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Exploring the effects of work zone on vehicular flow on ring freeways with a tunnel using a three-lane continuum model

Shan Li^a, M.N. Smirnova^{b,c,*}, Shanjun Yang^{a,e}, N.N. Smirnov^{b,c,d}, Zuojin Zhu^{a,b,*}^a CAS Key Laboratory of Mechanical Behavior and Design of Materials, Department of Modern Mechanics, University of Science and Technology of China, Hefei 230022, China^b Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Moscow 119992, Russia^c Scientific Research Institute for System Analysis, Russian Academy of Sciences, Moscow 117218, Russia^d Moscow Center for Fundamental and Applied Mathematics, Moscow 119991, Russia^e Physical Education Center, USTC, Hefei 230026, China

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ABSTRACT

Freeway work zone forms as a result of traffic crash or road rehabilitation. To ascertain the effects of work zone with lane II completely blocked on vehicular flow on ring freeways with a tunnel, a three-lane continuum model is put forward. The mandatory net lane-changing rate from lane II to lane I or III just upstream of the work zone is described by a random number model, with the random number being produced within a small range around a median based on a golden section analysis. The net-changing rate between adjacent lanes is described using a lane-changing time on the basis of an assumption: the time ratio to relaxation time equals infinity when the absolute value of traffic densities between the two adjacent lanes is less than 1 veh/km, implying that the net-changing rate is zero; otherwise, the time ratio is inversely proportional to the vehicular spatial headway, which is equal to unity for traffic flow at saturation state, but infinity when the traffic flow is completely jammed. It is assumed that the freeway is a three lane ring with a total length of 100 km, and has a tunnel with a speed limit of 60 km/h and a length of 1.6 km located downstream the work zone with a length of 0.16 km. The free flow speeds on lanes I, II, and III are 120 km/h, 100 km/h, and 85 km/h, respectively. For the vehicular flow on the ring freeway with a tunnel, numerical simulations based on the three-lane continuum model are carried out with a reliable numerical method of high accuracy. It is found that the vehicular flow has two thresholds of traffic jam formation, one depending upon the tunnel and the other upon the work zone. The tunnel triggers a traffic jam when the initial density normalized by jam density is equal to the first threshold 0.15, and the work zone originates another traffic jam when the normalized initial density equals the second threshold 0.19. The freeway tunnel plays a dominant role in the prediction of mean travel time as soon as the tunnel has generated a traffic jam at the tunnel entrance. For the vehicular flow at unsaturated state, the average speed through the tunnel is about 26.67 km/h. When the normalized initial density exceeds the second threshold 0.19, the mean travel time through every lane increases with the initial density linearly. Vehicle fuel

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* Corresponding authors.

E-mail addresses: wonrims@inbox.ru (M.N. Smirnova), zuojin@ustc.edu.cn (Z. Zhu).<https://doi.org/10.1016/j.ijtst.2023.03.004>

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consumption can be estimated by interpolation with the time averaged grid traffic speed and an assumed vehicle performance curve. It is found that the vehicle fuel consumption is lane number dependent, and distributes with the initial density concavely, as well as has a value in the range of 6.5 to 8.3 l.

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1. Introduction

The research background includes four aspects, i.e., macroscopic models, analysis of work zone crashes, and crash injury severity in different work zone configurations, tunnel bottlenecks, and lane changing models. It is well-known that a lot of models have been developed for vehicular flows, among which are the LWR model (Lighthill and Whitham, 1955; Richards, 1956), the Euler model (Payne, 1971), the gas-kinetic-based model (Helbing and Treiber, 1998; Hoogendoorn and Bovy, 2000), the Navier–Stokes like model (Kerner and Konhäuser, 1993), second order models (Greenberg, 2004; Borsche et al., 2012), and generic models (Lebacque and Khoshyaran, 2013).

A review of investigations on multi-lane vehicular flows were reported by Michalopoulos et al. (1984). A macroscopic multi-lane model was put forward by Helbing and Greiner (1997) by deducing from a Boltzmann-like gas-kinetic approach. A multi-lane model was proposed by Chang and Zhu (2006) to analyze the dynamic traffic properties of a highway segment under a lane-closure operation that often incurs with complex interactions between mandatory lane-changing vehicles and traffic at unblocked lanes. For homogeneous multi-lane freeways, a macroscopic behavior theory of traffic dynamics was proposed by Daganzo (2002a). Focusing on the onset of congestion, the behavior of multi-lane freeway traffic on entering road crossings was examined by Pestov et al. (2019). In the multi-lane continuum model proposed by Smirnova et al. (2014b), an expression of the parameter to describe the acceleration component in the orthogonal direction in relation to lane changing was derived. In addition, several investigations relating to visco-elastic traffic modelling have been reported (Bogdanova et al., 2015; Smirnova et al., 2016; Smirnova et al., 2017).

Theofilatos et al. (2017) conducted a meta-analysis on the effect of road work zones on crash occurrence. To provide overall estimates of those effects on crash frequencies, the relationship between the number of crashes and the work zone duration and length was estimated. It was found that similar to previous meta-analyses, the effect of duration after correcting for publication bias was not significant, while the effect of length was significant at a 95% level. The main factors influencing the overall estimates of the beta coefficients are the study year and region for duration and the study year and model specification for length. For work zone crashes, using the ordered Probit model with factor analyses in Egypt, an analysis was carried out by Zhang et al. (2018a). The work-zone data crashes from 2010 to 2015 that occurred in Egyptian long-term highway maintenance and rehabilitation projects were analyzed. To examine the contribution of common factors in the severity of work-zones, an ordered probit model was developed on the basis of the results of factor analysis scores. It was found that the most influential factors contributed to work-zone crashes were the weather condition, number of lane closures, type of surface construction, road character, day of week, etc.

Analyzing the passenger-car crash injury severity, in different work zone configurations, Osman et al. (2018) found that for contributing to the likelihood of severe outcomes, the key factors were the airbag deployment, alcohol involvement, ejection, seat belt use, and partial control of access. Based on logistic speed-density models, Lu et al. (2018) found that the predicted lifetime distribution could capture well the tendency of the observed work zone capacities. The random effect-based Poisson inverse Gaussian (REPIG) models provided the most accurate crash predictions (Park et al., 2019). The random parameter-based logit models could be used to study the unobserved heterogeneity and temporal instability in the analysis of severities of work zone crash-injury (Islam et al., 2020).

Goñi-Ros et al. (2016) found that to reduce congestion at sags is a potentially highly effective and innovative way, and could be implemented using cooperative adaptive cruise control systems. Jin (2018) developed a behavioral kinematic wave model to develop better design and control strategies to improve the performance of tunnel bottlenecks. Lane-changing is a difficult traffic phenomenon in modelling (Cassidy and Rudjanakanoknad, 2005). It is a primary trigger of oscillations, and has negative impact on traffic breakdowns and bottleneck discharge rate reduction at the onset of congestion (Zheng et al., 2011). Zheng (2014) reviewed several modules of lane-changing, such as the multi-lane kinematic wave module (Laval and Daganzo, 2006) and the extended kinematic wave theory (Jin, 2010; Jin, 2013). Recently, Ali et al. (2019) proposed a game theory-based approach for modelling mandatory lane-changing behaviour in a connected environment, where drivers could make more informed and safe lane-changing decisions to help them navigate through current and upcoming driving situations, especially the provision of possible unseen hazards.

To describe urban commuters' predictions of travel time as well as the adjustments of departure time in response to unacceptable arrivals in their daily commute under limited information, two heuristic rules were proposed and examined by Chang and Mahmassani (1988). They found that the magnitude of the predicted travel time depended on each commuter's own experience, including recallable travel time, schedule delay, and difficulties in searching for a satisfactory departure time. Considering that if the travel time information of each traffic stream was given, the results of dynamic traffic assign-

ment models could be made much more accurate and the effect of signal controls improved, Wang et al. (2016) conducted a comparison analysis of travel time estimation methods to verify the significant difference in the link travel time of different traffic streams, and redefined the link travel time in consideration of traffic stream directions. As existing estimation methods cannot reflect real values or fluctuations of travel time in sampling intervals without any valid observational data, a regression model was developed and integrated into a method of travel time estimation. It was found that the method improved the error analysis of several links on two different days.

Different from the work of Chang and Mahmassani (1988) and Wang et al. (2016), in this paper, travel time is estimated using the grid traffic speed as reported recently by Zhang et al. (2018b). The objective of this paper is to explore the effects of work zone with lane II completely blocked on a vehicular flow in the freeway with a tunnel, hence a three-lane continuum model is proposed. The work zone forms as a result of traffic crash or road rehabilitation. The work zone on lane II is chosen rather than the other lanes as it is more common for traffic crashes or road rehabilitations on lane II. The three-lane continuum model is characterized by the vehicular lane-changing time model for the net lane-changing rate between adjacent lanes, the random number model for the mandatory lane-changing rate just upstream of the work zone. The random number is generated in a small range around a median based on the golden section analysis. The model uses traffic pressure and sound speed in algebraic form rather than partial differential equations as reported (Helbing and Treiber, 1998; Hoogendoorn and Bovy, 2000). The model complexity is lower than that developed by Helbing and Greiner (1997). The lane changing model is simpler than that developed by Laval and Daganzo (2006). However, the continuum model has not taken account of the variations of vehicular engine type and the effects of lane width, shoulder width, and traffic crashes.

To simulate vehicular flows on a ring freeway with a tunnel having a length of 1.6 km located downstream of a work zone with a length of 0.16 km where lane II is completely blocked, the equations of the three-lane continuum model are solved numerically, for which a 3rd-order Runge–Kutta scheme (RK3) (Shu, 1988; Shu and Osher, 1989) is employed to handle the time derivative term, and a 5th-order weighted essentially non-oscillatory scheme (WENO5) (Jiang and Shu, 1996; Henrick et al., 2005) is used to calculate the numerical flux. From simulation, spatiotemporal evolutions of spatial headway and grid traffic speed may be obtained. The grid traffic speed on different lanes is used to predict the mean travel time and its fluctuation. Using a vehicle performance curve showing the dependence of vehicle fuel consumption on the vehicular speed, both the vehicle fuel consumption through each lane of the ring freeway and the more consumed vehicle fuel consumption in comparison with that when the work zone vanishes are predicted.

In this paper, the model equations are presented in Section 2, the numerical method is described in Section 3, the numerical results are discussed in Section 4, and some conclusions are provided in Section 5.

2. Equations of the three-lane model

In this section, the equations of a three lane continuum model is presented, to describe the mandatory net lane-changing rates from lane II onto lanes I and III just upstream of the roadway work zone, a random number lane changing model is put forward; while a lane-changing time model is proposed for the net lane changing rate in the normal segment and tunnel, by emphasizing the role of the local homogeneity of vehicular traffic flow in the orthogonal direction.

As shown in Fig. 1(a), the three lanes are labeled as I, II, and III, corresponding to traffic densities ρ_l and traffic speeds u_l for $l \in \{1, 2, 3\}$. Generally, the free flow speed of vehicles has the highest on lane I, but the lowest on lane III, with an intermediate value between the two on lane II. However, in the roadway work zone, lane II is assumed to be completely blocked so that the traffic density equals the jam density and the traffic speed is zero, and a single lane continuum model has to be used for the vehicular flow on lanes I and III. As shown in Fig. 1(b), the ring road has one initial jam at x_A , with the cordon having a length L_{WZ} with two boundaries at x_{WZ1} and x_{WZ2} . There are two ramp flow intersections connected with lane III at x_{R1} and x_{R2} . In the normal segment and tunnel, any vehicle attempting to run off the ring road makes at first a lane change and shifts onto

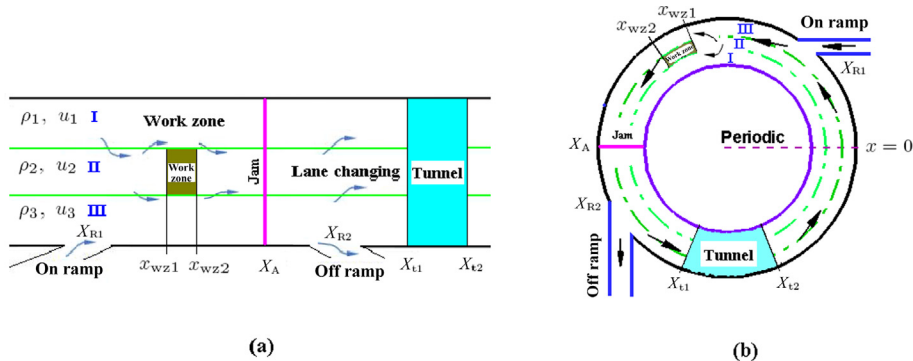


Fig. 1. (a) Illustration of a three lane vehicular flow. (b) Schematic of the vehicular traffic flow in a ring freeway with a tunnel, a work zone, and an initial jam at x_A . The work zone length is $L_{WZ} = x_{WZ2} - x_{WZ1}$.

lane III, except the mandatory lane change at the point just upstream of the work zone. Lane-changing in the normal segment and tunnel occurs spontaneously to keep the *local homogeneity* in the orthogonal direction, irrespective of whether lane changing is mandatory or discretionary. While the mandatory lane changing rates caused by the existence of work zone are analyzed alone.

To describe the three lane traffic flows in the normal segment and tunnel, for $l \in \{1, 2, 3\}$, traffic densities ρ_l and traffic flow rates $q_l (= \rho_l u_l)$ are taken as the main variables. Labeling the lane average density $\rho = (\rho_1 + \rho_2 + \rho_3)/3$, for any vehicle on lane I attempting to shift onto lanes II and III, its density should be $(\rho_1 - \rho)$ in unit of km. Labeling the traffic relaxation time on lane I as τ_l , the average relaxation time is $\tau = (\tau_1 + \tau_2 + \tau_3)/3$. Assume that the vehicular lane changing time is $(\tau\beta_\star)$. Then, the net lane-changing rate for the governing equation of ρ_1 can be approximated by $-(\rho_1 - \rho)/(\tau\beta_\star)$. Using the randomly generated ramp parameter σ that can be generated in a small range around a median, the equations of the three-lane traffic model may be written as

$$\begin{cases} \rho_{lt} + q_{lx} = -(\rho_l - \rho)/(\tau\beta_\star) + \sigma q_3 \delta_{l3}/l_0 + (-\delta_{l2}/l_0 + \delta_{l1}/l_0) \cdot [q_{wz1}]_{x_{wz1w}} \\ \quad + (-\delta_{l2}/l_0 + \delta_{l3}/l_0) \cdot [q_{wz3}]_{x_{wz1w}}, \\ u_{lt} + u_l u_{lx} = R_l/\rho_l. \end{cases} \quad (1)$$

where l_0 is the length scale of the traffic flow, the subscript $x_{wz1w} (= x_{wz1} - l_0/2)$ denotes the location, and q_{wz1} and q_{wz3} are the mandatory net lane changing rates. However, for traffic flows in the work zone, the lane average density is $\rho = (\rho_1 + \rho_3)/2$, for $l \in \{1, 3\}$, the relevant equations are

$$\begin{cases} \rho_{lt} + q_{lx} = 0, \\ u_{lt} + u_l u_{lx} = R_l/\rho_l, \end{cases} \quad (2)$$

where δ_{l2} and δ_{l3} are, respectively, the Kronecker deltas. Eqs. (1) and (2) indicate that there is a mass conservation of vehicular motion among lanes I, II, and III. Similarly, (Zhu and Yang, 2013; Bogdanova et al., 2015; Ma et al., 2018), R_l can be expressed as

$$R_l = (q_{el} - q_l)/\tau_l - p_{lx} + [\rho_l v_l u_{lx}]_x. \quad (3)$$

On lane $l \in \{1, 2, 3\}$, q_{el} and R_l/ρ_l are, respectively, the equilibrium flow rate and the acceleration, v_l and τ_l are, respectively, the traffic kinematic viscosity and the relaxation time, traffic pressure $p_l = c_l^2 \rho_l$, and c_l is the traffic sound speed. To simulate ramp flow, the three lane traffic model uses a random number σ produced in the small range $\Delta\sigma$ around an artificially median σ_{av} . Similar to the work of Huang et al. (2023), adopting $\tau\beta_\star$ to represent the lane changing time according to real traffic observation, we assume that the ratio of lane changing time to relaxation time can be written as

$$\beta_\star = \begin{cases} \infty, & |\rho_1 - \rho_2|, \text{ or } |\rho_2 - \rho_3| < 1 \text{ veh/km}, \\ (e-1)\rho/(\rho_m - \rho), & \text{otherwise,} \end{cases} \quad (4)$$

indicating that when the absolute value of traffic densities between the two lanes is below 1 veh/km, as vehicular drivers have not made any attempt to carry out lane-changing whether mandatory or discretionary, the ratio $\beta_\star = \infty$; otherwise, it is assumed to be inversely proportional to the vehicular spatial headway, equal to unity when the traffic flow is at the saturation with $\rho_s = \rho_m/e$. When traffic flow is completely jammed, $\beta_\star = \infty$, indicating that lane changing is hard to happen for congested flow. For unsaturated traffic flows, $\beta_\star < 1$. Particularly, for vehicular flows on lane II at the first critical density ρ_{*2} , using the data shown in Table 1, $\rho_{*2} = 0.0819$, one obtains $\beta_\star = 0.1533$, that means the lane-changing time is about 15.33%

Table 1
Parameters of vehicular flow on ring freeway.

$v_{f1}/(\text{km/h})$	120	ρ_{*1}	0.067 6	L_t/km	1.6
$v_{f2}/(\text{km/h})$	100	ρ_{*2}	0.081 9	l_{veh}/m	5.8
$v_{f3}/(\text{km/h})$	85	ρ_{*3}	0.096 0	$u_{c2}/(\text{km/h})$	18
$v_{f4}/(\text{km/h})$	60	ρ_{*4}	0.135 5	$\rho_m/(\text{veh/km})$	172
x_{br1}/m	80	ρ_{c21}	0.667 6	l_0/m	80
x_{br2}/m	65	ρ_{c22}	0.637 4	$v_0 = \rho_{*2} v_{f2}/(\text{m/s})$	2.275 6
x_{br3}/m	54.6	ρ_{c23}	0.608 8	t_0/s	35.156
x_{br4}/m	37	ρ_{c24}	0.549 0	L/km	100
$c_{\tau1}(v_0)$	5.437	x_{R1}/km	12	$I_{\max}$	1 251
$c_{\tau2}(v_0)$	4.879	x_{R2}/km	60	γ	0.191
$c_{\tau3}(v_0)$	4.428	x_A/km	50	$g_0 = 1 - 2\gamma$	0.618
$c_{\tau4}(v_0)$	3.664	x_{wz1}/km	39.96		
τ_1/s	5.388	x_{wz2}/km	40.12		
τ_2/s	7.206	L_{wz}/km	0.16		
τ_3/s	9.340	x_{t1}/km	80		
τ_4/s	15.989	x_{t2}/km	81.6		

¹ ρ_{*j} and ρ_{c2j} are normalized by ρ_m .

² Freeway work zone length $L_{wz} = x_{wz2} - x_{wz1}$.

of the average relaxation time. For traffic flow on empty road, the lane changing time is zero. This is of course not consistent with traffic reality.

In comparison with existing modules of lane-changing, the present model is simpler than the model proposed by [Laval and Daganzo \(2006\)](#), which has four easy-to-measure parameters, and reproduces two bottleneck phenomena: (i) the drop in the discharge rate of freeway bottlenecks when congestion begins, and (ii) the relationship between the speed of a moving bottleneck and its capacity.

In the normal road segment excluding the tunnel and work zone, lane numbers are $l \in \{1, 2, 3\}$, vehicles have different free flow speeds and braking distances on different lanes, indicating that the equilibrium traffic flow rate is lane-dependent, as shown in [Fig. 2](#). Let the jam density be ρ_m , in the normal road segment, the equilibrium traffic flow rate can be written as ([Zhang et al., 2018b](#))

$$q_{el} = \begin{cases} \rho_l v_{fl}, & \rho_l \leq \rho_{sl}, \\ -c_{\tau l} \rho_l \ln(\rho_l / \rho_m), & \rho_{sl} < \rho_l \leq \rho_{c2l}, \\ B_l \rho_l \{1 - \text{sech}[\Lambda_l \ln(\rho_l / \rho_m)]\}, & \rho_{c2l} < \rho_l \leq \rho_m. \end{cases} \quad (5)$$

In the work zone, the lane number is limited to $l \in \{1, 3\}$. While in the tunnel, as the free flow speed is just the same as the tunnel speed limit v_{f4} , the equilibrium traffic flow rate is

$$q_{el} = \begin{cases} \rho_l v_{f4}, & \rho_l \leq \rho_{sl} \\ -c_{\tau 4} \rho_l \ln(\rho_l / \rho_m), & \rho_{sl} < \rho_l \leq \rho_{c2l} \\ B_4 \rho_l \{1 - \text{sech}[\Lambda_l \ln(\rho_l / \rho_m)]\}, & \rho_{c2l} < \rho_l \leq \rho_m, \end{cases} \quad (6)$$

where the definitions of parameter B_j and the equilibrium traffic speed at saturation state c_{tj} for $j \in \{1, 2, 3, 4\}$ have been described in the work of [Zhang et al. \(2018b\)](#). The average length of vehicles l_{veh} has been used in the definition of c_{tj} .

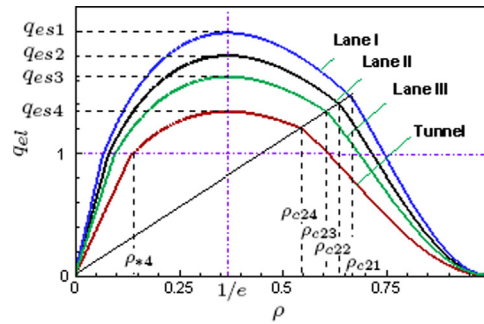


Fig. 2. Fundamental diagram (FD) for a three lane vehicular flow. ρ is measured by the jam density ρ_m , the traffic flow rate has the unit of $q_0 = \rho_m v_0$, and $q_{esj} = c_{tj}/e \cdot [\rho_{sj} v_{fj} / \rho_{s2} v_{f2}]$, $j \in \{1, 2, 3, 4\}$.

In particular, q_{wz1} and q_{wz3} at $x = x_{wz1w}$ are approximated by the random number model based on the golden ratio analysis: the sum of the mandatory net lane-changing rates to q_2 is approximately equal to $1 - g_0$, here $g_0 (= 0.618)$ is the golden ratio. Hence, assuming γ_1 and γ_2 are two random numbers generated by different seeds with the same median $\gamma = (1 - g_0)/2 = 0.191$ and the number range $\Delta\gamma = 0.05$, the net changing rates at $x = x_{wz1w}$ are assumed to be

$$q_{wz1} = \gamma_1 q_1(t, x_{wz1w}), \quad q_{wz3} = \gamma_2 q_3(t, x_{wz1w}). \quad (7)$$

Obviously, there is a demand of further analysis for the choice of random number median. As an example, the evolution curves of q_{wz1} and q_{wz3} during the period of $t \in (0.2, 0.22)$ h for the initial density normalized by the jam density $\rho_0 = 0.25$ are shown in [Fig. 3](#).

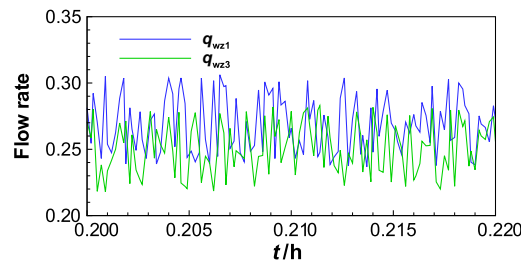


Fig. 3. Mandatory lane-changing rates q_{wz1} and q_{wz3} just upstream of the work zone during the period of $t \in (0.2, 0.22)$ h for $\rho_0 = 0.25$ in the case of RF1. The unit of the lane-changing rate is $\rho_m v_0$.

3. Numerical method

As details of the right and left characteristic matrices are crucial in building simulation platform, we will show the expressions of matrix elements explicitly.

In the normal road segment, the pressure gradient p_{lx} is related to sound speed as

$$p_{lx} = c_l^2 \rho_{lx}.$$

Then, using

$$R_{l*} = R_l + p_{lx} + u_l \cdot \left\{ -(\rho_l - \rho)/(\tau\beta_\star) + \sigma q_3 \delta_{l3}/l_0 + (\delta_{l2}/l_0 - \delta_{l3}/l_0) \cdot [q_{wz1}]_{x_{wz1w}} + (-\delta_{l2}/l_0 + \delta_{l3}/l_0) \cdot [q_{wz3}]_{x_{wz1w}} \right\},$$

instead of R_l for $l \in \{1, 2, 3\}$, Eqs. (1) can be written as

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} = \mathbf{S}, \quad (8)$$

where $\mathbf{U} = (\rho_1, q_1, \rho_2, q_2, \rho_3, q_3)^\top$, $\mathbf{F}(\mathbf{U}) = (q_1, q_1^2/\rho_1 + p_1, q_2, q_2^2/\rho_2 + p_2, q_3, q_3^2/\rho_3 + p_3)^\top$, and

$$\mathbf{S} = \begin{pmatrix} -(\rho_1 - \rho)/(\tau\beta_\star) \\ R_{1*} \\ -(\rho_2 - \rho)/(\tau\beta_\star) + [q_{wz1}]_{x_{wz1w}}/l_0 \\ R_{2*} \\ -(\rho_3 - \rho)/(\tau\beta_\star) - [q_{wz1}]_{x_{wz1w}}/l_0 + \sigma q_3/l_0 \\ R_{3*} \end{pmatrix}.$$

On lanes I and III in the work zone $x \in [x_{wz1}, x_{wz2}]$, for $l \in \{1, 3\}$, according to Eq. (2),

$$R_{l*} = R_l + p_{lx}.$$

Take $\mathbf{U} = (\rho_1, q_1, \rho_3, q_3)^\top$, $\mathbf{F}(\mathbf{U}) = (q_1, q_1^2/\rho_1 + p_1, q_3, q_3^2/\rho_3 + p_3)^\top$, Then, the source term becomes

$$\mathbf{S} = \begin{pmatrix} 0 \\ R_{1*} \\ 0 \\ R_{3*} \end{pmatrix}.$$

The WENO5 scheme has been reported in a detail by Zhang et al. (2022). To handle the time derivative term, the RK3 scheme (Shu, 1988; Shu and Osher, 1989) is used. For the convenience of description,

$$\mathcal{L}(\mathbf{U}) = -\frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} + \mathbf{S}. \quad (9)$$

To seek the numerical solution of

$$\frac{\partial \mathbf{U}}{\partial t} = \mathcal{L}(\mathbf{U}), \quad (10)$$

the RK3 can be written as

$$\begin{cases} \mathbf{U}_i^{(1)} &= \mathbf{U}_i^n + \Delta t \mathcal{L}(\mathbf{U}^n), \\ \mathbf{U}_i^{(2)} &= (3\mathbf{U}_i^n + \mathbf{U}_i^{(1)})/4 + \Delta t \mathcal{L}(\mathbf{U}^{(1)})/4, \\ \mathbf{U}_i^{n+1} &= (\mathbf{U}_i^n + 2\mathbf{U}_i^{(2)})/3 + 2\Delta t \mathcal{L}(\mathbf{U}^{(2)})/3, \end{cases} \quad (11)$$

where the superscript n denotes the time level.

The Courant–Friedrichs–Lewy (CFL) condition of the numerical method is

$$CFL = \frac{\Delta t}{\Delta x} \cdot \max |a_{k,i}| < 1, \quad i = 0, 1, 2, \dots, I_{\max} - 1, \quad (12)$$

where $a_{k,i}$ represents the k th eigenvalue for $\mathbf{A}$ at x_i , $I_{\max}$ is the maximum number of grid, and CFL (Shui, 1998) is fixed at 0.3 to ensure the numerical stability and calculate the time step Δt .

4. Results and discussion

4.1. Simulation parameters

As shown in Table 1, the free flow speeds on lanes I, II, and III are $v_{f1} = 120$ km/h, $v_{f2} = 100$ km/h, and $v_{f3} = 85$ km/h, respectively, the corresponding braking distances x_{br1} , x_{br2} , and x_{br3} are 80 m, 65 m and 54.6 m, the equilibrium speeds at saturation state on lanes I, II, and III $c_{\tau1}$, $c_{\tau2}$, and $c_{\tau3}$ are 5.437, 4.879, and 4.428 in the unit of $v_0 = \rho_{s2} v_{f2}$, and the relaxation times of vehicular flow on lanes I, II, and III at the second critical points (ρ_{c21} , ρ_{c22} and ρ_{c23}) τ_1 , τ_2 , and τ_3 are respectively, 5.388s, 7.206s, and 9.340 s. The tunnel speed limit is v_{f4} , correspondingly, the braking distance is $x_{br4} = 37$ m, the equilibrium speed at saturation state $c_{\tau4} = 3.664$ v_0 , $\tau_4 = 15.989$ s.

From the second column of Table 1, one can also see the first and second critical densities ρ_{s1} and ρ_{c21} . Besides, the positions of the first, second and the third ramp intersections are $x_{R1} = 12$ km, $x_{R2} = 60$ km, and the position of initial jam is $x_A = 50$ km.

The third column of Table 1 shows the ahead boundary of roadway work zone $x_{wz1} = 39.96$ km, the end boundary of roadway work zone $x_{wz2} = 40.12$ km, and the roadway work zone length $L_{wz} = 0.16$ km. The column also gives the second critical speed u_{c2} , the jam density ρ_m , the average length of cars l_{veh} , the scales of length, speed, and time l_0 , v_0 , t_0 , the total length of the ring freeway $L = (I_{max} - 1)l_0 = 100$ km, and the maximum number of grid $I_{max} = 1251$. In the numerical tests, the Reynolds number for the vehicular flow $Re = l_0 v_0 / \nu$ is 64. The parameters of random number generator for ramp flows are shown in Table 2.

Table 2
Parameters of random number generator for ramp flows.

Case	σ_{1av}	σ_{2av}	$\Delta\sigma$
RF0	0	0	0
RF1	0.12	-0.12	0.05
RF2	0.24	-0.24	0.05

In the numerical simulation of ring road traffic flow as shown in Fig. 1(b), the fundamental diagram is shown in Fig. 2. It is assumed that the initial density is given by

$$\rho_l(0, x) = \begin{cases} 1, & x \in [x_A - 1/2x_A + 1/2], \\ \rho_0(1 + 0.05\delta_{l2}), & \text{otherwise,} \end{cases} \quad (13)$$

with $q_l(0, x) = q_{el}(\rho(0, x))$. The density on lane II is slightly higher than those on lanes I and III, so that lane-changing can occur. The propagation of traffic jams depends closely on the value of ρ_0 , the effects of roadway work zone and ramp flow, and the traffic elasticity (Smirnova et al., 2016; Smirnova et al., 2017).

To examine the macroscopic model characterized by the ratio of lane-changing time to relaxation time β_* , a numerical study has been carried out by Li et al., (2022) to seek the effects of tunnel speed limit on traffic flow, where the observed speed data (Koshi et al., 1992) and the calculated speed on the basis of a behavioral kinematic wave model developed by Jin (2018) were used to make a comparison, which has shown that the predicted speeds based on a three lane traffic model agree quite well with the published data.

4.2. Threshold of traffic jam formation

By investigating the spatiotemporal evolution of vehicular spatial headway $h_{d3} [= (1 - \rho_3)/(\rho_m \rho_3)]$ (m), whether a traffic jam is triggered by the tunnel or work zone can be ascertained. From Fig. 4(a), it can be seen that the first threshold of traffic jam formation ρ_{th} is 0.15, at which the tunnel mean travel time may occur a tuning point, across which the tunnel mean travel time appears a platform, as can be seen in Table 3.

From Figs. 4(b) and 4(c), it can be seen that a blue line occurs in front of the work zone, indicating that a traffic jam is induced, here we call the relevant initial density ρ_0 as the second threshold $\rho_{th}^* = 0.19$. Such a threshold cannot be obtained by examining the variation of the tunnel mean travel time with the increase of ρ_0 . Comparing the evolution shown in Fig. 4(c) with that in Fig. 4(b), one can see that the on ramp flow at the first intersection $x_{R1} = 12$ km causes the decrease of h_{d3} in the downstream segment near the intersection, and the off ramp flow at the second intersection $x_{R2} = 60$ km leads to the increase of h_{d3} in the downstream segment near the intersection. Note that the ramp flow has not changed the value of the second threshold of traffic jam formation ρ_{th}^* .

In Figs. 4(b) and (c), it can be seen that the work zone has resulted in the light red region downstream of the end boundary at x_{wz2} , where traffic density is lower, and the traffic speed is higher, suggesting that in the light red region, the travel time is shorter.

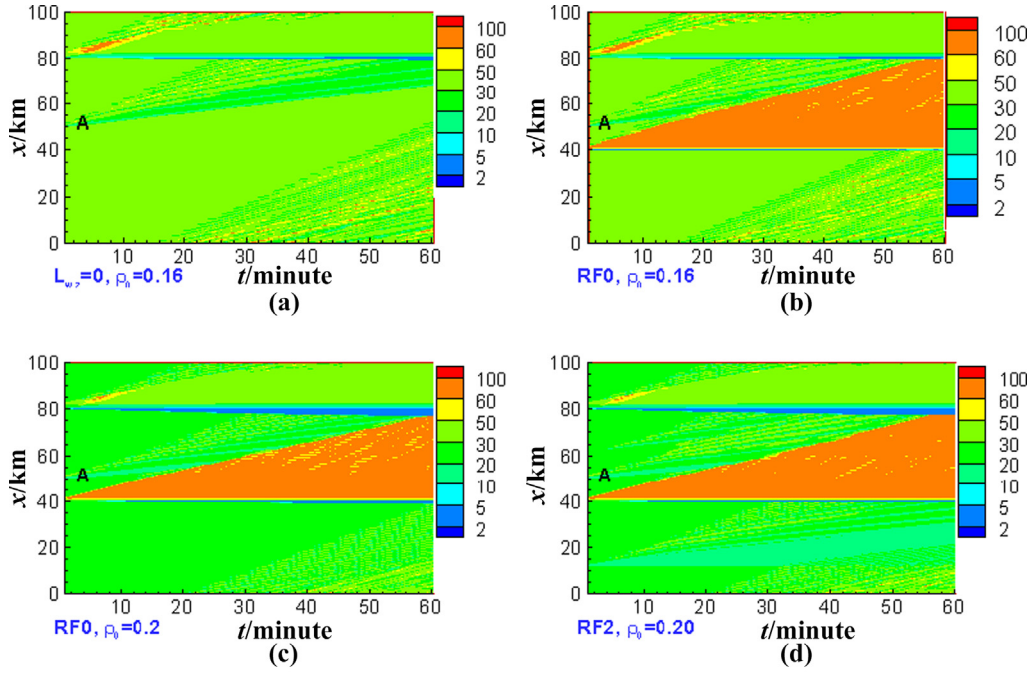


Fig. 4. Spatiotemporal evolution of spatial headway $h_{d3} [= (1 - \rho_3)/(\rho_m \rho_3)]$ (m) on the ring.

Table 3

Distributions of $T_{t,1av}$ and $T_{tu,1av}$ for the case of RF0.

ρ_0	$T_{t,1av}/h$	$T_{t,2av}/h$	$T_{t,3av}/h$	$T_{tu,1av}/h$	$T_{tu,2av}/h$	$T_{tu,3av}/h$
0.07	0.869 4	1.020 4	1.190 9	0.020 9	0.022 8	0.024 3
0.08	0.903 1	1.035 2	1.199 7	0.021 8	0.023 2	0.024 7
0.09	0.916 3	1.052 5	1.202 0	0.022 0	0.023 5	0.024 7
0.10	0.950 7	1.084 6	1.220 4	0.022 7	0.024 0	0.025 0
0.11	0.989 8	1.121 3	1.253 3	0.024 4	0.025 4	0.026 3
0.12	1.032 7	1.154 9	1.289 1	0.028 1	0.028 0	0.028 5
0.13	1.073 0	1.197 1	1.329 8	0.033 4	0.033 2	0.033 3
0.14	1.109 7	1.235 6	1.368 4	0.042 9	0.042 7	0.042 6
0.15	1.154 5	1.282 0	1.414 8	0.052 1	0.051 8	0.051 6
0.16	1.204 7	1.333 9	1.467 6	0.057 8	0.057 6	0.057 3
0.17	1.251 7	1.384 9	1.521 8	0.058 6	0.058 4	0.058 1
0.18	1.291 0	1.431 8	1.573 9	0.059 0	0.058 8	0.058 6
0.19	1.351 3	1.492 3	1.642 7	0.059 3	0.059 0	0.058 8
0.20	1.404 6	1.550 3	1.707 3	0.059 4	0.059 2	0.059 0
0.21	1.459 2	1.610 8	1.773 4	0.059 6	0.059 3	0.059 1
0.22	1.513 8	1.670 9	1.840 0	0.059 7	0.059 4	0.059 2
0.23	1.568 5	1.731 7	1.906 3	0.059 8	0.059 5	0.059 3
0.24	1.623 0	1.792 6	1.972 9	0.059 9	0.059 6	0.059 4
0.25	1.677 5	1.852 6	2.039 4	0.060 0	0.059 7	0.059 5
0.26	1.733 5	1.915 1	2.107 5	0.060 1	0.059 8	0.059 6
0.27	1.789 4	1.977 4	2.175 8	0.060 2	0.059 9	0.059 7
0.28	1.846 2	2.041 3	2.245 5	0.060 3	0.060 0	0.059 8
0.29	1.903 1	2.104 9	2.314 8	0.060 4	0.060 1	0.059 9
0.30	1.960 2	2.168 0	2.384 4	0.060 5	0.060 3	0.060 1
0.31	2.017 7	2.232 7	2.454 4	0.060 6	0.060 4	0.060 2
0.32	2.075 5	2.297 6	2.524 9	0.060 8	0.060 6	0.060 4
0.33	2.132 7	2.362 8	2.595 4	0.061 0	0.060 8	0.060 6
0.34	2.192 6	2.431 0	2.668 6	0.061 4	0.061 1	0.060 9
0.35	2.251 8	2.498 1	2.742 1	0.061 8	0.061 6	0.061 4
0.36	2.313 8	2.569 1	2.818 3	0.062 5	0.062 3	0.062 1
0.368	2.365 6	2.628 2	2.881 3	0.063 4	0.063 2	0.063 0
0.38	2.444 5	2.714 8	2.975 9	0.065 7	0.065 5	0.065 4
0.39	2.510 9	2.788 1	3.056 1	0.070 2	0.069 9	0.069 8

4.3. Variable distributions

For $\rho_0 = 0.25$ in the case of RF1, when $t = 1$ h, the distributions of ρ_l and u_l , $l \in \{1, 2, 3\}$, and s_1 are shown in Figs. 5(a), 5(b), and 5(c). As a result of traffic jam formed at the beginning of roadway work zone at x_{wz1} , in the segment close to x_{wz1} , ρ_l is higher, u_l is correspondingly lower. Similar property occurs in the segment close to the tunnel entrance, suggesting that a traffic jam has been triggered by the tunnel. The density in the traffic jam induced by the tunnel is slightly higher than that of the upstream traffic jam triggered by the work zone.

The spatially oscillations of ρ_l upstream of the work zone originated from traffic jam reflect the influence of the propagation and interaction of the initial and spontaneously generated jams. In the work zone, $s_1 = \rho_1/(\rho_1 + \rho_3)$, hence s_1 has a value around 0.5. Otherwise, $s_1 = \rho_1/(\rho_1 + \rho_2 + \rho_3)$, as the result of local vehicular homogeneity, s_1 has a value around 1/3. Theoretically, the equation of s_1 derived from Eq. (1) has the form

$$\frac{\partial s_1}{\partial t} + \frac{1}{3\rho} \left[(1 - s_1) \frac{\partial q_1}{\partial x} - s_1 \frac{\partial (q_2 + q_3)}{\partial x} \right] = \frac{1/3 - s_1}{\tau\beta_\star} - \frac{s_1\sigma q_3}{3\rho l_0},$$

where the source term $(1/3 - s_1)/(\tau\beta_\star)$ indicates that the desired value of s_1 is 1/3, another source term $-s_1\sigma q_3/(3\rho l_0)$ reflects the ramp-flow effects.

Correspondingly, Table 4 shows the values of ρ_l , u_l , and s_1 near the roadway work zone. It can be seen that in the traffic jam triggered by the work zone, ρ_l is generally over 0.5, excluding the grid on lane II at $x = 39.92$ km which is nearest to x_{wz1} . In particular, on lanes I and III, the jam terminates at the end boundary of the work zone x_{wz2} .

As can be seen in Table 4, in the jam for $x \in [39.44, 39.92]$ km, with the increase of x , ρ_2 decreases from 0.590 7 to 0.395 6, ρ_1 and ρ_3 have an increasing tendency: ρ_1 changes from 0.612 7 to 0.649 3 and ρ_3 varies from 0.607 5 to 0.618 7. In the downstream segment close to x_{wz2} , ρ_1 and ρ_3 maintain a decreasing trend: at first when x increases from 40.16 km to 40.64 km, ρ_1 drops from 0.197 2 to 0.096 5, ρ_3 drops from 0.208 8 to 0.101 1, and then in the downstream range with a length of 1.04 km, ρ_1 and ρ_3 gradually decrease to 0.079 6 and 0.086 5 at $x = 41.68$ km, respectively.

4.4. Travel time

As in the work zone on lane II $x \in [x_{wz1}, x_{wz2}]$, the traffic speed is zero, being not appropriate for the prediction of travel time through lane II using the grid traffic speed. Alternatively, the density-averaged grid traffic speed is used, as the work zone length is shorter, $L_{wz} = 0.16$ km, which is just 0.16% of the total road length $L = 100$ km (as shown in Table 1).

On the basis of grid speed from numerical simulation with a reliable scheme with 3rd order accuracy, it is found in the absence of work zone ($L_{wz} = 0$), as shown by the coarse solid blue curve in Fig. 6(a), the mean travel time through lane I $T_{t,1av}$ is slightly higher than that estimated for the cases with the work zone having a length of $L_{wz} = 0.16$ km, implying that the tunnel with a length of 1.6 km has played a dominant role in the prediction of $T_{t,1av}$.

The thin solid blank line in Fig. 6(a) shows $T_{t,1av}$ for RF0 without the influence of ramp flow, the corresponding values are shown in the second column of Table 3. $T_{t,1av}$ increases with the initial density ρ_0 . When $\rho_0 = 0.07$, $T_{t,1av} = 0.8694$ h, slightly over the travel time predicted by v_{f1} , 0.83 h. When ρ_0 increases from 0.25 to the saturation density $\rho_s = 1/e \approx 0.368$, $T_{t,1av}$

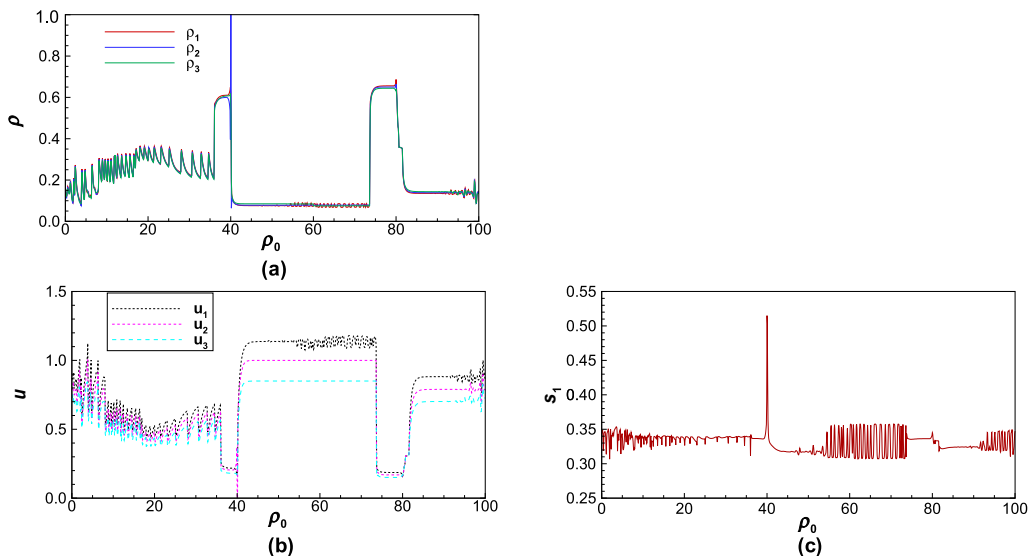
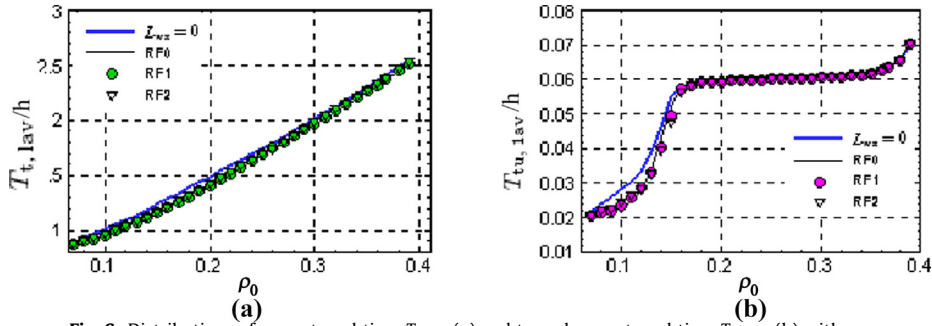


Fig. 5. Distributions of (a) traffic density, (b) traffic speed, and (c) density fraction $s_1 [= \rho_1 / \sum \rho_l]$ at $t = 1$ h on the ring road for $\rho_0 = 0.25$ in the case of RF1.

Table 4Predicted (ρ_l, u_l) for $l \in \{1, 2, 3\}$ and s_l near the roadway work zone at $t = 1$ h for the case of RF1 and $\rho_0 = 0.25$.

x	ρ_1	u_1^1	ρ_2	u_2	ρ_3	u_3	s_1
39.44	0.612 7	0.213 6	0.590 7	0.216 8	0.607 5	0.179 9	0.338 3
39.52	0.614 9	0.210 3	0.586 1	0.222 0	0.607 6	0.179 9	0.340 0
39.60	0.618 0	0.206 1	0.579 7	0.229 1	0.607 5	0.179 8	0.342 3
39.68	0.622 4	0.200 4	0.570 6	0.239 2	0.608 0	0.178 5	0.345 6
39.76	0.628 6	0.192 3	0.557 3	0.254 1	0.609 3	0.176 0	0.350 2
39.84	0.636 8	0.181 3	0.535 9	0.274 3	0.611 4	0.172 1	0.356 9
39.92	0.649 3	0.172 2	0.395 6	0.371 7	0.618 7	0.170 1	0.390 3
40.00	0.623 6	0.175 8	1.000 0	0.000 0	0.587 2	0.186 7	0.515 0
40.08	0.514 4	0.213 1	1.000 0	0.000 0	0.507 9	0.215 8	0.503 2
40.16	0.197 2	0.533 0	0.063 8	0.593 4	0.208 8	0.465 4	0.419 8
40.24	0.131 4	0.656 9	0.093 4	0.663 7	0.135 8	0.538 9	0.364 5
40.32	0.120 8	0.714 7	0.107 2	0.713 0	0.122 6	0.589 5	0.344 5
40.40	0.111 9	0.771 7	0.104 8	0.746 5	0.115 1	0.628 2	0.337 3
40.48	0.105 2	0.820 2	0.100 2	0.781 3	0.109 3	0.661 7	0.334 4
40.56	0.100 3	0.860 8	0.096 7	0.809 4	0.104 8	0.690 1	0.332 3
40.64	0.096 5	0.894 9	0.093 9	0.833 5	0.101 1	0.714 1	0.330 9
40.72	0.093 4	0.924 0	0.091 6	0.854 5	0.098 2	0.736 1	0.329 9
40.80	0.090 9	0.949 0	0.089 7	0.873 0	0.095 8	0.755 6	0.329 0
40.88	0.088 9	0.970 7	0.088 0	0.889 3	0.093 8	0.770 0	0.328 5
40.96	0.087 2	0.989 5	0.086 6	0.903 7	0.092 1	0.784 5	0.328 0
41.04	0.085 8	1.006 0	0.085 4	0.916 4	0.091 1	0.795 2	0.327 1
41.12	0.084 6	1.020 4	0.084 4	0.927 7	0.090 0	0.802 7	0.326 7
41.20	0.083 6	1.033 1	0.083 5	0.937 8	0.089 1	0.810 6	0.326 3
41.28	0.082 7	1.044 3	0.082 7	0.946 8	0.088 6	0.817 4	0.325 6
41.36	0.081 9	1.054 3	0.082 0	0.955 0	0.088 0	0.821 5	0.325 1
41.44	0.081 2	1.063 1	0.081 4	0.962 0	0.087 4	0.826 0	0.324 8
41.52	0.080 6	1.071 0	0.080 9	0.967 8	0.087 1	0.830 2	0.324 2
41.60	0.080 1	1.077 9	0.080 5	0.972 9	0.086 8	0.832 8	0.323 7
41.68	0.079 6	1.084 2	0.080 1	0.977 0	0.086 5	0.835 3	0.323 3

¹ u_l is normalized by v_{l2} .**Fig. 6.** Distributions of mean travel time $T_{t,1av}$ (a) and tunnel mean travel time $T_{tu,1av}$ (b) with ρ_0 .

grows from 1.677 5 h to 2.365 6 h. For the over saturated case $\rho_0 = 0.39$, the mean travel time $T_{t,1av}$ is 2.510 9 h. The green filled circles and the gradient symbols represent $T_{t,1av}$ for the cases RF1 and RF2 that agree well with that for the case RF0, indicating that the influence of ramp flows on the mean travel time is very small.

In Fig. 6(b), the dependence of tunnel mean travel time $T_{tu,1av}$ through lane I is shown, where the thin solid black curve corresponds to the values given by the fifth column of Table 3, the coarse solid blue curve illustrates $T_{tu,1av}$ for the case without work zone $L_{wz} = 0$. There exists a turning point being relevant to the threshold of traffic jam formation, $\rho_{th} = 0.15$, beyond which the blue curve occurs a plateau that ends at the saturation point ρ_s . For over saturated traffic flow, $T_{tu,1av}$ increases with ρ_0 . As can be seen in Fig. 6(b), when ρ_0 is lower than the turning point density $\rho_{th} = 0.15$, $T_{tu,1av}$ for the case of $L_{wz} = 0$ is slightly longer, indicating that the existence of work zone can slightly increase the traffic speed in the tunnel.

As shown in the fifth column of Table 3, the plateau height is about 0.06 h, it means that as soon as a traffic jam appears and propagates in the upstream direction, the time averaged traffic speed through the tunnel ($L_t/T_{tu,1av}$) is around 26.67 km/h, which agrees well with the recorded speed 28.9 km/h (Koshi et al., 1992) and the predicted speed 33.8 km/h (Jin, 2018).

The mean travel time through lane l $T_{t,lav}$ for $l = 1, 2, 3$ and the root mean square values $T'_{t,l}$ are shown in Figs. 7(a) and 7(b). For the case of RF0, $T_{t,lav}$ for $l = 1, 2, 3$ are illustrated by the coarse solid black curve, thin solid purple curve, and thin solid blue curve, respectively. While for RF1, the median σ_{1av} is set as 0.12 (Table 2), $T_{t,lav}$ and $T'_{t,l}$ for $l = 1, 2, 3$ are

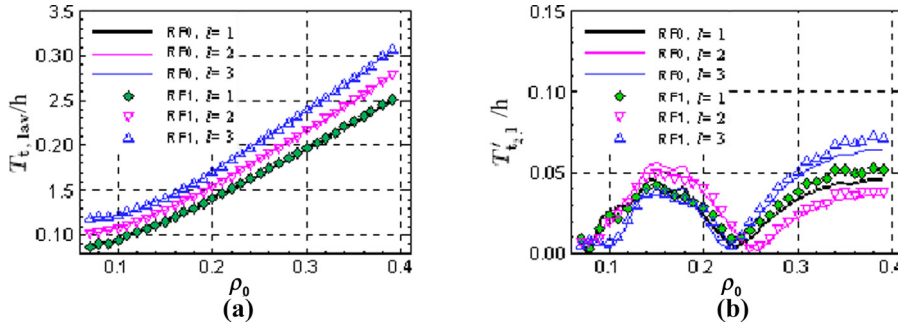


Fig. 7. Distributions of mean travel time $T_{t,lav}$ (a), and rms of travel time $T'_{t,lav}$ (b) with ρ_0 .

demonstrated by green filled circles, gradient and delta symbols, respectively, the corresponding values are shown in Table 5. It can be seen that ramp flow has brought small influence on $T_{t,lav}$, but observable influence on $T'_{t,l}$.

When ρ_0 is equal to the second threshold of traffic jam formation $\rho_{th}^* = 0.2$, $T_{t,lav}$ for $l = 1, 2, 3$ are 1.404 6, 1.550 3, and 1.707 3 h in the case of RF1, as shown in Table 5. From Fig. 7(a), for $\rho_0 < \rho_{th}^*$, with the increase of ρ_0 , $T_{t,lav}$ has a growing trend in a power law form, as shown by Zhang et al. (2022); Otherwise, for $\rho_0 \geq \rho_{th}^*$, but less than 0.39, $T_{t,lav}$ depends upon ρ_0 linearly.

As the free flow speed v_{fl} determines the equilibrium traffic speed on lane l , i.e., q_{el}/ρ_l , with the value of v_{fl} given in the first column of Table 1, for a given initial density ρ_0 , it is reasonable that $T_{t,3av}$ takes the largest value among the three mean travel times, as shown in Fig. 7(a).

From Fig. 7(b), it can be seen that $T'_{t,l}$ occur local crests with values generally below 0.051 h at $\rho_0 = 0.15$, and three local valleys with values generally below 0.011 h at $\rho_0 \approx 0.235$. As shown in Table 5, for RF1, the three troughs of $T'_{t,l}$ appear at $\rho_0 = 0.23, 0.24$, and 0.23 for $l = 1, 2$, and 3 respectively. After passing through the local troughs, the travel time fluctuations $T'_{t,l}$ enhance with the increase of ρ_0 in a way at first rapidly and then gradually to a value below 0.065 h at $\rho_0 = 0.39$.

Table 5
Distributions of $T_{t,lav}$ and $T'_{t,l}$ for the case of RF1.

ρ_0	$T_{t,1av}/h$	$T_{t,2av}/h$	$T_{t,3av}/h$	$T'_{t,1}/h$	$T'_{t,2}/h$	$T'_{t,3}/h$
0.07	0.869 4	1.020 4	1.190 9	0.004 1	0.004 1	0.003 3
0.08	0.903 1	1.035 2	1.199 7	0.002 0	0.006 6	0.007 7
0.09	0.916 3	1.052 5	1.202 0	0.017 7	0.010 0	0.006 4
0.1	0.950 7	1.084 6	1.220 4	0.025 7	0.021 5	0.003 8
0.11	0.989 8	1.121 3	1.253 3	0.028 5	0.027 8	0.008 5
0.12	1.032 7	1.154 9	1.289 1	0.028 8	0.034 4	0.019 1
0.13	1.073 0	1.197 1	1.329 8	0.034 9	0.042 4	0.029 7
0.14	1.109 7	1.235 6	1.368 4	0.043 6	0.052 5	0.039 6
0.15	1.154 5	1.282 0	1.414 8	0.044 8	0.055 5	0.042 7
0.16	1.204 7	1.333 9	1.467 6	0.040 0	0.053 2	0.041 1
0.17	1.251 7	1.384 9	1.521 8	0.037 2	0.052 0	0.039 1
0.18	1.291 0	1.431 8	1.573 9	0.040 2	0.053 2	0.039 5
0.19	1.351 3	1.492 3	1.642 7	0.030 4	0.045 4	0.029 8
0.2	1.404 6	1.550 3	1.707 3	0.024 0	0.038 1	0.021 8
0.21	1.459 2	1.610 8	1.773 4	0.016 3	0.030 5	0.012 8
0.22	1.513 8	1.670 9	1.840 0	0.009 9	0.022 8	0.005 1
0.23	1.568 5	1.731 7	1.906 3	0.004 8	0.016 3	0.005 1
0.24	1.623 0	1.792 6	1.972 9	0.005 3	0.010 1	0.011 2
0.25	1.677 5	1.852 6	2.039 4	0.009 4	0.004 3	0.017 6
0.26	1.733 5	1.915 1	2.107 5	0.014 9	0.002 4	0.024 1
0.27	1.789 4	1.977 4	2.175 8	0.020 1	0.007 8	0.030 8
0.28	1.846 2	2.041 3	2.245 5	0.025 2	0.013 5	0.037 6
0.29	1.903 1	2.104 9	2.314 8	0.029 5	0.017 8	0.043 0
0.3	1.960 2	2.168 0	2.384 4	0.033 2	0.022 0	0.047 4
0.31	2.017 7	2.232 7	2.454 4	0.036 4	0.025 1	0.051 3
0.32	2.075 5	2.297 6	2.524 9	0.038 9	0.027 8	0.054 4
0.33	2.132 7	2.362 8	2.595 4	0.040 2	0.029 4	0.056 5
0.34	2.208 1	2.436 4	2.682 0	0.051 2	0.036 7	0.067 5
0.35	2.265 8	2.504 0	2.755 8	0.051 7	0.038 6	0.069 5
0.36	2.313 8	2.569 1	2.818 3	0.043 0	0.033 8	0.061 6
0.368	2.365 6	2.628 2	2.881 3	0.044 3	0.035 3	0.063 2
0.38	2.444 5	2.714 8	2.975 9	0.045 4	0.035 9	0.064 2
0.39	2.510 9	2.788 1	3.056 1	0.045 5	0.035 6	0.063 9

4.5. Vehicle fuel consumption

Any vehicle has its own performance of fuel consumption. To predict vehicle fuel consumption on the basis of the time averaged grid traffic speed $\bar{u}_l$, a reference curve as shown in Fig. 8 is needed. It is seen that the curve has steps, showing the dependence of fuel consumption F_c on vehicular speed u . The four points 'a, b, c, d' are used to identify the changes of vehicular gear number, with the relevant vehicular speeds given by

$$u_a = 0.077 \text{ 4}, u_b = 0.290 \text{ 3}, u_c = 0.367 \text{ 7}, u_d = 0.4 \text{ in the unit of } v_{f2}.$$

As shown Fig. 9(a), the time averaged grid traffic speed $\bar{u}_l$ have two troughs caused by the work zone and the tunnel. In the case of RF2, as illustrated by the dash-dotted curves, the on-ramp flow at x_{R1} makes $\bar{u}_l$ drop suddenly and then increase gradually in the downstream segment near x_{R1} ; as a result of the off-ramp flow at x_{R2} , $\bar{u}_l$ are slightly higher in the downstream segment near x_{R2} as compared with that for RF0. Using $\bar{u}_l$ in the interpolation with the green solid curve shown in Fig. 8, the fuel consumption through a grid length $F_{cl}(x_i)$ can be calculated, as shown in Fig. 9(b). Because the grid length $\Delta x_i (= l_0)$ in the numerical simulation of this paper is selected as $l_0 = 80 \text{ m}$ (as shown Table 1), being close to 100 m, while the unit of $F_c(\bar{u}_l(x_i))$ is $1/100 \text{ km}$, therefore ml is selected as the unit of F_{cl} . It can be seen that not only in the upstream segment near the ahead boundary of work zone but also in the upstream segment near the tunnel entrance, the grid fuel consumption f_{cl} takes a value around 8.5 ml. Otherwise, f_{cl} is usually below 6 ml, except for f_{cl} in the segment near the intersection of on ramp flow at $X_{R1} = 12 \text{ km}$, as shown by the blue dashed curve in Fig. 9(b).

Taking the summation of $f_{cl}(x_i)$, the total vehicle fuel consumption through the ring freeway is given by

$$F_{ul} = 10^{-3} \cdot \sum_{i=1}^{l_{\max}} f_{cl}(x_i).$$

The unit of F_{ul} is taken as l by using the factor 10^{-3} .

The fuel consumption through the ring freeway F_{ul} is plotted as a function of initial density ρ_0 , as shown in Fig. 10, where the solid curves are obtained in the case of RF0, with squares, purple-filled circles, and blue-filled delta symbols corresponding to the case of RF2. Comparing the fuel consumption between the two cases, it can be seen that ramp flow can lead to an observable deviation of F_{ul} when $\rho_0 > 0.27$. F_{ul} is lane dependent, and concavely distributed in the range of $\rho_0 \in [0.07, 0.39]$, implying that there exists a local minimum. For the fuel consumption on lane I F_{u1} , as shown by the coarse solid black curve and in the fifth column of Table 6, the local minimum of F_{u1} 6.724 3 l appears at $\rho_0 = 0.22$, with the two terminal values: 7.514 1 l at $\rho_0 = 0.07$, and 7.153 7 l at $\rho_0 = 0.39$. As shown in the sixth and seventh columns of Table 6, F_{u2} for RF0 has a minimum 6.622 4 l at $\rho_0 = 0.19$, F_{u3} for RF0 takes its minimum of 6.525 7 l at $\rho_0 = 0.14$. From Fig. 10, it can be seen that in the range of $\rho_0 \in [0.07, 0.39]$, F_{ul} is below 8.2 l.

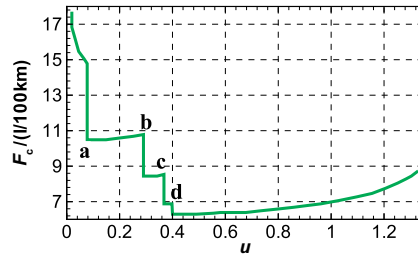


Fig. 8. Fuel consumption F_c as a function of the vehicular speed. The volume unit $l = 1000 \text{ cm}^3$, the vehicular speed has the unit of v_{f2} .

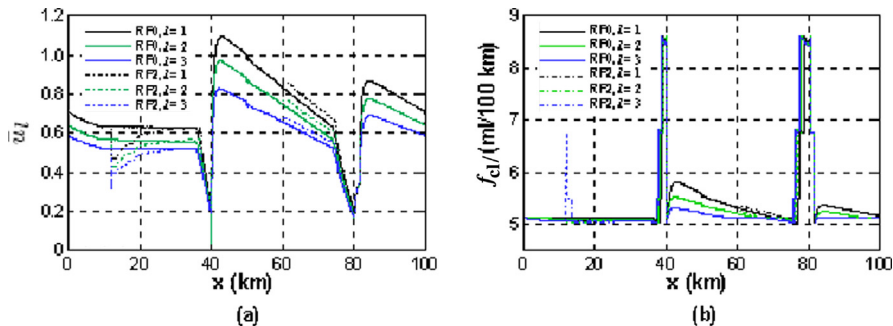


Fig. 9. Distributions of (a) time averaged traffic speed $\bar{u}_l$ and (b) fuel consumption through a grid length f_{cl} for $\rho_0 = 0.25$.

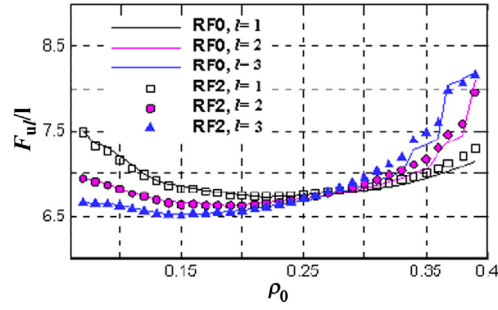
Fig. 10. Fuel consumption Fu_l as a function of ρ_0 .

Table 6

Distributions of Fu_l for the cases of $L_{wz} = 0$ and RF0.

ρ_0	$L_{wz} = 0$			RF0		
	Fu_1/l	Fu_2/l	Fu_3/l	Fu_1/l	Fu_2/l	Fu_3/l
0.07	7.424 8	6.940 8	6.655 8	7.514 1	6.950 0	6.659 5
0.08	7.254 1	6.891 0	6.643 4	7.345 7	6.915 3	6.650 2
0.09	7.150 5	6.830 1	6.636 7	7.306 6	6.878 7	6.648 1
0.1	7.012 0	6.762 7	6.606 9	7.211 9	6.830 7	6.628 0
0.11	6.900 6	6.699 3	6.564 2	7.103 4	6.777 2	6.595 1
0.12	6.826 7	6.649 1	6.528 3	6.989 5	6.738 8	6.570 1
0.13	6.757 7	6.597 5	6.490 0	6.926 5	6.695 7	6.539 8
0.14	6.730 1	6.582 0	6.480 3	6.882 2	6.669 0	6.525 7
0.15	6.700 2	6.564 5	6.469 6	6.861 0	6.665 2	6.531 7
0.16	6.671 7	6.545 3	6.469 1	6.831 3	6.658 2	6.532 4
0.17	6.642 8	6.522 0	6.478 1	6.798 5	6.639 0	6.534 4
0.18	6.614 4	6.510 6	6.489 2	6.787 2	6.625 4	6.540 9
0.19	6.592 3	6.514 2	6.500 4	6.763 2	6.622 4	6.559 6
0.2	6.570 4	6.522 2	6.508 6	6.746 2	6.626 8	6.578 3
0.21	6.555 1	6.530 6	6.510 4	6.732 4	6.638 6	6.594 0
0.22	6.547 5	6.540 0	6.512 2	6.724 3	6.656 1	6.611 4
0.23	6.556 0	6.544 9	6.516 4	6.729 3	6.671 3	6.631 5
0.24	6.564 3	6.544 6	6.523 1	6.737 5	6.687 7	6.653 9
0.25	6.570 3	6.545 8	6.532 7	6.745 2	6.701 0	6.680 0
0.26	6.583 0	6.555 6	6.549 5	6.758 6	6.722 5	6.706 9
0.27	6.589 6	6.568 8	6.575 6	6.772 0	6.741 3	6.744 3
0.28	6.579 5	6.568 5	6.591 6	6.7810	6.766 3	6.781 0
0.29	6.587 3	6.593 4	6.629 9	6.793 7	6.796 1	6.822 1
0.3	6.604 6	6.627 1	6.678 6	6.810 8	6.827 3	6.865 3
0.31	6.612 4	6.658 2	6.720 7	6.828 8	6.864 9	6.9153
0.32	6.620 4	6.687 5	6.752 2	6.851 9	6.906 4	6.964 7
0.33	6.642 1	6.724 4	6.808 5	6.877 2	6.949 9	7.024 5
0.34	6.632 4	6.703 4	7.183 3	6.910 6	6.999 2	7.277 3
0.35	6.690 6	6.800 2	7.247 9	6.945 6	7.051 0	7.344 4
0.36	6.732 8	7.061 1	7.326 0	6.987 6	7.246 7	7.419 9
0.368	6.716 2	7.207 5	8.305 2	7.024 1	7.365 2	8.038 0
0.38	6.806 4	7.311 0	8.354 5	7.089 7	7.455 5	8.122 0
0.39	6.880 2	8.301 3	8.405 2	7.153 7	8.119 6	8.190 3

To see the work zone effect on fuel consumption through the ring freeway more clearly, the difference of fuel consumption to that consumed in the case without a work zone,

$$\delta F_{ul} = F_{ul} - [F_{ul}]_{L_{wz}=0},$$

is calculated and shown in Fig. 11. Here we call the difference δF_{ul} as the addition fuel consumption, as the difference value is generally positive.

Examining the distribution of the additional fuel consumption δF_{ul} in the range of $\rho_0 \in [0.07, 0.39]$, it can be seen that for unsaturated traffic flows $\rho_0 < \rho_s = 0.368$, δF_{ul} is positive for $l \in \{1, 2, 3\}$; otherwise, as shown by the black, purple, and blue curves and by comparing the values of F_{ul} as given in several rows at the bottom part of Table 6, δF_{u3} changes to negative in the range of $\rho_0 \in [\rho_s, 0.39]$, and δF_{u2} changes from positive to negative at $\rho_0 = 0.39$. Being consistent with the distribution of F_{ul} shown in Fig. 10, ramp flow affects the additional fuel consumption δF_{ul} more evidently when $\rho_0 > 0.27$.

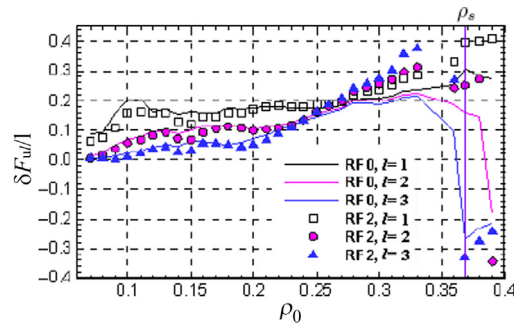


Fig. 11. Distribution of δF_{ul} with different ρ_0 .

From Fig. 11, it is seen that for the case of RF0 without any ramp flow effect, δF_{ul} is above 0.1867 l for the unsaturated traffic flow in the range of initial density $\rho_0 \in (0.27, 0.33]$, also this can be seen more clearly from the values of F_{ul} shown in Table 6. However when $\rho_0 \in (0.27, 0.33]$, in the case of RF2, as illustrated by the squares, purple-filled circles, and the deltas in Fig. 11, δF_{ul} is above 0.2 l.

5. Conclusions

A three-lane continuum model is developed to explore the effect of work zone with completely blocked lane II due to traffic crash or road rehabilitation. Based on the continuum model, numerical simulations of vehicular flow on ring freeway with a tunnel are carried out. Numerical results show the following:

- (i) The three-lane continuum model has a potential to analyze work zone effects on vehicular flow numerically to obtain simulations that are obviously helpful for the understanding of work zone bottlenecks. It can be easily extended to describe multi-lane traffic flows macroscopically with more work zones.
- (ii) In the continuum model, the net rates of mandatory net lane changing just upstream of the work zone are approximated by a random number model, with the number median chosen based on the golden ratio analysis. The lane changing time is defined by a ratio to lane average relaxation time, with the ratio described algebraically. The time averaged grid speed can be employed to estimate travel time and vehicle fuel consumption through every lane by interpolation with a vehicle performance curve.
- (iii) Under suitable conditions, there exist two density thresholds of traffic jam formation for traffic flow in the ring freeway. The first density threshold of traffic jam formed by the freeway tunnel is 0.15 in the unit of jam density, the second density threshold of traffic jam formed by the work zone is 0.19. The work zone affects the formation of traffic flow pattern significantly.
- (iv) In the absence of the work zone, the mean travel time is slightly longer than that estimated for the case with the work zone with a length of 0.16 km, implying that the tunnel with a length of 1.6 km plays a dominant role in the prediction of mean travel time. As soon as a traffic jam occurs at the tunnel entrance, the tunnel mean travel time becomes 0.06 h, suggesting that the average traffic speed through the tunnel is around 26.67 km/h, which agrees well with the tunnel mean speed of 28.9 km/h recorded by Koshi et al. (1992) and 33.8 km/h calculated by Jin (2018).
- (v) As the initial density is beyond the second threshold of traffic jam formation, the mean travel time through every lane increases with increasing initial density linearly. The fuel consumption, while lane dependent, distributes concavely with initial density, having a value in the range of 6.5 l to 8.3 l. Comparing the fuel consumption with the case without work zone, when there is no ramp flow effect, the normalized initial density is in the range of 0.27 to 0.33, with additional fuel consumption not below 0.1867 l. However, for the case of RF2, in the same initial density range, the additional fuel consumption is above 0.2 l.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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