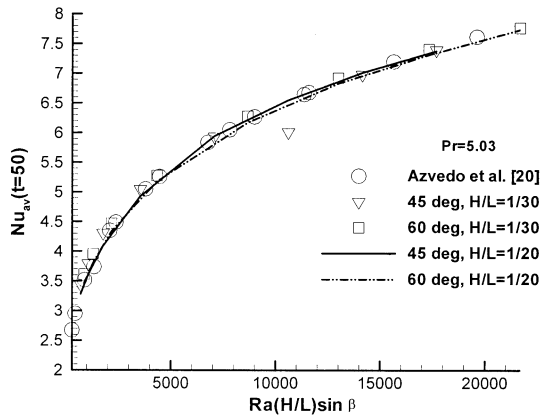

FIG. 2
Comparison of numerical calculation results and experimental results for inclination angles of 45 and 60 degrees when Pr=5.0 and H/L<1/20

The behavior of the time oscillation of the overall Nusselt number under relatively large values of the $Ra(H/L)$ produces the peak deviation of about 9% as shown in Table 2. This can be observed from Figures 3 and 4, which illustrate the evolution of the overall Nusselt numbers when $Ra(H/L)$ equals to

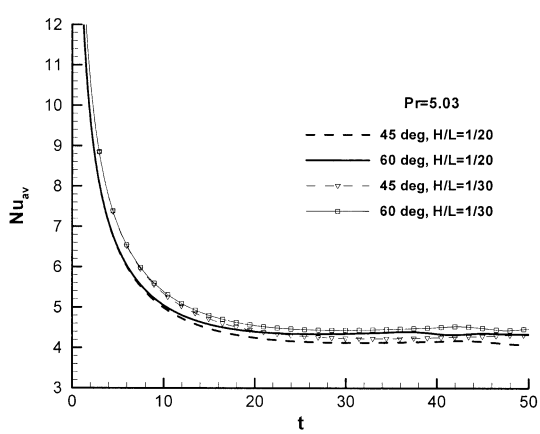

FIG. 3
Evolution of the calculated overall Nusselt numbers for four different cases when $Ra(H/L)$=2,500

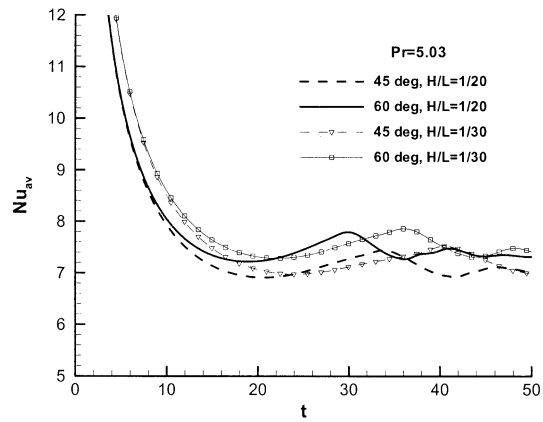

FIG. 4
Evolution of the calculated overall Nusselt numbers for four different cases when $Ra(H/L)$=20,000

2,500 and 20,000, respectively. For smaller values of $Ra(H/L)$, the oscillation has smaller magnitude, as shown in Figure 3. However, for larger values of $Ra(H/L)$, the magnitude of the Nusselt number oscillation becomes more significant. For the dashed curve labeled by gradient symbols, as shown in Figure 4, the overall Nusselt number for $t = 50$ takes a smaller value.

Conclusions

A numerical study on the laminar natural convection of water in an inclined parallel-walled channel was investigated for two different inclination angles, 45 and 60 degrees. The governing equations were solved by an advanced numerical method, the PmIII, where the Poisson equation of pressure potential was approximated by numerical algorithm AF1. A good agreement is achieved between the calculated overall Nusselt numbers and the published experimental results. It was also found that, for the laminar natural convection of water in case of heating from the bottom wall, when the wall temperature difference is maintained at constant and for water with Pr =5 and spacing ratio less than 1/20, the overall Nusselt number can be expressed as a monotonic function of the parameter $Ra(H/L)\sin\beta$.

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Nomenclatures

β	inclination angle
β_T	coefficient of fluid thermal expansion, 1/K
g	gravitational acceleration, m/s²
H	gap height, m
L	gap length, m
p	pressure, Pa
Nu	Nusselt Number
Pr	Prandtle number
Ra	Rayleigh number
Ra_{av}	average Rayleigh number
ΔT_w	temperature difference, $T_w - T_\infty$
u	velocity component in x direction, m/s
v	velocity component in y direction, m/s

Subscript:

w	*wall*
T	temperature
∞	ambient

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