



Letter

Traffic flow sensitivity to visco-elasticity

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HIGHLIGHTS

- Visco-elasticity effect on traffic flows.
- Self-organization is a crucial feature in traffic flow pattern formation.
- Optimization of traffic control regulations is necessary.

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ABSTRACT

This letter reports traffic flow sensitivity to visco-elasticity, with the traffic flow modeling briefly described at first and then used to do traffic flow simulations whose results can reflect the properties of spatial-temporal evolution of ring traffic flow. It reveals that visco-elasticity plays crucial role in formation of traffic flow patterns, implying that self-organization of traffic flow is crucial in determining traffic flow status.

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Traffic flows have been extensively studied due to its significant impacts on work and life in modern society. Many models have been developed to ascertain traffic flow characteristics and understand intrinsic properties of traffic wave propagation, among which is vehicular mass conservation based Lighthill, Whitham, and Richards (LWR) model [1,2], probably the simplest one being able to capture some crucial flow features on highways, and predict traffic shock waves with relatively steep wave fronts [3]. Although results of LWR traffic modeling are not completely favorable for predicting traffic waves spreading on highways, its extensions can predict traffic hysteresis [4], evolution of density waves [5], and critical transition of bottleneck in traffic flows [6].

In traffic modeling, further involving momentum conservation leads to the occurrence of high-order models, among which are

the Euler model [7], the gas-kinetic-based model [8,9], the cluster effect model [10], and the generic model [11–13]. High-order models can explain the vehicular stop and moving phenomena and predict traffic wave spreading successfully. Although a critic comment [14] has deeply suspected high-order models, there are still many remarkable applications [15–17] and further developments [18–28].

In traffic systems, spatial-temporal pattern formations have a surprisingly rich spectrum, which can be described by the car-following models, the cellular automaton models [29–31], the gas-kinetic models, or the fluid-dynamical models [32]. Using methods in statistical physics and nonlinear dynamics to self-driven many-particle systems, Helbing has answered some flow questions [33].

As soon as instantaneous traffic flow rate q is not equal to equilibrium flow rate q_e , time headway will automatically adjust to approach $1/q_e$ [34]. By introducing traffic viscosity and pressure in some high-order models [10,35], this self-organization of traffic flow has been considered. In congested traffic flows where traffic density is larger than saturation density $\rho_s (= \rho_m/e)$ as shown in Fig. 1, it is common to observe instantaneous variation of flow rate.

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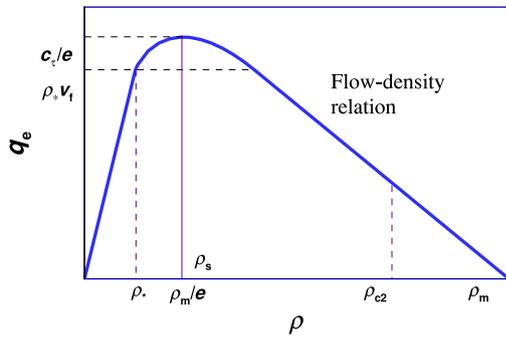


Fig. 1. Fundamental diagram used in Ref. [21].

The interaction of traffic flow waves is more intensive, causing the occurrence of a synchronized flow regime or a jam existing regime [36]. However, the flow is homogeneous and stable for the denser traffic flow where traffic density is larger than the second critical density ρ_{c2} [37].

Since relaxation and elastic processes are intrinsically related from points of view in fluid mechanics, relaxation time has been used for denoting external force of traffic flows, visco-elastic traffic flow models have some grounds of fluid mechanics. For simplicity, we assume: (1) ramp flow effect is negligible; (2) road capacity is insensitive to vehicular drivers; (3) traffic flow satisfies linear viscoelastic constitutive relation. The main reason for the 3rd assumption has been reported in Ref. [22].

Let q_e be traffic flow rate under equilibrium flow state, using traffic fundamental diagram as shown in Fig. 1, it has the form

$$q_e = \begin{cases} v_f \rho & \text{for } \rho \leq \rho_*, \\ -c_\tau \rho \ln(\rho/\rho_m) & \text{for } \rho_* < \rho \leq \rho_m. \end{cases} \quad (1)$$

As reported previously [27], traffic pressure p is proportional to density, with the proportional coefficient being the square of traffic sound speed c^2 . Using jam density and jam pressure of traffic flow, p has the form

$$p = p_m(1 - \alpha)(\rho/\rho_m)/[1 - \alpha(\rho/\rho_m)], \quad (2)$$

where

$$\rho_* = \rho_m \exp(-v_f/c_\tau) = \rho_m[1 + X(v_f)/l]^{-1}$$

and

$$c_\tau = v_f / \ln[1 + X(v_f)/l],$$

as shown in Fig. 1. Note that $\alpha = l/\rho_m$, l is average vehicular length, $X(v_f)$ is braking distance depending merely on free-flow speed v_f . The fundamental diagram is driver-dependent [23]. Clearly, it has significant impact on traffic road operation [38]. While the jam pressure is

$$p_m = c_0^2 \rho_m, \quad (3)$$

where

$$c_0^2 = \frac{(1 - \alpha\rho_*/\rho_m)\rho_*/\rho_m}{2(1 - \rho_*/\rho_m)} v_f^2 \quad (4)$$

with the sound speed in traffic flows given by

$$c = c_0 \sqrt{1 - \alpha/(1 - \alpha\rho/\rho_m)}. \quad (5)$$

As expressed by Eq. (5), traffic sound speed is proportional to free flow speed v_f . To express relaxation time τ explicitly, supposing $(c \cdot \tau) = \text{const}$, and denoting traffic jam relaxation time by $\tau_0 = l_0/c_\tau$ with traffic length scale l_0 , we have [27]

$$\tau = \tau_0(1 - \alpha\rho/\rho_m)/\sqrt{1 - \alpha}. \quad (6)$$

Table 1
Traffic operation parameters on the ring road.

v_f (km·h ⁻¹)	ρ_m (veh·km ⁻¹)	$X(v_f)$ (m)	l (m)
110	150	50	5.8

Obviously, the relaxation time τ decreases linearly with traffic density, implying that equilibrium flow state can play a more important role under congested flow conditions, as external traffic force is explicitly τ -dependent.

Therefore, the governing equations of viscoelastic traffic flows are (see also in Ref. [27])

$$\begin{cases} \rho_t + q_x = 0, \\ \rho(u_t + uu_x) = R \end{cases} \quad (7)$$

with R satisfying the expression

$$R = \rho(u_e - u)/\tau - c^2 \rho_x + [(2G\tau)u_x]_x, \quad (8)$$

where x is space coordinate, G is the modulus of traffic flow elasticity, $2G\tau = \rho\nu$, with ν denoting the kinematic viscosity of traffic flow. The inclusion of elasticity allows the acceleration speed varying with viscoelastic dependent force, implying the traffic flow model has intimately involved the influences of neighborhood traffic operations.

To verify the proposed model briefly described above, numerical tests are carried out to show the visco-elastic effect on ring road traffic flows. The road length is assumed to be $860l_0$, with a length unit $l_0 = 160$ m, and a velocity scale given by $v_0 = v_f\rho_*/\rho_m \approx 3.176 \text{ m}\cdot\text{s}^{-1}$, and time scale $t_0 = l_0/v_0 \approx 50.377$ s. The initial density condition is given by

$$\rho(0, x) = \begin{cases} 1.0 & \text{for } x/l_0 \in [429, 431], \\ 1/3 & \text{otherwise} \end{cases} \quad (9)$$

with $q(0, x) = q_e(\rho(0, x))$. The flow on the ring road is explicitly congested, as traffic density in some regions of the road is higher than normalized saturation density $\rho_s/\rho_m = 1/e$. The numerical tests use fundamental diagram given by Fig. 1, with traffic operation parameters on the ring road given in Table 1. It is assumed that average vehicular length l is 5.8 m, jam density is 150 m, free flow speed is 110 km·h⁻¹, the braking distance is 50 m. If the traffic length scale l_0 is fixed at 160 m, the relaxation time $\tau_0 = l_0/c_\tau$ is $0.2353t_0 \approx 11.854$ s. To seek the viscoelastic impact on ring traffic flows, viscoelastic parameter $\gamma = \hat{G}_{\tau_0} = \frac{2G(\tau_0 v_0)}{l_0^2} \cdot \frac{t_0}{q_0}$ is assumed to be 0.03125, 0.0625, and 0.125, respectively.

The comparison of ring traffic patterns is illustrated in Fig. 2(a–c), where the ring traffic density contours are shown in flood-type form and labeled by values of 0.309, 0.35, 0.618, and 0.7, respectively. This means the density in the blue region is below 0.309, in the red region is larger than 0.7. While traffic density in the cyan region has a value in the range of [0.309, 0.35], with the green and yellow regions being relevant to the density range [0.35, 0.618], and [0.618, 0.7], respectively. From Fig. 2(a–c), it can be seen that with the increase of viscoelastic parameter γ , self-organization of traffic flows increases. Since the flow pattern has become more regular as a result of interaction of traffic shock and deflation waves, it reveals that self-organization can impact traffic flow pattern formation significantly.

To illustrate traffic sensitivity to visco-elasticity, distributions of time-average based mean density and speed are given in Fig. 3(a–b), where green-solid, blue-dash, and dash-dot black curves are relevant to $\gamma = 0.03125, 0.0625, \text{ and } 0.125$, respectively. The mean traffic density and speed on the ring road are both clearly sensitive to parameter γ , a smaller γ value can lead to a larger variation range of traffic density and speed. As seen in

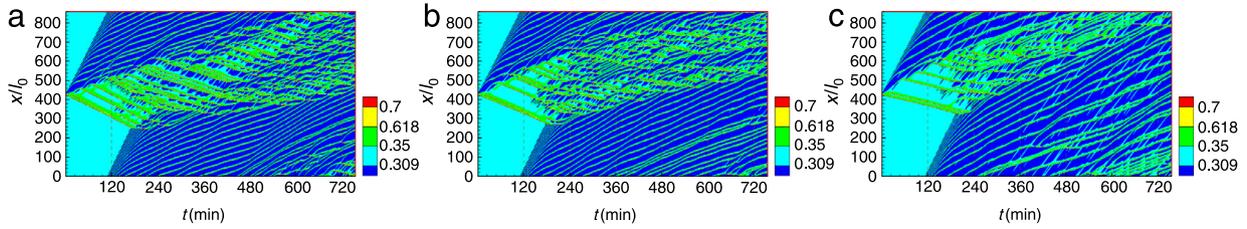


Fig. 2. (Color on online) Comparison of ring traffic density contours in the $t - x/l_0$ plane, (a) $\gamma = 0.03125$, (b) $\gamma = 0.0625$, and (c) $\gamma = 0.125$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

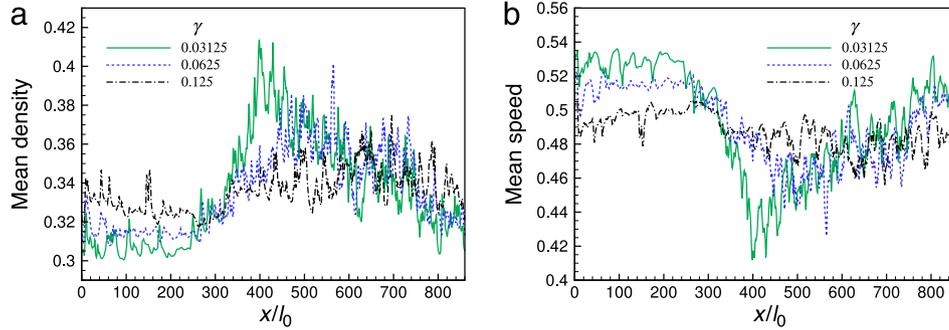


Fig. 3. Distributions of (a) mean density and (b) mean speed on the ring road. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

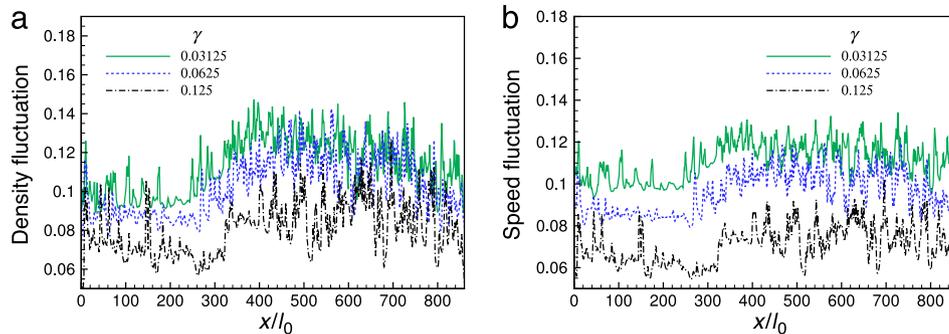


Fig. 4. Distributions of (a) density and (b) speed fluctuations on the ring road.

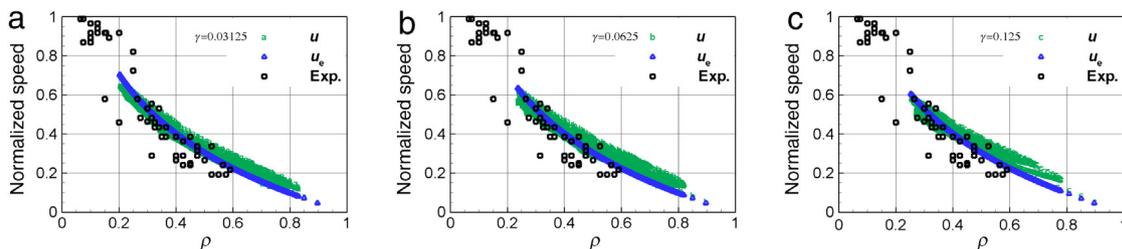


Fig. 5. Comparison of traffic speed with existing measured data at $x/l_0 = 430$, (a) $\gamma = 0.03125$, (b) $\gamma = 0.0625$, and (c) $\gamma = 0.125$. The observation data are obtained from Ref. [39], and the jam density for normalization is supposed to be $200 \text{ veh}\cdot\text{mile}^{-1}$.

Fig. 2(a–c), self-organization represented by γ , can bring about significantly difference of traffic wave structures. The traffic wave interaction has resulted in oscillated distributions of mean density and speed, with the oscillation mode being γ -dependent.

The distributions of density and speed fluctuations are shown in Fig. 4(a–b), where the so-called fluctuation is just time-average based root mean square (RMS) value of some variable. Totally, the smaller the value of visco-elastic parameter γ , the larger RMS values are for traffic density and speed. Since the distributions intrinsically depend on the traffic wave propagation on the ring

road, at a given spatial point, the γ induced difference of RMS values of density and speed has some uncertainty.

As shown in Fig. 5(a–c), the instantaneous speed u recorded at the section of $x = 430$ plotted as a function of density is labeled by green alphabets a, b, and c, respectively. The instantaneous equilibrium speed u_e at the section of $x = 430$ determined by fundamental diagram Eq. (1) is labeled by non-filled blue triangles, with existing measured data [39] labeled by non-filled black squares. The comparison of speed–density relation at a given observing road section with measured data, not only shows the traffic sensitivity to visco-elasticity γ , but also reflects to some

extent the potential of viscoelastic traffic flow model, and the reliability of simulation results.

The viscoelastic traffic model introduced above has potential in traffic flow simulations. Numerical tests revealed that viscoelastic sensitivity of traffic flows is certainly larger, indicating that self-organizing traffic flows play a significant role in flow pattern formation, and it is crucial in determining traffic flow status. This suggests that optimization of traffic control regulations is necessary in reality as traffic flow has become a key feature of modern society.

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