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Mathematical modeling of air pollution in city tunnels and evaluating mitigation strategies

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ABSTRACT

A mathematical model has been developed that is able to describe the environment of the city tunnels being affected by the road traffic, natural and forced air convective flows. The mathematical model describes the peculiarities of the traffic flows on one-lane roads with a satisfactory accuracy. The model well matches experimental data on traffic flows. Multidimensional calculations of the influence of cars on the airflow in tunnels are performed. The numerical model for simulating exhaust gas emissions by automobiles and their accumulation in a tunnel and evolution with traffic induced air flow was developed. The results of numerical investigations make it possible providing recommendations for transport researchers and policy-makers. In particular, it was shown that in the presence of long tunnels on automobile roads it is necessary to choose the traffic arrangement avoiding the necessity for vehicles to come to a full stop and then accelerate in tunnels. This could happen in the presence of traffic lights or other type of traffic regulation near the exit of the tunnel. Thus it is necessary in arranging traffic regulation avoiding placing traffic lights in the proximity of tunnel exit for the waiting cars line to be shorter than the distance from the tunnel exit. In venting the tunnel, the direction of the wind should coincide with the direction of traffic flow. Attempts to arrange venting in the opposite direction for high blockage ratio of a tunnel by vehicles could result in bringing to a worse situation with air pollution.
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1. Introduction

The problem of quality of an outdoor atmosphere in big urban agglomerations is an indicator of the total urban environmental quality. Contribution of science and research to solving problem of creating clean atmosphere in big cities can hardly be underestimated. The reduction of exhaust emission is the main goal for many researchers (Raeesi and Zografos, 2019, Carroll et al., 2019). The World Health Organization in 2014 recognized air pollution as the world's "largest single environmental health risk" (Barnes et al., 2019).

Present paper is aimed at developing a mathematical model describing how air pollution in city tunnels is affected by road traffic self-organization. This is a critical step to develop effective mitigation strategies. This can be regarded as a part of an overall basic research to improve the air quality. This approach is concentrated on the fundamental aspects of mathematical modeling of the road traffic intensity and the induced drift and dispersion of contaminants in stratified turbulent flows.

Most of pollution in urban territories comes from road traffic. Traffic induced pollution depends on regime of flow: irregular traffic modes, associated with accelerations and decelerations, long standing in traffic jams produce maximal pollution. One of the methods of traffic exhaust reduction was considered decreasing the traffic irregularity by reducing the number of traffic lights and building automobile tunnels. That brought the problem of air quality in city tunnels on top of current needs for research. Previous mathematical models of air pollution by automobile exhaust rely heavily on the assumption that traffic exhaust is a function of the car flux, and take into account the winds and the height of the buildings in an urban environment. These models usually take into account the intensity of traffic, the air flows, the heights of the buildings, but disregard the effect of moving cars on polluting exhaust gases propagation and mixing with air. This approach is well grounded for being applied in an open space, while in city tunnels the situation is quite different: the cars motion plays an essential role in the dispersion of air pollutants.

For contaminants dispersion in tunnels modeling it is necessary to take into account the influence of moving cars and fans on the air flow. All the existing models operate with a given traffic intensity as an external parameter, while traffic flows are a self-regulating system. Nevertheless, a number of attempts have been made to develop mathematical models for road traffic (Lighthill and Whitham, 1955; Richards, 1956; Greenberg, 1959;

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Prigogine and Résibois, 1962; Smirnov et al., 2000; Kiselev et al., 2000) based on a continua approach.

Multiphase traffic models based on microscopic and macroscopic approaches were compared by Borsche et al. (2012). Helbing (2001) used methods of statistical physics and nonlinear dynamics. Very wide spectrum of spatial-temporal pattern formation in automobile traffic flows was described by Nagatani (2002). Different approaches were used for describing traffic flows: Lagrange type car following models (Brackstone and Mcdonald, 1999), cellular automats approach (Nagel and Schreckenberg, 1992; Helbing and Huberman, 1998; Chowdhury et al., 2000), fluid or gas dynamical models (Helbing and Treiber, 1998, Hoogendoorn and Bovy, 2000, Kerner and Konhäuser, 1993).

Currently, continuum traffic flow modeling is developing in a number of directions. Some of the new models concentrate on methods that determine optimal adjusting parameters for the model equations (Ngoduy and Maher, 2012; Zheng et al., 2017). Other developed models take into account different types of vehicles or different driving styles (Gupta and Katiyar, 2007) or models with deeper view on weak disturbances propagation across the traffic (Zheng et al., 2015). The traffic of public transport and its interaction with passengers is discussed in the papers (Nagatani, 2001; Regirer et al., 2007). Finally, some models expanded to two-dimensions to describe the traffic flow on multilane roads (Sukhinova et al., 2009).

Macroscopic models describing fluxes of cars in roads are present in papers (Ngoduy, 2013; Tordeux et al., 2014; Zhu and Yang, 2013; Costeseque and Lebacque, 2014), mathematical models based an approach similar to a visco-elastic one is discussed in papers by Smirnova et al. (2015, 2016, 2017). Accent on travel time prediction for a single vehicle within the traffic flow was made in papers (Zhang et al., 2018a, 2018b), latest results having been described in (Kumar et al., 2017, Ladino et al., 2017, Ma et al., 2017, Rahmani et al., 2017). Influence of avenue trees on traffic pollutant dispersion in asymmetric street canyons was studied by Sun and Zhang (2018).

The novelty of the suggested approach was in development of a coupled model, which describes dynamics of automobiles being a function of traffic regulating strategy, dynamics of air flows in tunnels induced by motion of cars, and dynamics of exhaust pollutants emission and dispersion in the atmosphere of a tunnel being a function of both traffic flow dynamics and air flows in turbulent atmosphere of a tunnel. Multi-phase models for continuum mechanics are used, wherein one of the phases (gaseous) is considered containing multiple components. The next section describes a numerical model, which can be used to forecast the accumulation of exhaust gases in a tunnel under different traffic flow conditions. The model makes it possible to evaluate the effect of different traffic regulating strategies on the accumulation of exhaust gases in tunnels, which is a way to achieve optimal traffic regulation maximizing tunnel capacity and minimizing pollutants accumulation.

2. Mathematical modeling of unsteady-state traffic flows

Mathematical model for unidirectional unsteady traffic flows developed in the present paper relies heavily on preceding studies (Smirnov et al., 2000; Kiselev et al., 2000), but contains some new methodological approaches, which do not limit the model by one continuity equation and empirical relationships for flux versus density dependence.

Let the X-axis be oriented in the direction of cars motion, denoting the temporal axis with t. Density of cars in the road $n(x,t)$ is defined as the ratio of the area covered by cars in the road to the area of the control section of the road:

$n = \frac{S_{veh}K}{S}$, where S_{veh} is the dynamic area of an average vehicle, S – is the area of a control zone in the road, K is the number of vehicles in the control zone. The density defined above is a dimensionless variable changing in the interval: $0 < n < 1$.

The traffic flow velocity $v(x,t)$ is introduced; it can vary within the following limits:

$$0 < v < v_{max},$$

where v_{max} is the speed limit applied to the lane.

The density can vary within the range from zero to unity: the maximal density $n = 1$ is reached on cars occupying the whole road standing packed one by one (at $v = 0$) in a traffic jam.

On determining the value $m = \int_0^L n dx$ as the generalized "mass" in the road section L, one could write the mass conservation law for the road. In case of continuous traffic flow, the continuity equation will take place:

$$\frac{\partial n}{\partial t} + \frac{\partial nv}{\partial x} = 0. \tag{1}$$

On writing down the equation of motion we'll take into account, that variations of flow velocity are limited by effectiveness of the car engine and braking distance on one hand, and by drivers' reaction on variation of traffic conditions ahead of them on the other hand. Increase or decrease of flow density ahead of the driver motivates him to accelerate or slow down the car. Then the equation of motion takes the form:

$$\frac{dv}{dt} = a, \text{ where } a = \begin{cases} a^+, & -\frac{k^2 \partial n}{n\,\partial x} > a^+ \\ -\frac{k^2 \partial n}{n\,\partial x}, & -a^- < -\frac{k^2 \partial n}{n\,\partial x} < a^+ \\ -a^-, & -\frac{k^2 \partial n}{n\,\partial x} < a^- \end{cases}$$
$$\text{or } \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{k^2 \partial n}{n\,\partial x} = 0 \tag{2}$$

where a^+ – maximal acceleration, a^- - maximal deceleration. Both a^+ and a^- are non-negative ($a^+, a^- \geq 0$); they are limited by relevant characteristics of cars. The factor k is positive and has the dimension of speed. It is dependent on human factors as well (the drivers' reaction delays, etc.). Factor k physically accounts for the velocity of weak disturbances and their propagation in the counter flow direction ("speed of sound") (Smirnov et al., 2000; Smirnova et al., 2015, 2016, 2017).

3. Modeling of gas dynamical flows in tunnels induced by the motion of vehicles

When modeling air pollution in big cities the influence of winds and thermal convective air flows on dispersion and drift of atmospheric contaminants are usually taken into account (Smirnov et al., 1996a, 2002). The role of traffic induced air flow perturbations is usually disregarded being negligibly small. While in long tunnels the situation is absolutely different: a moving vehicle acts as a piston bringing the air into a motion and turbulizing it essentially. These effects turn out to be much stronger than the atmospheric air flows (winds). The longer is the tunnel, the larger is the role of traffic induced flow perturbations. For example, Metro in Moscow being hidden deep underground does not have any other vent and air supply mechanisms but for the forced convection induced by moving trains.

The characteristic cross-sections ratio of the moving car and the tunnel (the blockage ratio BR: $S = \frac{S_{car}}{A}$) is usually rather small as compared to that of the metro. Thus the multidimensional flow simulation is essential. Turbulent diffusion and vorticity cause the dispersion of the exhausted pollutants and their lifting to the top of the tunnel. The atmosphere of the tunnel is disrupted by successive waves of cars moving through the tunnel. Thus, the atmosphere encountered by cars entering the tunnel is affected by the movement of cars ahead of them in the tunnel.

Mathematical modeling of the local problem for an air flow perturbations and turbulization, and dispersion of contaminants caused by successively moving vehicles will provide the data to be incorporated into a meso-scale model for the traffic induced air flow in the whole tunnel.

The flow of an initially quiescent viscous and heat-conductive multi-component gas mixture bounded by rigid walls is regarded. For description of turbulence effects we use k-□ model, which is upgraded to take into account the near-wall damping effects. The Lam-Bremhorst low Reynolds model is used to describe gas flow near the walls (Smirnov et al., 2002). The value Y_k stands for the mass fraction of the k-th contaminant (automobile exhausts could contain different types of contaminants), its amount increases due to automobile exhaust and could be reduced due to ventilation. We define δ the mass concentration of oxygen in the air inside the tunnel, its amount decreases due to consumption by the engines and it could have to be restored by ventilation. The gas mixture equation of state is as follows (perfect gaseous mixture):

$$e_k = c_{vk}T + h_{0k}; p_k = \rho_k \frac{R}{W_k} T,$$

where c_{vk}- the heat capacity of the k-th fraction, e_k is the specific internal energy, W_k - the molar mass, h_{0k}- the chemical enthalpy, T – temperature, R - universal gas parameter.

The governing equations after Favre procedure (averaging bars are omitted) has the following form:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\left(\rho\, \vec{u}\right) = W, \tag{3}$$

$$\frac{\partial \delta\rho}{\partial t} + \nabla\cdot\left(\rho\, \vec{u}\delta\right) = -\dot{w}_\delta - \nabla\cdot\left(\vec{I_\delta} + \vec{I_\delta^t}\right), \tag{4}$$

$$\frac{\partial \rho Y_k}{\partial t} + \nabla\cdot\left(\rho\, \vec{u} Y_k\right) = \nu_k W_k \dot{w} - \nabla\cdot\left(\vec{I_k} + \vec{I_k^t}\right), \tag{5}$$

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla\cdot\left(\rho\, \vec{u}\,\Box\,\vec{u}\right) = -\nabla p + \nabla\cdot\left(\tau + \tau^t\right), \tag{6}$$

$$\frac{\partial \rho E}{\partial t} + \nabla\cdot\left(\rho\, \vec{u}E\right) = -\nabla\rho\, \vec{u} - \nabla\cdot\left(\vec{I_q} + \vec{I_q^t}\right) + \nabla\cdot\left(\left(\tau + \tau^t\right)\cdot\vec{u}\right), \tag{7}$$

where ρ, $\vec{u}$, p are the averaged mixture density, velocity, pressure of the air flow; $E = \Sigma_k Y_k e_k + \frac{u^2}{2} + k$ - the total specific energy; k - kinetic energy of turbulence; $\Box_k$ - stoichiometric coefficient, $\dot{w}_\delta$ is the rate of oxygen consumption by the engine, $\dot{w}$ is the rate of the generalized contaminant component production, W is the mass difference between the gas emission and consumption by the engine. The source terms differ from zero only in specifically designated zones of the computational domain simulating the respective devices of the vehicle. The fluxes present in Eqs. (3)–(7) look as follows:

$$\tau = \left(\mu + \rho\nu^t\right)\left(\nabla\vec{u} + \nabla\vec{u}^T - (2/3)\left(\nabla\vec{u}\right)U\right) - (2/3)\rho k U$$

$$\vec{I_k} = -\rho\left(D + \left(\nu^t/\sigma_d\right)\right)\nabla\cdot Y_k$$

$$\vec{I_q} = \vec{J_q} + \Sigma_k\left(c_{pk}T + h_{0k}\right)\vec{I_k}$$

$$\vec{J_q} = -\left(\lambda + \Sigma_k c_{pk} Y_k \rho\left(\nu^t/\sigma_t\right)\right)\nabla\cdot T$$

where U is a unit tensor, $\nabla\vec{u}^T$ is the transposed matrix $\nabla\vec{u}$; ν - molecular kinematic viscosity; ν^t- turbulent kinematic viscosity; D - mean molecular diffusion coefficient; λ - thermal conductivity.

The turbulent viscosity is developed using the formula:

$$\nu^t = C_\mu {}^{k^2}\!\big/_{\varepsilon}.$$

where ε stands for the pulsations dissipation, which can be obtained solving the following equations.

$$\frac{\partial \rho k}{\partial t} + \nabla\cdot\left(\rho\vec{u}k\right) = \nabla\cdot\left(\rho\left(\nu + \frac{\nu^t}{\sigma_k}\right)\nabla k\right) + \tau^t : \nabla u - \rho\,\varepsilon, \tag{8}$$

$$\frac{\partial \rho \varepsilon}{\partial t} + \nabla\cdot\left(\rho\vec{u}\varepsilon\right) = \nabla\cdot\left(\rho\left(\nu + \frac{\nu^t}{\sigma_\varepsilon}\right)\nabla\varepsilon\right) + \frac{\varepsilon}{k}\left(C_{1\varepsilon}\tau^t : \nabla u - C_{2\varepsilon}\rho\,\varepsilon\right), \tag{9}$$

The model Eq. (8), (9) is modified near the walls using low Reynolds flows approach (Smirnov et al., 2002). The original coefficients $C_\Box$, $C_{1\Box}$, $C_{2\Box}$ in Eqs. (8), (9) are multiplied by the wall functions:

$$C_\mu = C^0_\mu f_\mu; C_{1\varepsilon} = C^0_{1\varepsilon}\frac{f_1}{f_\mu}; C_{2\varepsilon} = C^0_{2\varepsilon} f_2,$$

where $f_\Box$, f_1, f_2 are positive functions: $0 < f_\Box \le 1$; $f_1 \ge 1$; $0 < f_2 \le 1$.

The model parameters were developed based on comparing simulation results with experiments for model problems of turbulent flows. The parameters have the following values (Smirnov et al., 1996a, 2002):

$$C^0{}_\mu = 0.09, \quad C^0{}_{1\varepsilon} = 1.45, \quad C^0{}_{2\varepsilon} = 1.92,$$
$$\sigma_d = 1, \quad \sigma_t = 0.9, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1.3.$$

The boundary conditions for the volume G are the following:

$$\partial G : \vec{u} = u_b; k = 0; \frac{\partial\varepsilon}{\partial\vec{n}} = \frac{\partial\sigma}{\partial\vec{n}} = \frac{\partial Y_k}{\partial\vec{n}} = \frac{\partial T}{\partial\vec{n}} = 0 \tag{10}$$

where $\vec{n}$ is a vector normal to the boundary. Special volumes of a moving vehicle are assigned for the oxygen consumption (front part) and contaminants production (the rear part).

The initial conditions for the system of Eqs. (1)–(8) are the following:

$$t = 0, G\backslash G_r : \vec{u} = 0; \rho = \rho_0; T = T_0; \delta = \delta_0; Y_k = Y_{k0}; k = k_0; \varepsilon = \varepsilon_0; \tag{11}$$

$$t = 0, G_r : \vec{u} = u_v; \rho = \rho_0; T = T_0; \delta = \delta_0; Y_k = Y_{k0}; k = k_0; \varepsilon = \varepsilon_0; \tag{12}$$

To solve the system of Eqs. (3)–(9) splitting by physical processes and by coordinates is used. Each operator is also split into two parts: the parabolic and the hyperbolico-parabolic including the source terms. The computational technique is described in details in (Tannehill et al., 1997).

To study the influence of car motion on the dynamics of air in a tunnel, we performed a multidimensional modeling of gas-dynamic flows induced in the tunnel in the presence of cars based on the model described above. An axis-symmetrical problem within a half a cylinder with obstacles was solved. The computational domain simulated a cylindrical tunnel having a hemi-circle in the cross-section. The cars present in the tunnel were simulated by central obstacles composed of cylinders of different radii. The length of the computational domain could be variable as the computational domain was assembled of the attached sections each containing a single obstacle simulating one vehicle of a definite type. The characteristics of a tunnel and of model vehicles could be variable.

A typical computational domain can be seen in Fig. 1. The use of an axis-symmetrical approximation decreases the number of independent variables and the dimension of space thus decreasing the computer time necessary to perform one calculation and providing the possibility to perform parametric studies within a reasonable time.

In numerical simulations the air properties were as follows: air density ρ_1 = 1,29 kg/m^3; air initial temperature T = 300 K; tunnel radius R_T = 6 m; tunnel length L = 100 m; car maximal height H = 3 m; car front part height h = 1.5 m; the wind velocity in a tunnel was varied in the range between 1 m/s and 40 m/s.

The variable parameters in numerical simulations were the mean flow velocity (or the pressure drop) in the tunnel, the number and size of vehicles inside the tunnel, and the initial level of turbulence in the tunnel as that parameter could influence the drag forces and the overall pressure drop.

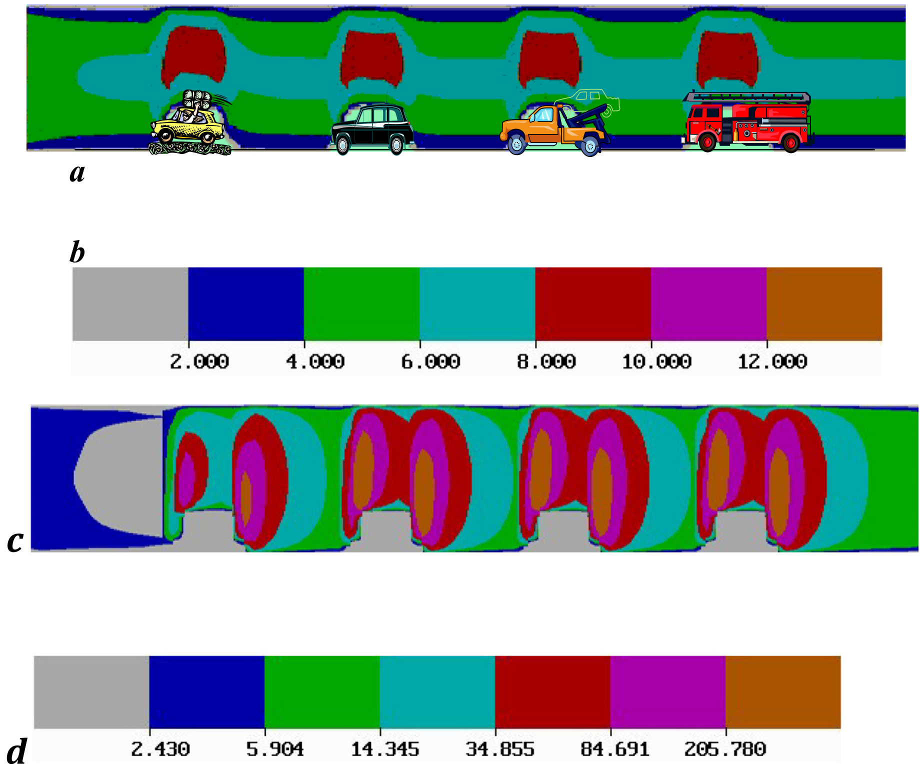

Fig. 1. Steady flow velocities (*a*) and turbulent kinetic energy distributions (*c*) in a tunnel 100 m long and 5 m high in the presence of four model vehicles 7 m long and 1.5 m high each. The scaling maps for velocities in m/s (*b*) and turbulent kinetic energy in m^2/s^2 (*d*). Mean airflow velocity 5 m/s.

The typical steady flow pictures obtained for different flow velocities and vehicle characteristics in the tunnel are shown in Figs. 2 and 3. The zone of flow induced vortices behind the vehicles is clearly seen in the figures. It is seen from the figures, that a bigger car provides a higher blockage ratio to the tunnel, which leads to the increase of turbulence and vorticity. The blockage ratio (BR: $S = \frac{S_{car}}{A}$) is defined as a ratio of a vehicle cross section to the cross section of the tunnel.

To investigate the influence of cars inside the tunnel on the air flow a series of numerical simulations was performed providing the dependence of flow velocity on the pressure drop in the tunnel for different values of the blockage ratios. (By the blockage ratio here we mean the ratio of an obstacle area to the tunnel area.) At a zero blockage ratio one has a tunnel in the absence of cars. The increase of the blockage ratio characterizes the presence of larger vehicles in the tunnel. To investigate the influence of initial flow turbulization we also varied the initial RMS values.

Fig. 4 shows the plots of mean flow velocity in a tunnel versus a pressure drop for variation of the blockage ratio. Fig. 4 allows to compare the air flow in the tunnel in the absence of cars (BR $=$ 0) with that in the presence of cars (BR $\neq$ 0). The results show that for the same pressure drop the mean

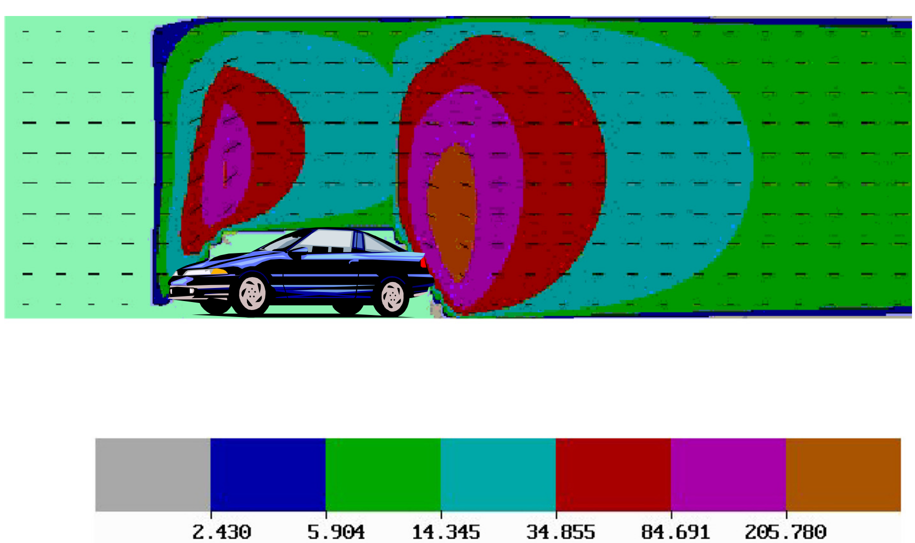

Fig. 2. Turbulent kinetic energy distributions and scaling maps (in m^2/s^2) in a section (41 m $< X <$ 68 m) of a tunnel 100 m long and 5 m high containing one model vehicle 7 m long and 1.5 m high. Mean airflow velocity 5 m/s. Initial RMS $= 0.35u_0$.

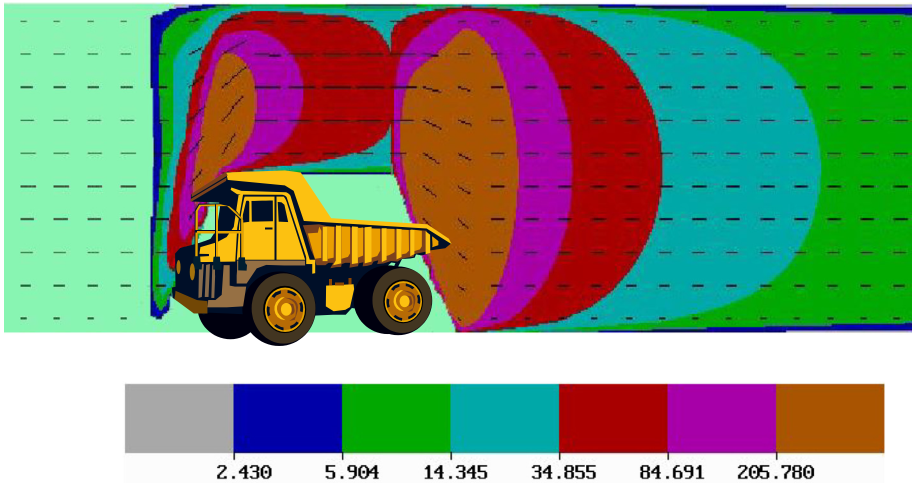

Fig. 3. Turbulent kinetic energy distributions and scaling maps (in m^2/s^2) in a section (41 m < X < 68 m) of a tunnel 100 m long and 5 m high containing one model vehicle 7 m long and 2.5 m high. Mean air flow velocity 5 m/s. Initial RMS = $0.35u_0$.

velocity difference increases proportionally to the blockage ratio. On the other hand, for the same blockage ratio the air flux variation in the tunnel is proportional to relative velocities of cars and the air flow. The velocities are provided in the moving system of co-ordinates linked to vehicles velocity, which means that in the system of coordinates linked with the tunnel, gas velocity in the direction of moving cars increases with the increase of the blockage ratio.

The obtained data of direct numerical simulations of the air flow in tunnels in presence of cars show that large variations of initial level of turbulence in the interval 0 < RMS < 0.3 does not cause a big divergence of the obtained curves. That testifies to the fact that the initial level of turbulence does not play an essential role in determining final results as in all the cases the final turbulization of the flow is determined rather by actual flow characteristics and channel geometry than by the initial level of turbulence.

Fig. 5 illustrates the results for the mean air flow velocity as a function of a pressure drop for the same blockage ratio BR = 0.6, but for different values of traffic flux density. The results show that increasing the traffic density also causes a decrease of relative mean velocity, which means increase of velocity in the tunnel in the direction of the cars motion.

4. The model for the tunnel air contaminants evolution caused by an unsteady traffic flow

The problem of the impact of vehicles on the tunnel environment that could be easily solved for the case of a single vehicle, turns to be hardly feasible for a traffic flow if one uses the same approach. The number of vehicles is very large and each one moves in a different environment perturbed by the previous one. Besides, the models for traffic flows provide

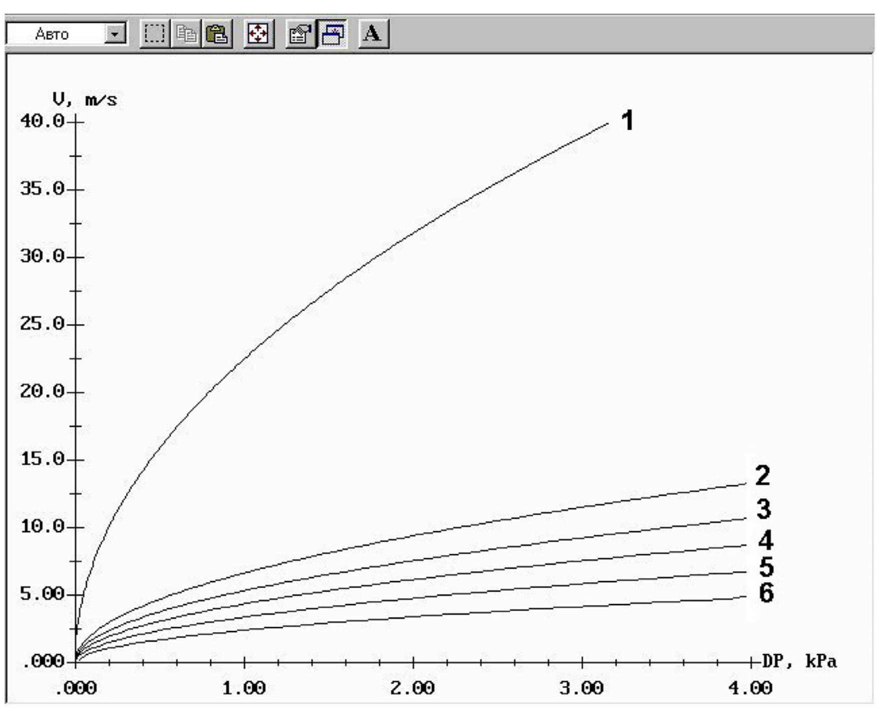

Fig. 4. The plots of mean flow velocity in a tunnel 100 m long and 5 m high versus a pressure drop for variation of the blockage ratio and constant traffic density n = 0.28 (4 cars in the tunnel): curve 1 - BR = 0; curve 2 - BR = 0.3; curve 3 - BR = 0. 4; curve 4 - BR = 0.5; curve 5 - BR = 0.6; curve 6 - BR = 0.7.

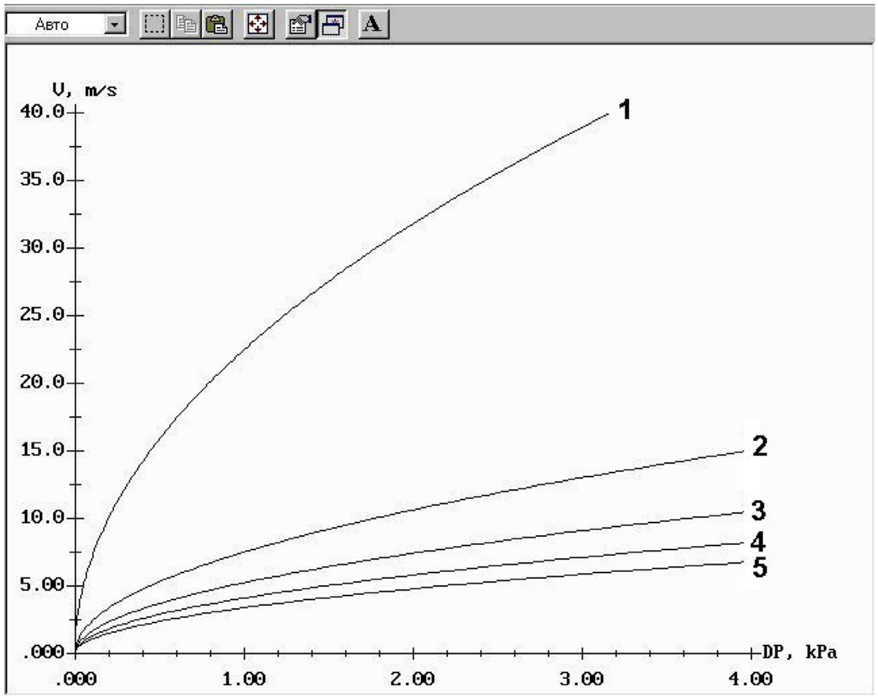

Fig. 5. The plots of mean flow velocity in a tunnel 100 m long and 5 m high as a function of a pressure drop for the same blockage ratio BR = 0.6 but for different values of traffic density: curve 1 - no cars in the tunnel (n ≡ 0); curve 2 - one car in the tunnel (n = 0.07); curve 3 - two cars in the tunnel (n = 0.14); curve 4 - three cars in the tunnel (n = 0.21); curve 5 - four cars in the tunnel (n = 0.28).

only averaged continuous characteristics. Thus it seems reasonable to apply a continuous model of the multi-phase mechanics for modeling the motion induced in the tunnel. The model would take into account the interaction of air and traffic and the accumulation or/and removal of pollutants under different traffic conditions.

The approach to be used is described in details in the paper (Smirnov et al., 1996b). The air in the tunnel would be assigned as a gaseous phase incorporating a number of gaseous components: oxidant, inert components (all other gases, such as nitrogen, water vapour, etc.) and contaminants (CO_2, CO, etc.). The flow of vehicles would be regarded as a condensed phase composed of rigid elements each characterized by its velocity and volume. The mean volume fraction of the condensed phase within the tunnel would be introduced as an additional unknown parameter to be determined by solving the traffic evolution equations. The traffic flow intensity and velocity variations could be determined using the Eqs. (1)–(2). The interaction of the solid phase elements with the gaseous phase (mass and momentum fluxes) could be determined by solving the Eqs. (3)–(9) for a single element and then introducing an appropriate approximation.

For long tunnels one could assume the flow one-dimensional that enables one to perform modeling within reasonable time scales. Meanwhile, all important multidimensional effects would be taken into account by using the solution of the local problem of a single vehicle interaction with the gas flow.

The results of direct numerical simulations of multidimensional air flow in a tunnel in presence of vehicles inside it show that the influence of vehicles is directly proportional to the velocity difference, cross-section area ratio and density of cars in the tunnel. Thus we suggest the following approximation providing a link between the velocity of air in the tunnel in absence of cars and in presence of moving cars:

$$u = u_0(1 - s) + sv \tag{13}$$

where $S = \dfrac{S_{car}}{A}$ is the cross-section area ratio of a car and a tunnel, u_0 is a velocity of an air flow in the tunnel in absence of moving cars.

The formula (13) gives a first order approximation. In case the cross-section area ratio is small the influence of moving vehicles in the tunnel on motion of air is negligible. If cross-section area ratio is close to unity the air velocity practically coincides with that of vehicles (the case of metro). Though being an approximation, the formula (13) gives us a big advantage allowing omitting momentum equation and keeping only mass equations for species Eqs. (4), (5). That reduces the overall number of differential equations incorporated in the model and simplifies the solution procedure.

Another simplification will be an assumption that we have only two components in the atmosphere of the tunnel: atmospheric air and gaseous pollutants. Let the mass concentration of gaseous pollutants be Y.

$$Y = \frac{\rho_{pol}}{\rho_1},$$

where ρ_{pol} is the density of gaseous contaminants, ρ_1 is the density of gas in a tunnel. Then the mass equation for gaseous contaminants takes the form:

$$\frac{\partial \rho_1 Y\alpha}{\partial t} + \frac{\partial \rho_1 Y\alpha u}{\partial x} = \dot{m}(x, t) \tag{14}$$

The volume fraction of gaseous phase in a tunnel α present in the Eq. (14) could be determined by the following formula

$$\alpha = 1 - ns \tag{15}$$

The contaminants mass emission function $\dot{m}(x,t)$ denoting the mass of gaseous contaminants emitted per time unit within an interval of a unit length having coordinate x at time moment t, could be determined by the following formula:

$\dot{m} = \dfrac{\dot{m}_1 n}{l \cdot A}$ (16)where $\dot{m}_1$is the mass of exhaust gases emitted by one car per time unit, $l\cdot$ is the dynamic length of the car. To determine the average emission we'll use the results of experiments. The average emission is closely connected with a number of revolutions per minute of the shaft of an engine. The experimental diagram showing the exhaust rate depending

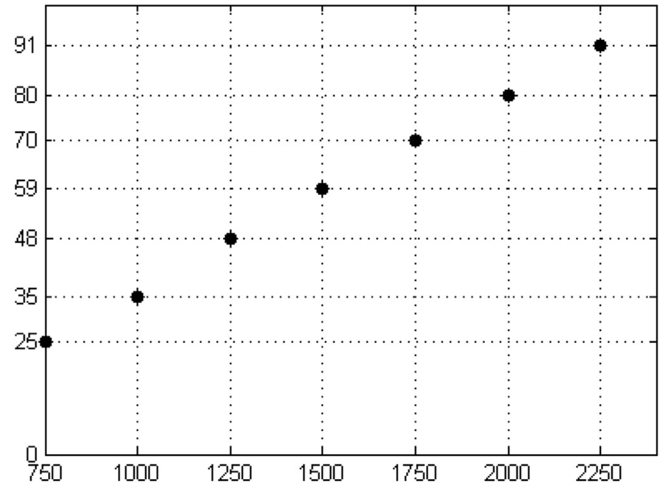

Fig. 6. Exhaust of an engine in kg per hour as a function of rotation per minute (R.P.M.) value.

on the engine R.P.M. can be found in Fig. 6. The dependence gives a practically linear proportionality.

Besides we assume that the concentration of toxic components (CO and CH) in the exhaust gases does not surpass the allowable values.

As the R.P.M. value is closely connected with acceleration of a car we'll assume that at a steady motion and deceleration the emission of toxic components is minimal $\dot{m}_1^0$. At a maximal acceleration the emission of toxic components is assumed to be maximal $\dot{m}_1^{max}$. Following the results of experiments, we'll assume that on increasing acceleration the emission of toxic components increases linearly. Based on the above assumptions the following formula is suggested:

$$\dot{m}_1 = \dot{m}_1^0 + \left(\dot{m}_1^{max} - \dot{m}_1^0\right)\frac{aH(a)}{a^+}, \tag{17}$$

which gives the dependence of automobile emission on the traffic conditions.

The notation a in Eq. (17) stands for actual acceleration of a car

$$a = \frac{dv}{dt}.$$

$H(a)$ is a Heavyside function $H(x) = \{1 \text{ at } x \geq 0; 0 \text{ at } x < 0\}$

Fig. 7 illustrates the accepted law of emission.

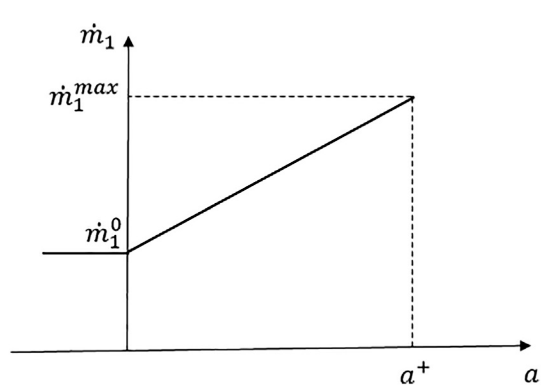

Fig. 7. Diagram of toxic components emission being a function of vehicle acceleration.

The developed model makes it possible to describe the emission of toxic components in an unsteady-state traffic flows in automobile tunnels and their transport with an air flow. It should be combined with a model for traffic flow evolution. Using this model in automobile tunnels free of intersections and traffic lights seems quite reasonable. Then the whole set of equations has the form:

$$\begin{cases} \dfrac{\partial n}{\partial t} + v\dfrac{\partial n}{\partial x} + n\dfrac{\partial v}{\partial x} = 0 \\ \dfrac{\partial v}{\partial t} + v\dfrac{\partial v}{\partial x} + \dfrac{k^2\partial n}{n\,\partial x} = 0 \\ \dfrac{\partial \rho_1 Y\alpha}{\partial t} + \dfrac{\partial \rho_1 Yu\alpha}{\partial x} = \dfrac{\dot{m}_1^0}{l{\cdot}S_{car}}n\left(1 + \dfrac{\dot{m}_1^{max} - \dot{m}_1^0}{a^+\dot{m}_1^0}aH(a)\right) \end{cases} \tag{18}$$

The first two equations of the system Eq. (18) allow us to determine the main characteristics of a traffic flow: velocity and density variations versus time and length of a tunnel. The third equation allows determining the toxic components concentration distribution in the tunnel and its evolution in time.

Numerical solution of the governing system of Eq. (18) was carried out using the three step algorithm of the TVD scheme of the first order of accuracy. The two model problems were chosen to serve the basis for numerical investigations. One of them investigates an essentially unsteady-state case, while the other deals with studying of automobile exhaust accumulation in a tunnel under the conditions of a quasi-steady motion of vehicles.

4.1. Automobile exhaust accumulation in the tunnel under the conditions of an essentially unsteady motion of cars

Let us regard a test section of a road having length L. The interval beginning from 0 up to $L/4$ is occupied by cars standing next to each other ($n = 1, v = 0$), the rest of the road is free ($n = 0, v = 0$). The part of the road from $L/5$ up to $L/2$ goes through a tunnel. At time $t > 0$ the front cars come into motion. The basic values of the governing parameters for the present problem are the following: $v_{max} = 80$ km/h = 22,22 m/s; $k = 35$ km/h = 9,72 m/s; $l = 5$ m; $L = 100$ m; $a^+ = 1{,}63$ m/s^2; $a^- = 5{,}5$ m/s^2; $u_0 = 5$ m/s, $\rho_1 = 1{,}29$ kg/m^3; $s = 0{,}2$; $\dot{m}_1^0 = 10^{-4}$ kg/s; $\dot{m}_1^{max} = 3.8{\cdot}10^{-4}$ kg/s (the values for $\dot{m}_1^0$ and $\dot{m}_1^{max}$ were determined based on the experimental data Fig. 6); $A = 10$ m^2.

Figs. 8 and 9 illustrate exhaust gases concentration profiles in the tunnel for different times after the beginning of motion. Thus these data show the accumulation of exhaust gases under essentially unsteady traffic conditions.

Concentration profiles shown in Fig. 8 correspond to the case when the velocity of the forced convection of the air in the tunnel ($u_0 = 5$ m/s) caused by fans and external wind goes in the same direction as the traffic flow. Fig. 9 illustrates the case when the forced convection of an air flow in the tunnel takes place in opposite direction to cars motion (the absolute value of convective flow velocity in the tunnel still remains the same: $u_0 = 5$ m/s).

As it is seen from Figs. 8 and 9 the unsteady-state exhaust concentration profiles in the tunnel appearing in the beginning of the process in the long run come to steady solutions. The limiting steady-state profiles, however, differ for the two different directions of wind in the tunnel. Comparing the two results one comes to a conclusion that the limiting steady concentration of exhaust gases in the tunnel is much less under the conditions the forced wind blows being a co-flow to cars motion in the tunnel as compared with the case wind blowing in the opposite direction. Thus it seems more effective to arrange venting of tunnels in the direction of the traffic flow.

On increasing the tunnel venting velocity u_0 up to 8 m/s the concentration of exhaust gases accumulating in the tunnel essentially decreases.

The concentration of exhaust gases accumulating in the tunnel is strongly affected by the cross-section area ratio: $S = \frac{S_{car}}{A}$. On increasing the cross-section area ratio accumulation of exhaust gases in the tunnel decreases.

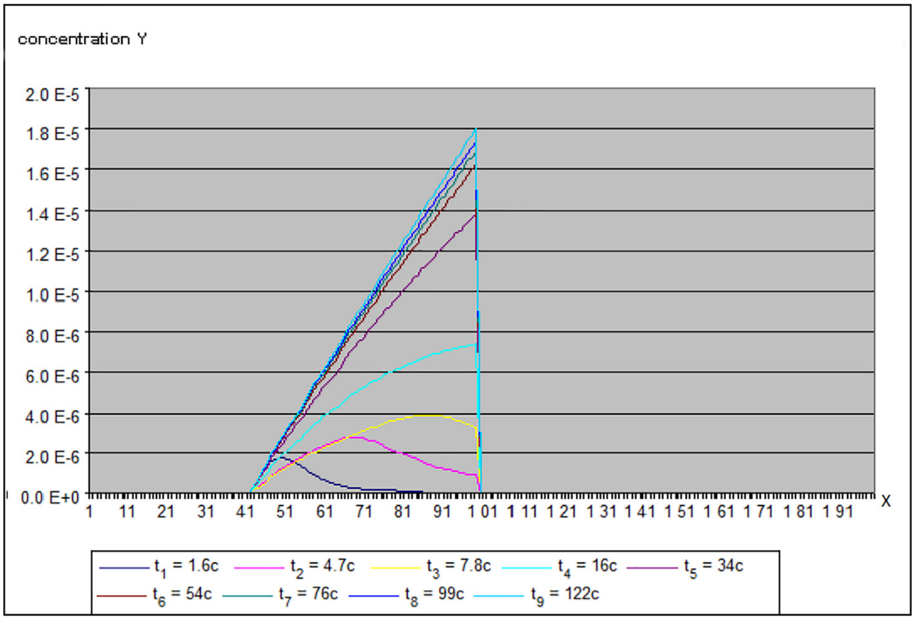

Fig. 8. Exhaust gases concentration Y profiles in the tunnel for successive times for the case velocity of air forced convection in the tunnel ($u_0 = 5$ m/s, from left to the right) coincides with the direction of traffic flow, $s = 0{,}2$.

4.2. Automobile exhaust accumulation in the tunnel under the conditions of a quasi-steady car flux

Let us regard a test section of a road having length L. Let the traffic flow at the initial instant be steady on the road characterized by density $n = ½$ and velocity $v = 80$ km/h $= 22{,}22$ m/s. The road at the interval beginning from $L/5$ up to $L/2$ passes through a tunnel. Let us investigate the accumulation of exhaust gases in the tunnel. The other values of governing parameters are similar to that regarded in the previous subsection: $k = 35$ km/h $= 9{,}72$ m/s; $l = 5$ m; $L = 100$ m; $a^+ = 1{,}63$ m/s^2; $a^- = 5{,}5$ m/s^2; $u_0 = 5$ m/s; $\rho_1 = 1{,}29$ kg/m^3; $s = 0{,}2$; $\dot{m}_1^0 = 10^{-4}$ kg/s; $\dot{m}_1^{max} = 3.8\cdot10^{-4}$ kg/s (the values $\dot{m}_1^0$ and $\dot{m}_1^{max}$ were determined based on the experimental data Fig. 6); $A = 10$ m^2.

Under the present conditions (Fig. 10) the maximal concentration of toxic exhaust gases is 3 times less than that for the case of essentially unsteady motion of cars discussed before. The reason is the following: amount of toxic exhaust gases increases on increasing acceleration of a car and has its minimum for a steady traffic mode.

Thus, the developed mathematical model is able to describe satisfactory atmospheric air pollution in the tunnels caused by traffic exhaust. The results of numerical investigations show that in the presence of long tunnels on automobile roads it is necessary to choose the traffic arrangement avoiding the necessity for vehicles to come to a full stop and then accelerate in tunnels. This could happen in the presence of traffic lights or other type of traffic regulation near the exit of the tunnel. The minimal safe distance from the tunnel, at which traffic regulation would not essentially increase

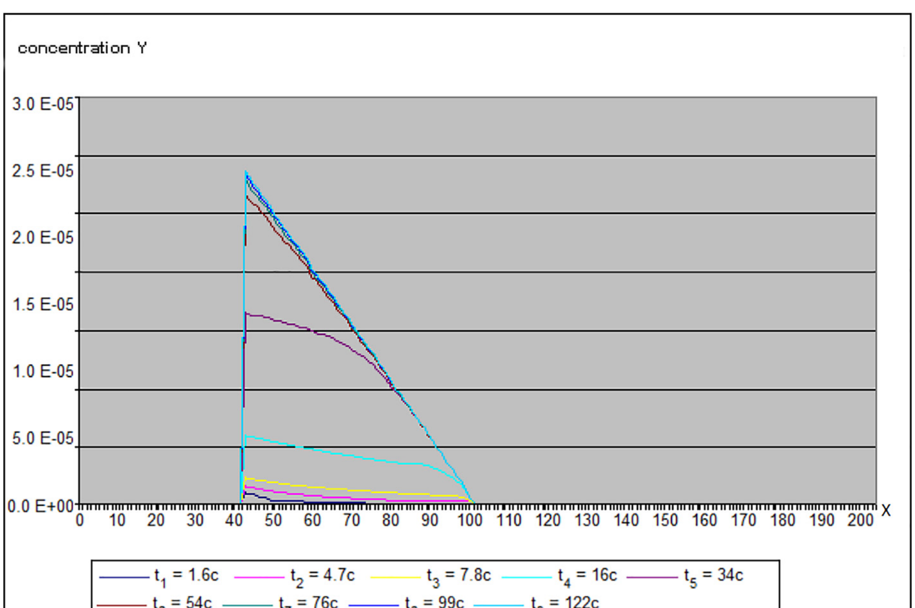

Fig. 9. Exhaust gases concentration Y profiles in the tunnel for successive times for the case velocity of air forced convection in the tunnel ($u_0 = 5$ m/s, from right to the left) is opposite direction for cars motion (from left to the right), $s = 0{,}2$.

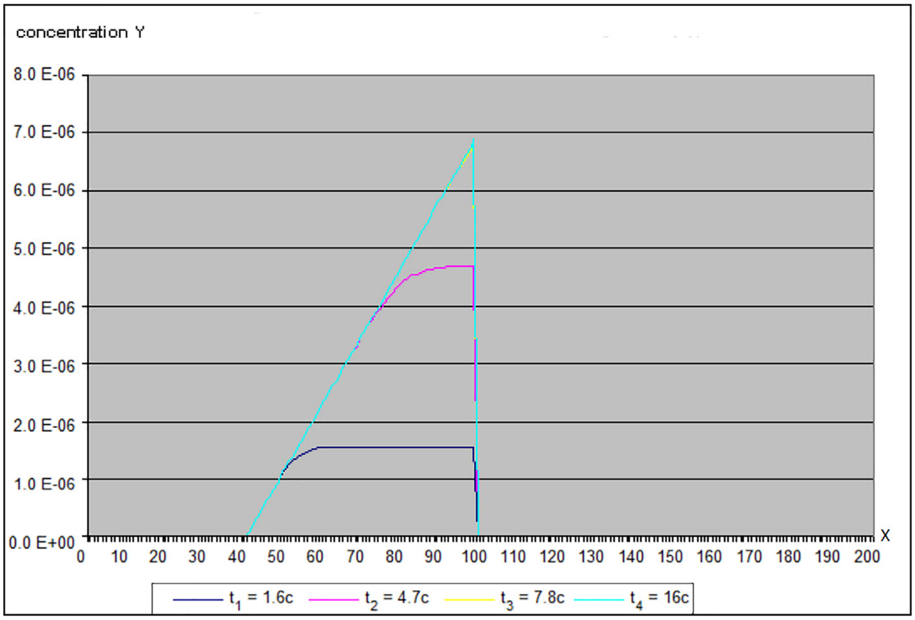

Fig. 10. Exhaust gases concentration Y profiles in the tunnel for successive times for steady traffic flow, velocity of air forced convection in the tunnel ($u_0 = 5$ m/s) coincides with the direction of traffic flow, $s = 0{,}2$.

the accumulation of exhaust gases in the tunnel depends upon the intensity of traffic, evolution of traffic jams opposite to the flow and the scenario of regulation.

The results show as well, that venting of tunnel using forced air convection could be an effective procedure reducing the air pollution if it is properly arranged. The direction of the wind in a tunnel should coincide with the direction of traffic flow. On the other hand, attempts to arrange venting in the opposite direction under certain conditions could result in bringing to a worse situation increasing the accumulation of toxic exhaust gases in a tunnel.

5. Conclusions

A mathematical model was worked out able to describe the environment of the city tunnels being affected by the road traffic, natural and forced air convective flows. The developed mathematical model takes into account:

- self-organization of the traffic flow depending on external regulations,
- handling capacity of the tunnel variations;
- the possibility of traffic jams origination and evolution;
- the coupled effects of the influence of the traffic intensity on the induced air flow in the tunnel and its turbulence;
- the role of additional forced air convection in the tunnel.

The analysis of the mathematical model for traffic flows and of the typical problems solution shows that the medium under consideration (traffic flow) has some basic peculiarities distancing it from substances traditionally regarded using the continua mechanics formalism, however some analogy with the theory of compressible gas motion still exists.

The mathematical model describing essentially unsteady one-lane traffic flows is constructed based on kinematic and dynamic equations. The model does not need empirical relationships to be incorporated in.

Multidimensional calculations of the influence of cars on the airflow in tunnels were performed. The results of direct numerical simulations of multidimensional airflow in a tunnel in the presence of vehicles inside it show that the influence of vehicles is directly proportional to the velocity difference, cross-section area ratio and density of cars in the tunnel. Based on the results of numerical modeling an approximation formula was suggested providing a link between the speed of air in the tunnel in absence of cars and the speed of air in the presence of moving cars in a tunnel.

The numerical model for simulating exhaust gases emission by automobiles and their accumulation in a tunnel and evolution with traffic induced airflow was developed.

Results of theoretical investigations made it possible developing basic recommendation for those designing traffic infrastructures in cities.

First, in the presence of long tunnels the traffic arrangement should avoid vehicles to come to a full stop and then accelerate in tunnels. No traffic lights or other type of traffic regulation should be present in the tunnel or near the exit of the tunnel. The minimal safe distance from the tunnel, could be developed based on estimates of the expected length of a traffic jam near the place of traffic regulation.

Second, venting of tunnel using forced air convection is effective for reducing the air pollution if the direction of the wind in a tunnel coincides with the direction of traffic flow.

Third, venting flow having opposite direction could result in accumulation of toxic exhaust gases in a tunnel for high blockage ratios.

The developed model makes it possible to forecast the air pollution in tunnels under different conditions of traffic regulations and intensity and determining maximal cars flux in a road under the condition not exceeding the critical level of air pollution.

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References

Barnes, J.H., Chatterton, T.J., Longhurst, J.W.S., 2019. Emissions vs exposure: increasing injustice from road traffic-related air pollution in the United Kingdom. Transp. Res. Part D: Transp. Environ. 73, 56–66 ISSN 1361-9209. https://doi.org/10.1016/j.trd.2019.05.012.

Borsche, R., Kimathi, M., Klar, A., 2012. A class of multi-phase traffic theories for microscopic, kinetic and continuum traffic models. Comput. Math. Appl. 64, 2939–2953.

Brackstone, M., Mcdonald, M., 1999. Car-following: a historical review. Transp. Res. Part F 2, 181–196.

Carroll, P., Caulfield, B., Ahern, A., 2019. Measuring the potential emission reductions from a shift towards public transport. Transp. Res. Part D: Transp. Environ. 73, 338–351 ISSN 1361-9209. https://doi.org/10.1016/j.trd.2019.07.010.

Chowdhury, D., Santen, L., Schadsschneider, A., 2000. Statistical physics of vehicular traffic and some related systems. Phys. Rep. 329, 199–329.

Costeseque, G., Lebacque, J.P., 2014. A variational formulation for higher order macroscopic traffic flow models: numerical investigation. Transp. Res. Part B 70, 112–133.
Greenberg, H., 1959. An analysis of traffic flow. Oper. Res. 7, 79–85.
Gupta, A.K., Katiyar, V.K., 2007. A new multi-class continuum model for traffic flow. Transportmetrica 3 (1).
Helbing, D., 2001. Traffic and related self-driven many-particle systems. Rev. Mod. Phys. 73, 1067–1141.
Helbing, D., Huberman, B.A., 1998. Coherent moving states in highway traffic. Nature 396 (6713), 738–740.
Helbing, D., Treiber, M., 1998. Gas-kinetic-based traffic model explaining observed hysteretic phase transition. Phys. Rev. Lett. 81, 3042–3045.
Hoogendoorn, S.P., Bovy, P.H.L., 2000. Continuum modeling of multiclass traffic flow. Transp. Res. Part B 34 (2), 123–146.
Kerner, B.S., Konhäuser, P., 1993. Cluster effect in initially homogemeous traffic flow. Phys. Rev. E 48, 2335–2338.
Kiselev, A.B., Nikitin, V.F., Smirnov, N.N., Yumashev, M.V., 2000. Irregular traffic flow on a ring road. Journal of Applied Mathematics and Mechanics (Engl. Transl.) 64 (4), 627–634.
Kumar, B.A., Vanajakshi, L., Subramanian, S.C., 2017. Bus travel time prediction using a timespace discretization approach. Transp. Res. Part C 79, 308–332.
Ladino, A., Kibangou, Y., Canudas de Wit, C., Fourati, H., 2017. Travel time prediction and departure time adjustment behavior dynamics in a congested traffic system. Transp. Res. Part C 80, 216–238.
Lighthill, M.J., Whitham, G.B., 1955. On kinetic waves II. A theory of traffic flow on long crowded roads. Proc. of the Royal Society. Ser. A. 229 (1178), 317–345 (London).
Ma, Z.L., Koutsopoulos, H.N., Ferreira, L., Mesbah, M., 2017. Estimation of trip travel time distribution using a generalized markov chain approach. Transp. Res. Part C 74, 1–21.
Nagatani, T., 2001. Bunching transition in a time-headway model of a bus route. Phys. Rev. E. 296 (1–2), 320–330.
Nagatani, T., 2002. The physics of traffic jams. Rep. Prog. Phys. 65, 1331–1386.
Nagel, K., Schreckenberg, M.A., 1992. Cellular automaton model for freeway traffic. J. De Phys. I 2 (12), 2221–2229.
Ngoduy, D., 2013. Platoon-based macroscopic model for intelligent traffic flow. Transportmetrica B: Transport Dynamics 1 (2), 153–169.
Ngoduy, D., Maher, M.J., 2012. Calibration of second order traffic models using continuous cross entropy method. Transportation Research Part C: Emerging Technologies 24, 102–121 ISSN 0968-090X. https://doi.org/10.1016/j.trc.2012.02.007.
Prigogine, I., Résibois, P., 1962. On a generalized Boltzman-like approach for traffic flow. Bull. Cl. Sci., Acad. Roy. Belg. 48 (9), 805–814.
Raeesi, R., Zografos, K.G., 2019. The multi-objective Steiner pollution-routing problem on congested urban road networks. Transportation Research Part B Volume 122, 457–485.
Rahmani, M., Koutsopoulos, H.N., Jenelius, E., 2017. Travel time estimation from sparse floating car data with consistent path inference: a fixed point approach. Transp. Res. Part C 85, 628–643.
Regirer, S.A., Smirnov, N.N., Chenchik, A.E., 2007. Mathematical model of moving collectives interaction: public transport and passengers. Autom. Remote. Control. 68 (7), 1225–1238.
Richards, P.L., 1956. Shock waves on the highway. Oper. Res. 4, 42–51.
Smirnov, N.N., Nikitin, V.F., Legros, J.C., 1996a. Mathematical modelling of aerosol evolution and sedimentation in the atmosphere of big cities. Int. Journal of Aerosol Science 27 (1), 5597–5598 sup..
Smirnov, N.N., Zverev, N.I., Tyurnikov, M.V., 1996b. Two-phase flow behind a shock wave with phase transitions and chemical reactions. International Journal of Experimental Thermal and Fluid Science. 13 (1).
Smirnov, N.N., Kiselev, A.B., Nikitin, V.F., Yumashev, M.V., 2000. Mathematical modelling of road traffic flows. Moscow University Mechanics Bulletin. 2000, № 4. Allerton Press, New York, pp. 39–44.
Smirnov, N.N., Nikitin, V.F., Legros, J.C., Shevtsova, V.M., 2002. Motion and sedimentation of particles in turbulent atmospheric flows. Aerosol Science and Technology. The Journ. of the American Association for Aerosol Research. 36, 101–122.
Smirnova, M.N., Bogdanova, A.I., Zhu, Z., Smirnov, N.N., 2015. Exploring peculiarities of traffic flows with a viscoelastic model. Transportmetrica A: Transport Science 11 (7), 561–578.
Smirnova, M.N., Bogdanova, A.I., Zhu, Z., Smirnov, N.N., 2016. Traffic flow sensitivity to visco-elasticity. Theor. Appl. Mech. Lett. 6 (4), 182–185. https://doi.org/10.1016/j.taml.2016.05.003.
Smirnova, M.N., Bogdanova, A.I., Zhu, Z., Smirnov, N.N., 2017. Traffic flow sensitivity to parameters in viscoelastic modeling. Transportmetrica B: transport Dynamics 5 (1), 115–131. https://doi.org/10.1080/21680566.2016.1142402.
Sukhinova, A.B., Trapeznikova, M.A., Chetverushkin, B.N., Churbanova, N.G., 2009. Two dimensional macroscopic model for traffic flows. Mathematical modeling. 21 (2), 118–126.
Sun and Zhang, 2018. Influence of avenue trees on traffic pollutant dispersion in asymmetric street canyons: numerical modeling with empirical analysis. Transp. Res. Part D: Transp. Environ. 65, 784–795.
Tannehill, J.C., Anderson, D.A., Pletcher, R.H., 1997. Computational Fluid Mechanics and Heat Transfer. Taylor & Frances, p. 1997 (792 p).
Tordeux, A., Roussignol, M., Lebacque, J.P., Lassarre, S., 2014. A stochastic jump process applied to traffic flow modelling. Transportmetrica A: Transport Science 10 (4), 350–375.
Zhang, Y., Smirnova, M.N., Bogdanova, A.I., Zhu, Z., Smirnov, N.N., 2018a. Travel time estimation by urgent-gentle class traffic flow model. Transp. Res. B 113, 121–142.
Zhang, Y., Smirnova, M.N., Bogdanova, A.I., Zhu, Z., Smirnov, N.N., 2018b. Travel time prediction with a viscoelastic traffic model. Appl. Math. Mech. -Engl. Ed. 39 (1), 1–26.
Zheng, L., Jin, P.J., Huang, H., 2015. An anisotropic continuum model considering bidirectional information impact. Transp. Res. B Methodol. 75, 36–57 ISSN 0191-2615. https://doi.org/10.1016/j.trb.2015.02.011.
Zheng, L., He, Z., He, T., 2017. An anisotropic continuum model and its calibration with an improved monkey algorithm. Transportmetrica A: Transport Science 13 (6).
Zhu, Z.J., Yang, C., 2013. Visco-elastic traffic flow model. J. Advanced Transp. 47, 635–649.