ON THE FILTRATION OF NON-NEWTONIAN FLUID IN POROUS MEDIA WITH A MULTIPLE PARAMETER MODEL

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ABSTRACT: A multiple parameter model to describe the Non-Newtonian properties of fluid filtration in porous media is presented with regard to the pressure gradient expression in terms of the velocity of filtration, where the multiple parameters should be determined by measurements. Based on such a model, an analysis was furnished to deduce the formula for the rate of production of a oil well, and the governing equations for single phase Non-Newtonian fluid fritration. In order to examine the effects of model parameters. the governing equations were numerically solved with the method of cross-diagonal decomposition ZG method. It is found that, for constant rate of production the power index n of the model influences the pressure distribution considerably, particularly in the vicinity of a single well. The well-bore pressure of Leibenzonian fluid is lower than that of the power-law fluid in the case of the same parameter B and the power index n = 0.5.

KEY WORDS: multiple parameter model, filtration, Non-Newtonian fluid

NOMEN CLATURE

A Empirical coefficient, kg/(sm) *B* Empirical coefficient, kgs^{n-2}/m C and C_1 Integral constant C_t Total compressibility, 1/Pa Cp Compressibility of fluid, 1/Pa Cr Compressibility of rock, 1/Pa g The gravitational acceleration, m/s^2 h effective thickness, m *H* Consistency of power-law fluid, kgs^{n-2}/m k Absolute permeability, m^2 *n* Power index P Pressure. Pa PoInitial pressure, Pa P_w Well-bore pressure, Pa P_e Oil discharge Pressure, Pa q Rate of mass production, kg/sr Ratial distance, coordinate, m r_w Well-bore radius, m

- r_e Oil discharge radius, m
- U Exponential function of pressure
- $w\,$ M agnitude of velocity vector of filtration, $\,$ m/s $\,$
- w max M aximum velocity of filtration, m/s
- \vec{w} Velocity vector of filtration, m/s
- \overrightarrow{x} Vector of a position in space, m
- $\overrightarrow{x_0}$ Vector of a position of a well space, m
- z Depth of the porous medium from the ground, m
- z_0 Reference depth, m

GREEK SYMBOLS

- $\begin{array}{l} \label{eq:product} \ensuremath{\mathsf{P}}\ensuremath{\mathsf{F}}\ensuremath{\mathsf{u}}\ensuremath{\mathsf{d}}\ensuremath{\mathsf{e}}\ensuremath{\mathsf{s}}\ensuremath{\mathsf{g}}\ensuremath{\mathsf{m}}\ensuremath{\mathsf{m}}\ensuremath{\mathsf{s}}\ensuremath{\mathsf{m}}\ensuremath{\mathsf{m}}\ensuremath{\mathsf{m}}\ensuremath{\mathsf{s}}\ensuremath{\mathsf{m}}\ensuremath{\mathsf{$
- $\nabla \phi$ Potential gradient

SUBSCRIPTS

0 Initial e Discharge eff Effective i Grid node number t T otal w Well-bore

1. INTRODUCTION

Available Non-Newtonian fluid models used to study the performance of Non-Newtonian fluid filtration in porous media are required for the recovery of subsurface petroleum reservoir. The performance of filtration is mainly determined by the relation of the filtration velocity and the pressure gradient, which was

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confirmed to be nonlinear for Non-Newtonian fluid flow in a porous medium by a large amount of experiments. It not aubtedly deviates from the well known Darcy's curve of filtration of a normal Newtonian fluid. However, in many previous papers, the nonlinear relationship between velocity and pressure gradient of Non-Newtonian fluid has been linearly approximated to result in the simplified fluid models, for example, the Bingham fluid model^[3, 10]. segmental linear fluid model^[8]. This kind of simplification truly introduces deviations under the circumstance of small pressure gradient. For Non-Newtonian fluid, a variety of rheological models were summarized by Bird et al^[2]., (1969) Recenty, Ma and Ruth performed a physical explanation of Non-Darcy effects, although a direct prediction of the fluid motion in a porous medium has not been furnished further in the reported work [7].

Notably, Gogarty^[4] found by experiments that effective viscosity of pseudoplastic fluid flow in a core depends upon the average shear rate, which is a singlevalued function of a velocity in a given porous medium. The experimental results have been utilized as a bridge in the theoretical work of Ikoku et al^[6]., and Wu^[11], in which the effects of power-law fluid behavior of filtation and displacement have been investigated analytically. However, despite of its obvious advantages in the description under the condition of small velocity. the power-law model is hardly appropriate to large pressure gradient. Consequently, Gurbanov et al^[5]., completed their analysis with Leibenzonian fluid model, and their solution indicated that empirical coefficients have a profound influence on the mass rate of a production well for heavy oil.

In this paper, a multiple parameter fluid model is proposed for Non-Newtonian fluid filtration, which is more complicated, but is anticipated to have wider feasibility of engineering application. The model is used to deduce the governing equations of fluid flow in a porous medium which are numerically solved with a cross-diagonal decomposition $ZG^{[12]}$ method to find the effects of Non-Newtonian behavior. It is found that the expression for the mass rate of production of a single oil well in steady filtration is really different from Dupui's^[5] expression based on Darcy's law.

2. MULTIPLE PARAMETER FLUID MODEL

Consider the filtration of Non-Newtonian fluid flow in a porous medium. The results of experiments showed that the non-linear of filtration in porous media just strongly appeares in the case of comparatively small pressure gradient. Subsequently, experimental data indicates that if the velocity in porous media is larger, the relationship between the pressure gradient and the velocity tends to be linear, thus, instead of Darcy's law, the Non-Newtonian fluid flow in a porous medium is considered to adherence the following law:

where ρ , g, z, \overrightarrow{w} , w are fluid density, gravitational acceleration, vertical depth from the point considered to the ground surface, velocity vector and its magnitude respectively, and ∇ is the Hamiltonian operator. Eq. (1) defines a multiple parameter fluid. As shown in Fig. 1, the filtration curve in the $(\omega \nabla P/10^6)$ plane is closely related to the parameters selected. Thus by determining the multiple parameter in laboratory. The performance of Non-Newtonian fluid filtration can be understood more accurately. In addition, k is the permeability of the porous medium, A, B and n are three empirical parameters to be determined by experiments. From Eq. (1). It is seen that as long as the parameter B = 0, $A = \mu$, the model reduces to the well known formulation of Darcy, i. e., defines the fluid to be Newtonian type. Subsequently, if the third parameter $n = \frac{1}{2}$, the multiple parameter model keeps the same form as presented by Leibenzon (see Ref. [5]). Which is suitable for non-Newtonian fluid motion in porous media, and the values of A and B can be found in Ref. [1]. But in Ref. [5] there is just a table giving values of A and B for some kinds of Non-Newtonian fluids under normal conditions. In Eq. (1), the unit of A and B are respectively kg/(sm) and kgsⁿ⁻²/m. As A = 0, for power-law fluid, it is seen that

$$B = k^{\frac{n-1}{2}} \mu_{df} \tag{2}$$

Oil of lubrication	<i>k</i> m/ d	Temperature /°C	<i>A</i> / kg• s ^{- 1.5} • m ^{- 1}	<i>B</i> ∕kg•sm ⁻¹	V ariation range of velocity gradient/s ⁻¹
<i>n</i> = 0.5	12	28	0. 0076	0. 00974	0.020.79
		35. 6	0.00079	0. 00394	0.060.5

*Cited from Ref. [5]



Fig. 1 The relation between pressure gradient and velocity of filtration, for the case of $B = 20 \text{kgs}^{n-2} \text{m}$, where *P* is in the unit of 10⁶ Pa, and *w* with its maximum value w_{max}

where μ_{gf} is the effective viscosity, and from the work of Savins^[9], it is defined as

$$\mu_{gf} = \frac{H}{12} (9 + \frac{3}{n})^n (150k \, \Phi)^{\frac{1}{2}}$$
(3)

in which H represents the consistency of power-law fluid and φ is the porous medium. Obviously, the parameter n to be measured is rehabilitated to the power-law index, the power-law model is just a simplified model of the present one. The combination of relations (2) and (3) gives

$$B = \frac{H}{12}(9 + \frac{3}{n})^{n}(150 \, \varphi)^{\frac{1-n}{2}} \tag{4}$$

which is only applicable to power-law fluid filtration. For Leibenzonian fluid, the measured results for the temperature and permeability in experiments. cited from Ref. [5], are indicated in Table 1, where the variation range of velocity gradient of filteration is shown as well. In the next section, attention is focused on deducing the expression of production rate of single well.

3. RATE OF PRODUCTION

Consider one-dimensional radial flow under steady condition. Assume that the rock non-deformed, thus the continuity equation can be written as

$$\nabla \cdot (\overrightarrow{w}) = \frac{1}{r} \frac{d(rw)}{dr} = 0$$
⁽⁵⁾

By intergration, one gets

$$rw = C \tag{6}$$

Since the mass rate q, of a productive oil well with the effective thickness of the porous medium could be expressed by

$$q = 2 \, \Pi hr \, w \tag{7}$$

by comparing (7) with (6), it is found that

$$C = \frac{q}{2\pi h}, \quad w = \frac{C}{r} \tag{8}$$

On the other hand, we define a flow potential φ which satisfies the following expression:

$$\nabla \phi = \frac{\nabla P}{\rho g} - \nabla z \tag{9}$$

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$$\phi = \phi_{0+} \int_{P_0}^{p} \frac{dP}{Q} - (z - z_0)$$
(10)

in which ϕ_0 is the reference flow potential for p = P_0 . and $z = z_0$.

$$\rho = \rho_0 e^{C_{\rho}(P - P_0)}$$
(11)

then by some algebraic operations, yields

$$\phi = \phi_{\theta} (z - z_0) + \left(\frac{1 - e^{C_{\rho}(P - P_0)}}{\rho_0 g C_{\rho}}\right)$$
(12)

Substituting (9) into (1) and integrating under the assumption of incompressible fluid, one could obtain the following expression:

$$\phi = C_1 + \left(\frac{Q_2}{k}\right)^{-1} \cdot \left(\frac{A}{k}C\ln r + \frac{B}{\sqrt{k}}\left(\frac{C}{\sqrt{k}}\right)^n \frac{r^{-n}}{1-n}\right) (13)$$

where C_1 is a constant of integration.

The boundary conditions are

where r_w , r_e are the well radius and oil discharge radius respectively, while ϕ_{u} , ϕ_{e} are respectively the well-bore flow potential and discharge flow potential. The substitution of (14) into equation (13) gives

$$C1 = \frac{1}{2} (\phi_{ut} \phi_{et} \frac{1}{2} (\rho_g)^{-1} \cdot \left(\frac{A}{k} C \ln(r_w, r_e) + \frac{B}{\sqrt{k}} (\frac{C}{\sqrt{k}})^n \frac{r_w^{1-n} + r_e^{1-n}}{1-n}\right)$$
(15)

and

$$(Pg) \cdot (\Phi_{e^{-}} \Phi_{p}) = \frac{A}{k} C \ln(\frac{r_{e}}{r_{w}}) + \left(\frac{B(r_{e^{-}}^{1-n} - r_{w}^{1-n})}{(\sqrt{k})^{1+n}(1-n)}\right) \cdot C^{n}$$
(16)

the application of Eq. (8) yields the relation of flow potential difference and the rate of oil production

$$(\rho_g) \cdot (\phi_e - \phi_w) = \frac{Aq}{2 \pi h k} \ln(\frac{r_e}{r_w}) +$$

$$\left(\frac{q}{2\pi\hbar}\right)^{n} \left(\frac{B\left(r_{e}^{1-n}-r_{w}^{1-n}\right)}{\left(\sqrt{k}\right)^{1+n}\left(1-n\right)}\right)$$
(17)

from which the rate of production can be evaluated with the Newton-Raphson iteration when the remained variables are given. It is also applicable to the problems of multiphase flow and displacement in porous media, provided that some considerations of effects associated with relative permeability and saturation are included.

GOVERNING EQUATION OF FILTRATION 4.

Consider the problem of the single phase transient. flow of the Non-Newtonian fluid in a porous medium. C_r is assumed to be the compressibility of the rock, thus, the porosity could be written as

$$\varphi = \varphi_0 \ e^{C_r (P - P_0)} \tag{18}$$

where φ_{dis} the initial porosity. In order to linearize the time dependent term in the equation of filtration, we assume that there is a new variable U, defined by the exponential function

$$U = e^{(C_r + C_p)(P - P_0)} = e^{C_t(P - P_0)}$$
(19)

Applying Eq. (12) yields the relation between U and φ

$$U = \frac{1}{\sqrt[m]{1 - (\rho_0 g C \rho)(\varphi + z - \varphi_0 - z_0)}}$$
(20)

Since the governing equation of filtration has the following form:

$$\frac{\partial(\varphi p)}{\partial t} = - \nabla \cdot (\rho \vec{v}) + f(t, \vec{x}, \vec{x}_0)$$
(21)

Appling Eqs. (1) and (20). One can obtain the following governing equation with the variables U and w:

$$\begin{pmatrix}
\frac{\partial}{\partial t} = \varphi_0^{-1} \nabla \cdot \left(\frac{k(U^{-1} \nabla U - C_t U^m \nabla Z)}{C_t (A + B(w / \sqrt{k})^{n-1})} \right) + \\
\rho_0 g \left(- U^m \nabla Z + \frac{U^{-1} \nabla U}{C_t} \right) = - \\
\left(\frac{A}{k} w + \frac{B}{\sqrt{k}} (\frac{w}{\sqrt{k}})^n \right) (\frac{w}{w})
\end{cases}$$
(22)

where $f(t, \vec{x}, \vec{x}_0)$ is the intensity of oil production, © 1994-2011 China Academic Journal Electronic Publishing House. All forms reserver on, hear be represented as

$$f(t, \overrightarrow{x, x_0}) = q(t) \cdot \delta(\overrightarrow{x - x_0})$$
(23)

For steady filtration, the production rate can be calculated with Eq. (16). However it should be reasonably modified by careful analysis. $m = (C_p/C_t)$ is the ratio of compressibility, which is zero for incompressible filtration. q(t) is the time dependent rate of production of a well at $\overrightarrow{x_0}$. It is seen that, although, the transient term on the left hand side has been simplified, the space dependent term becomes more complicated. Thus quantitative analytical work would encounter rather large difficulties. Therefore, as an example numerical solution of the governing equation for Non-Newtonian fluid flow in porous media is given in the next section.

5. NUMERICAL SIMULATION

one-dimensional single well problem А is numerical simulated by solving Eq. (22) given in the foregoing section. which is first discretized by use of a staggered grid system. where the variable U at i node is stored at the center point between w_i and w_{i+1} , positioned at $r_{i-1/2}$ and $r_{i+1/2}$. The discretization of (22) yields the following finite difference e

for i = 1

$$\begin{cases} \frac{(r_{i-1}^2 - r_i^2)}{\Delta} (U_i^{l+1} - U_i^l) + \\ w_{i+1}^{l+1} r_{i+1} U_{i+1}^{ml+1} + \frac{q}{\rho_0} = 0 \\ w_i^{l+1} r_i U_i^{ml+1} = -\frac{q}{\rho_0} \end{cases}$$
(24)

for
$$2 \leq i \leq N - 1$$

$$\frac{\frac{1}{2}(r_{i+1}^{2} - r_{i-1}^{2})}{\Delta t}(U_{i}^{l+1} - U_{i}^{l}) + w_{i+1}^{l+1}r_{i+1}U_{i+1}^{ml+1} - w_{i}^{l+1}r_{i}U_{i}^{ml+1} = 0$$

$$\frac{U_{i}^{l+1} - U_{i-1}^{l+1}}{C_{t}(r_{i} - r_{i-1})} + \frac{1}{2}(U_{i}^{l+1} + U_{i-1}^{l+1}) \cdot \left(\frac{A}{k} + \frac{B}{k}\left(\frac{1 - w_{i}^{l+1}}{\sqrt{k}}\right)^{n-1}\right)w_{i}^{l+1} = 0$$
for $i = N$

$$\begin{cases} \frac{(r_i^2 - r_{i-1}^2)}{\Delta t} (U_i^{l+1} - U_i^l) - w_i^{l+1} r_i U_i^{ml+1} = 0\\ \frac{U_i^{l+1} - U_{i-1}^{l+2}}{C_t (r_i - r_{i-1})} + \frac{1}{2} (U_i^{l+1} + U_{i-1}^{l+1}) \\ \left(\frac{A}{k} + \frac{B}{k} \left(\frac{1 w_i^{l+1}}{\sqrt{k}}\right)^{n-1}\right) w_i^{l+1} = 0 \end{cases}$$
(26)

in which (24) and (26) are the discretized forms of the boundary conditions given by

$$rw \rho = q = \text{Constant}, \text{ for } r = r_w$$
 (27)

$$w = 0, \text{ for } r > r_N \tag{28}$$

accompanied with the initial conditions:

$$w(r, t) = 0, U(r, t) = 1, \text{ for } t = 0$$
 (29)

and Δt , N and l are the time interval, total grid number and time level respectively. In addition. it is selected that $r_1 = r_w$, and $r_{i+1} = 1.05r_i$. Note that the variable U can be transformed to pressure Pimmediately in terms of Eq. (20). The matrix form of finite difference equations can be written as

$$\begin{array}{c} \begin{array}{c} (24) \\ (24) \\ (25) \end{array} \begin{cases} a_{1}^{(1)} & 0 & \cdots & \cdots & b_{1}^{(2)} & a_{1}^{(2)} \\ 0 & a_{2}^{(1)} & \cdots & \cdots & a_{2}^{(2)} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{3}^{(3)} & a_{2}^{(3)} & \cdots & \cdots & a_{2}^{(4)} & 0 \\ a_{1}^{(3)} & 0 & \cdots & \cdots & 0 & a_{1}^{(4)} \end{array} \\ \end{array} \right)$$

where the supersripts (k), k = 1, 2, 3, 4 is used to indicate the difference of the matrix elements. From (24), (25) and (26). it is seen that

for i = 1.

$$\begin{cases} a_i^{(1)} = \frac{(r_{i+1}^2 - r_i^2)}{\Delta t}, \ a_i^{(2)} = 0\\ a_i^{(3)} = 0, \ a_i^{(4)} = r_i U_i^{m_{l+1}}\\ b_i^{(2)} = r_{i+1} U_{i+1}^{m_{l+1}}, \ b_i^{(k)} = 0 \text{ for } k = 1, 3, 4\\ c_i^{(k)} = 0 \text{ for } k = 1, 2, 3, 4\\ d_i^{(1)} = a_i^{(1)} U_i^l - \frac{q}{\rho_0}, \ d_i^{(2)} = -\frac{q}{\rho_0} \end{cases}$$

for $2 \leq i \leq N - 1$

$$\begin{cases} a_{i}^{(1)} = \frac{(r_{i+1}^{2} - r_{i-1}^{2})}{2\Delta t}, \\ a_{i}^{(2)} = -r_{i}^{l+1}U_{i}^{ml+1} \\ a_{i}^{(3)} = \frac{1}{C_{l}(r_{i} - r_{i-1})}, \\ a_{i}^{(4)} = \frac{1}{2}(U_{i}^{l+1} + U_{i-1}^{l+1})\left(\frac{A}{k} + \frac{B}{k}\left(\frac{|w_{i}^{l+1}|}{\sqrt{k}}\right)^{n-1}\right) \\ b_{i}^{(2)} = r_{i+1}U_{i+1}^{ml+1}, \\ b_{i}^{(k)} = 0 \text{ for } k = 1, 3, 4 \\ c_{i}^{(3)} = -a_{i}^{(3)}, c_{i}^{(k)} = 0 \text{ for } k = 1, 2, 4 \\ d_{i}^{(1)} = a_{i}^{(1)}U_{i}^{l}, d_{i}^{(2)} = 0 \end{cases}$$
(32)

and for i = N.

$$\begin{cases} a_{i}^{(1)} = \frac{(r_{i}^{2} - r_{i-1}^{2})}{\Delta t}, \ a_{i}^{(2)} = -r_{i}^{l+1}U_{i}^{ml+1} \\ a_{i}^{(3)} = \frac{1}{C_{t}(r_{i} - r_{i-1})}, \\ a_{i}^{(4)} = \frac{1}{2}(U_{i}^{l+1} + U_{i-1}^{l+1}) \left(\frac{A}{k} + \frac{B}{k} \left(\frac{|w_{i}^{l+1}|}{\sqrt{k}}\right)^{n-1}\right) \\ b_{i}^{(k)} = 0 \text{ for } k = 1, 2, 3, 4 \\ c_{i}^{(3)} = -a_{i}^{(3)}, \ c_{i}^{(k)} = 0 \text{ for } k = 1, 2, 4 \\ d_{i}^{(1)} = a_{i}^{(1)}U_{i}^{l}, \ d_{i}^{(2)} = 0 \end{cases}$$
(33)

 Table 2
 Parameters for Non-Newtonian fluid flow*

|--|

* 1 Darcy= $1.02 \cdot 10^{-12} \text{ m}^2$

(31)

then by choosing the simple iteration approach, to overcome the non-linearity of equation (30), the algebraic equations are solved by a compact ZG cross-diagonal decomposition algorithm (see Ref. [12]).

The numerical results, with respect to the parameters illustrated in Table 2 for the flow in a single well of Non-Newtonian fluid, have been obtained which are found to be grid number independent when the total grid number is 100.

As shown in Fig. 2. For constant rate of production, the power index has a profound influence on the pressure distribution in the vicinity of an oil well. The well-bore pressure, i.e., the pressure at r $= r_w$, decreases with increasing power index, regardless of the value of A being zero or non-zero. However, there is a region in which the value of pressure is larger when the power index is chosen to be a large one. For the multiple parameter Non-New tonian fluid, i. e., for the case of $A \neq 0$, the pressure drops more quickly than power-law Non-Newtonian fluid. Fig. 3 shows the evolution of pressure and velocity of filtration in the porous medium. For a given rate of production. The velocity variation as time increases is small, and it seems that the property of rheology almost has no effects upon the velocity distribution along the radial direction.

6. CONCLUSION

A multiple parameter Non-Newtonian fluid model for the investigation of filtration in a porous medium



Fig. 2 The influence of power index on the pressure distribution in the near well region, under the conditions of $B = 200 \text{ kgs}^{n-2}/\text{ m}$ and t = 150 d with the remained parameters shown in Tab. 1

has been proposed, which may be considered to be the generalization of Leibenzonian model.

It was found that, if one chooses A = 0, the model becomes the power-law fluid model; if B = 0, the model reduces to the Newtonian fluid model; and finally one defines the Leibenzonian fluid if $A \neq 0$, $B \neq 0$, $n = \frac{1}{2}$.

By using this multiple parameter model, an expression for the rate of production involved with the well-bore pressure has been derived, which is found to be different from the Dupui formula based on the linear Darcy's law. The expression for the rate of production is certainly of special importance in the application of the multiple model to engineering.

The equations governing the filtration of multiple parameter fluid has been derived, which can be used as the foundation of numerical simulation for the problem of single well. The numerical results obtained by use of the cross diagonal decomposition ZG algorithm, show that the power in dex has a strong effect on the pressure distribution in the near well region in the case of definite rate of production. It means that, for a large power index, in order to recover the same amount of Non-Newtonian oil. a comparatively large pressure gradient-should be maintained. By comparing



Fig. 3 Normalized pressure and velocity distribution for the case of n = 0.5 and $B = 20 \text{ kgs}^{-1.5}/\text{m}$, with the remained parameters shown in Tab. 1

the results in the cases of A = 0 and A = 5 kg/(sm), it is found that the increase of parameter A, leads to the greater difficulty of heavy oil recovery. However, to extend the applicability of this multiple parameter model in engineering, particular auxiliary experiments are needed to determine the multiple parameters.

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