Pergamon

Int. Comm. Heat Mass Transfer, Vol. 29, No. 4, pp. 497-508, 2002

0735–1933/02/$–see front matter

PII: S0735-1933(02)00347-0

NUMERICAL ANALYSIS OF HEAT TRANSFER IN A PHOTOVOLTAIC PANEL, I: INDOOR CASES

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(Communicated by J.P. Hartnett and W.J. Minkowycz)

ABSTRACT

This paper presents a numerical analysis of heat transfer in a photovoltaic (PV) panel for indoor cases. A mathematical model was first described from energy conservations, in which the effects of thermal storage, the absorption of optical energy, as well as the thermal contact resistances at interfaces between materials in the PV panel were taken into consideration. Based on this model, for indoor cases, numerical analysis of heat transfer was then carried out by using a fractional scheme under the assumptions of constant physical properties, and constant efficiency of the optical energy conversion for the panel working under indoor conditions. It was found that the thermal contact resistances at interfaces play a profound role of the temperature field in a PV panel. The effect of thermal storage is rather significant for the evolution of average surface temperature and average temperature of solar cells. However, the extinction coefficient of the transparent glass cover shows just a slight influence on temperature field. © 2002 Elsevier Science Ltd

Introduction

In the new century, photovoltaic (PV) application would appears a great potentiality in energy and building services engineering due to its direct electricity generation, further, this technology is helpful to the environmental protection of modern life. During the process of energy conversion, apart from electricity generation, when optical energy is imposing on a PV panel, it changes into heat in a great portion which is then dissipated away. So far, many models have been developed for the purpose of photovoltaic simulation. For example, Brinkworth et al. [1] proposed a model to study the effect of thermal regulation; Yoo et al. [2] reported their study in Korea for building integrated photovoltaics; Hirata et al. [3] reported a model to consider the output variation of photovoltaic panels including seasonal variation; Yang et al. [4] built a model to investigate the performance of crystalline silicon photovoltaics; Recently, Zhu et al. [5] has developed a heat transfer model to simulate a building integrated photovoltaic system. However, in order to analyze heat transfer in a PV panel in great detail, improvements of the model aforementioned is necessary to contain the effect of thermal storage and heat absorption in the transparent cover of a PV panel.

It is common knowledge that a PV panel is composed of three layers, *i. e.* the glass cover, the rear paper layer and the solar cell layer. The cover and the rear layer are used to avoid the oxidization of solar cells. The allowed space between the glass and the paper layers is vacuumized for the same reason.

In this paper, we presents a numerical analysis of heat transfer in a PV panel which is considered as a multiple-layer complex plate, the relevant schematic is shown in Figure 1. The incident optical energy flux with intensity I_0 are normal to the cover, apart from the negligible reflection, and most incident optical energy is directly into the transparent cover with layer thickness δ_1. Apart from the absorption of the cover, those portions of optical energy flux which has transmitted across the cover and arrived at the solar cell surface, is permissible to be converted into electricity. The operation of a PV penal is accompanied with heat transfer and heat loss. In view of these features, developing a mathematical model different from those mentioned previously is of great significance, since, it permits a numerical analysis which may provide detail solutions for temperature field in a PV panel, and show the effects of the thermal storage, absorption of optical energy in the transparent cover, and thermal contact resistances on the temperature field and its evolution as well

Mathematical Model

Suppose $\delta_i(i=1,2,3)$ denote the layer thickness of cover, PV cell, or paper, together with panel height H. Let K denote the extinction coefficient of the cover, $Cp_i(i=1,2,3)$ express the specific capacity

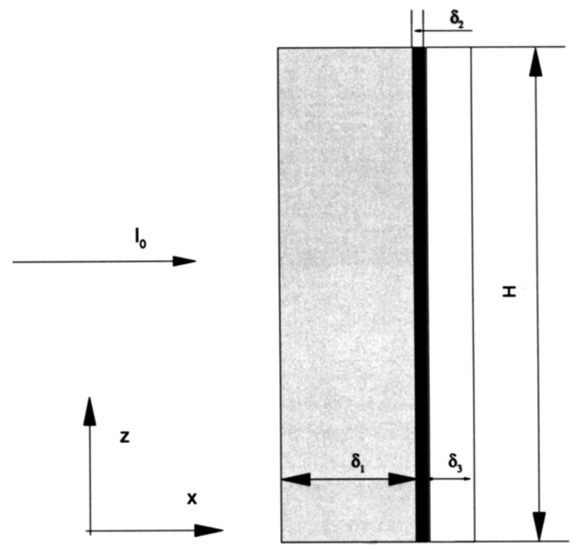

FIG. 1
The schematic of a PV panel working under indoor conditions

under constant pressure. Considering that δ_2 is generally small, and heat is generally well conducted, the temperature $T_2=T(x,y)$ defined in the PV cell (*i.e.* in R^2: $x\in(\delta_1+\delta_2)$, $y\in(0,H)$) is close to uniform in the normal direction of PV panel. This implies that $T_2\approx\int_{\delta_2}^{\delta_1}T(x,y)dx/\delta_2$ is held. Thus, if thermal physical properties of a PV panel are unchanged in our problem, following Mathey [6] and from energy conservation, we obtain: for $y\in[0,H]$,

$$\frac{\partial T_2}{\partial t}=\frac{q_2-q_1-q_3}{\delta_2\rho_2 Cp_2}+\alpha_2\frac{\partial^2 T_2}{\partial z^2},\quad \text{for } x\in(0,\delta_1) \tag{1}$$

$$\frac{\partial T_1}{\partial t}=\alpha_1\left[\frac{\partial^2 T_1}{\partial x^2}+\frac{\partial^2 T_1}{\partial z^2}\right]+\frac{KI_0}{\rho_1 Cp_1}\exp(-Kx),\quad \text{for } x\in(0,\delta_1) \tag{2}$$

$$\frac{\partial T_3}{\partial t}=\alpha_3\left[\frac{\partial^2 T_3}{\partial x^2}+\frac{\partial^2 T_3}{\partial z^2}\right],\quad \text{for } x\in(\delta_1+\delta_2,\delta_1+\delta_2+\delta_3) \tag{3}$$

where ρ_i(i=1,2,3) is material density of glass, PV cells, or paper; corresponding to the thermal diffusivity $\alpha_i = \lambda_i/\rho_i\, Cp_i$. $q_2 = (a\tau)I_0 - \eta I_0$ is the optical energy absorbed by PV cells; η is the efficiency of energy conversion, which is assigned to be 10% in the analysis; q_1 and q_3 are the heat flux conducted to the glass and paper, respectively, while $(a\tau)$ denotes the transmittance-absorptance product for the cover. There should have thermal contact resistance between two layers, this feature should be included in the analysis. For heat transfer in the multiple-layer PV panel the boundary conditions of thermal equilibrium must be involved with constant thermal resistances. Accordingly, we have, for $y \in [0,H]$,

$$-\alpha_1\rho_1 Cp_1 \frac{\partial T_1(x,y)}{\partial x} = (h_{r1} + h_{c1})(T_\infty - T_1(x,y)), \quad x = 0 \tag{4}$$

$$q_1(x,y) = \alpha_1\rho_1 Cp_1 \frac{\partial T_1(x,y)}{\partial x} = -\Delta T_{12}/R_{s1}, \quad \text{for } x = \delta_1 \tag{5}$$

$$q_3(x,y) = -\alpha_3\rho_3 Cp_3 \frac{\partial T_1(x,y)}{\partial x} = \Delta T_{23}/R_{s2}, \quad \text{for } x = \delta_1 + \delta_2 \tag{6}$$

$$\frac{\partial T_3(x,y)}{\partial x} = (h_{r3} + \mathrm{h}_{c3})(T_1(x,y) - T_\infty), \quad \text{for } x = \delta_1 + \delta_2 + \delta_3 \tag{7}$$

where $\Delta T_{12} = T_1(x)\,|\,x=\delta_1^- - T_2(x)\,|\,x=\delta_1^+$ is the contact temperature difference at the interfaces of glass and PV cells, and similarly ΔT_{23} is that at the interface of cell and paper. T_∞ is the ambient temperature. Clearly, R_{s1} and R_{s2} are the corresponding thermal contact resistances. In addition, the initial temperature field for the PV panel is supposed to be uniform.

Numerical Method

In our numerical treatment, a proper 2D computational domain are used, which links the physical domain by maps: $x_1 = x/\delta_1$, $x_2 = (x-\delta_1)/\delta_2$, $x_3 = (x-\delta_1-\delta_2)/\ \delta_3$, and $z_1 = z_2 = z_3 = z/\delta_1$. It is easy to see that x_i (i=1,2,3) are defined in the region $x \in [0,1]$, and z_i (i=1,2,3) are similarly defined together in $z \in [0, H/\delta_1]$. This means that we can use the 2D coordinates (x,y) defined in merely one domain to describe the three variables T_i (i=1,2,3). Choosing t_0 as the unit of time, we have the governing equations

$$\frac{\partial T_1}{\partial t} = \frac{t_0\alpha_1}{\delta_1^2}\frac{\partial^2 T_1}{\partial x^2} + \frac{t_0\alpha_1}{\delta_1^2}\frac{\partial^2 T_1}{\partial z^2} + \frac{t_0 K I_0}{\rho_1 Cp_1}\exp(-K\delta_1 x) \tag{8}$$

$$\frac{\partial T_2}{\partial t} = \frac{t_0(q_2 - q_1 - q_3)}{\delta_2\rho_2 Cp_2} + \frac{t_0\alpha_2}{\delta_1^2}\frac{\partial^2 T_2}{\partial z^2} \tag{9}$$

$$\frac{\partial T_3}{\partial t} = \frac{t_0\alpha_3}{\delta_3^2}\frac{\partial^2 T_3}{\partial x^2} + \frac{t_0\alpha_3}{\delta_1^2}\frac{\partial^2 T_3}{\partial z^2} \tag{10}$$

where the origin is placed on the cover surface. Using

$$A = \frac{t_0\alpha_1}{\delta_1^2}, B = \frac{t_0\alpha_2}{\delta_1^2}, C_1 = \frac{t_0\alpha_3}{\delta_3^2}, C_2 = \frac{t_0\alpha_3}{\delta_1^2} \tag{11}$$

the equations can be then re-written as

$$\frac{\partial T_1}{\partial t} = A\left[\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial z^2}\right] + a_1 \exp(-b_1 x) \tag{12}$$

$$\frac{\partial T_2}{\partial t} = \frac{t_0(q_2 - q_1 - q_3)}{\delta_2\rho_2 Cp_2} + B\frac{\partial^2 T_2}{\partial z^2} \tag{13}$$

$$\frac{\partial T_3}{\partial t} = C_1\frac{\partial^2 T_3}{\partial x^2} + C_2\frac{\partial^2 T_3}{\partial z^2} \tag{14}$$

where $a_{I=}t_0 KI_0/\rho_1\ Cp_1$, and $b_{I}= K\delta_1$. Accordingly, the boundary conditions takes the form as: for $y\in [0,H/\delta_1]$,

$$A\frac{\partial T_1(x,y)}{\partial x} = \frac{T_0}{\rho_1\delta_1 Cp_1}(h_{r1} + h_{c1})(T_1(x,y) - T_\infty), \quad x=0^+ \tag{15}$$

$$q_1(x,y) = -\Delta T_{12}/R_{s1}, \quad \text{for } x = 1^- \tag{16}$$

$$q_3(x,y) = -\Delta T_{23}/R_{s2}, \quad \text{for } x = 0^+ \tag{17}$$

$$-C_1\frac{\partial T_3(x,y)}{\partial x} = \frac{t_0}{\rho_3\delta_3 Cp_3}(h_{r3} + h_{c3})(T_1(x,y)\text{-}T_\infty), \text{ for } x = 1^- \tag{18}$$

Note that h_{r1} and h_{r3} are heat transfer coefficients of radiation on the outer surfaces of cover and the rear paper, while h_{c1} and h_{c3} are the corresponding heat transfer coefficients concerned with natural convection. According to the empirical expression of for a vertical plate given by McAdams [7]

$$\overline{Nu} = 0.59\, Ra^{1/4}, \qquad \text{for } Ra \in (10^4, 10^9) \tag{19}$$

$$\overline{Nu} = 0.13\, Ra^{1/3}, \qquad \text{for } Ra \in (10^9, 10^{12}) \tag{20}$$

for the Nusselt number *Nu(z)*, it is appropriate to take the form as below:

$$Nu(z) = 4/5 \times 0.59[Ra(2zW/(w+z))]^{1/4} \qquad \text{for } Ra \in (10^4, 10^9) \tag{21}$$

$$Nu(z) = 3/4 \times 0.13[\mathrm{Ra}(2\mathrm{z\ W}/(\mathrm{w}+\mathrm{z}))]^{1/3} \qquad \text{for Ra} \in (10^9, 10^{12}) \tag{22}$$

where W is the plate width. Thus, let λ_a denote the thermal conductivity of ambient air, we have

$$h_{c1}(z) = \lambda_a Nu(z)|_{T=T_1(0)} /(2zW/(w+z)) \tag{23}$$

$$h_{c3}(z) = \lambda_a Nu(z)|_{T=T_3(1)} /(2zW/(w+z)) \tag{24}$$

On the other hand, the radiative heat transfer coefficients h_{r1} and h_{r2} can be calculated by

$$h_{r1} = \varepsilon_g \sigma_0[(T_1(0)/100)^2 + (T_\infty/100)^2][(T_1(0)/100) + (T_\infty/100)] \tag{25}$$

$$h_{r3} = \varepsilon_p \sigma_0[(T_3(1)/100)^2 + (T_\infty/100)^2][(T_3(1)/100) + (T_\infty/100)] \tag{26}$$

where ε_g and ε_p are the specular reflectances of glass cover and rear paper. σ_0(=5.6703) is the radiation constant of a blackbody.

The initial condition is simply given by

$$T_i = T_0 \quad \text{for } i = 1,2,3,\ x \in [0,1], \text{ and } y \in [0, H/\delta_1] \tag{27}$$

The solution for the problem of heat transfer in a vertical PV panel as described can be obtained numerically. Since the mathematical model contains multi-parameters, such as thermal resistances at the interfaces of both materials, the natural convection coefficients, which require values to be assigned, it seems to be reasonable to use a numerical scheme with first order accuracy. To obtain the temperature field in the PV panel, special attentions are required for treatment the source terms in the governing equations, and the boundary conditions. Thus, the numerical techniques reported by Patankar [8] are appreciated and utilized in our analysis using a fractional method, in which the transient terms is discretized by first order backward difference scheme, and the space discretization is performed by finite volume scheme.

Results and Discussions

The numerical results were obtained with parameters shown in Table 1. Uniform grid 80 ×50 as well as uniform time interval δt=10^{-2} were employed. Numerical solutions were obtained by using a Langchao 466 type personal computer.

TABLE 1

The Parameters Used in Numerical Analysis

$\rho_1 = 2500\text{kg/m}^3$	$Cp_1 = 670\text{J/kgK}$	$\lambda_1 = 0.75\text{W/mK}$
$\rho_2 = 2325\text{kg/m}^3$	$Cp_2 = 677.8\text{J/kgK}$	$\lambda_2 = 83.7\text{W/mK}$
$\rho_3 = 750\text{kg/m}^3$	$Cp_3 = 1510\text{J/kgK}$	$\lambda_3 = 0.14\text{W/mK}$
$\rho_a = 1.205\text{kg/m}^3$	$Cp_a = 1005\text{J/kgK}$	$\lambda_a = 2.67\times10^{-2}\text{W/mK}$
$\delta_1 = 0.004\text{m}$	$\delta_2 = 0.0008\text{m}$	$\delta_3 = 0.0008\text{m}$
$t_0 = 200\text{s}$	$W = 0.8\text{m}$	$v_a = 1.056\times10^{-5}\text{m}^2/\text{s}$
$T_0 = 300\text{K}$	$H = 1.2\text{m}$	$K = 15\text{m}^{-1}$
$\varepsilon_g = 0.90$	$\varepsilon_p = 0.45$	$(\alpha\tau) = 0.90\exp(-K\delta_1)$

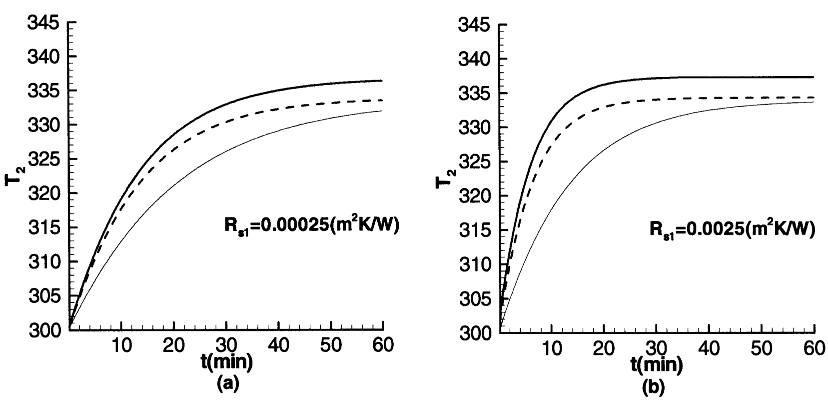

FIG. 2

The evolution of temperature T_2 for I_0=1000W/m^2 and K=20m^{-1}, where the thin solid curve, dashed curve and the coarse solid curve are compatible to the cases of R_{s2}=0.00025, 0.0025, and 0.025m^{2}K/W, respectively.

(a) R_{s1}=0.00025m^{2}K/W; (b) R_{s1}=0.0025m^{2}K/W.

The Effects of Thermal Contact Resistances

Thermal contact resistance affects the heat conduction rate and time required to arrive at steady state. For the problem on hand, from Figure 2 (a) and (b), the PV cell temperature increases as the thermal contact resistances R_{s1} and R_{s2} take relatively large values, and in the mean while, the time period required to approach the thermal equilibrium decreases. In the case of R_{s2}=0.025m^{2}K/W, if R_{s1} is 0.00025m^{2}K/W, to arrive at the thermal equilibrium from a uniform initial steady state under indoor normal irradiation I_0=

1000W/m^2, it needs about 55min; but if the value assigned for R_{s1} is 0.0025 M^{2}K/W, the time period is about 20min.

The effects of thermal contact resistances can also be found from Figure 3 (a) and (b), where PV cell temperature as well as surface temperatures for cover and rear paper layer are also illustrated. Again, Table 2 has shown the detail numerical solutions for nine cases, where the steady state temperatures are given.

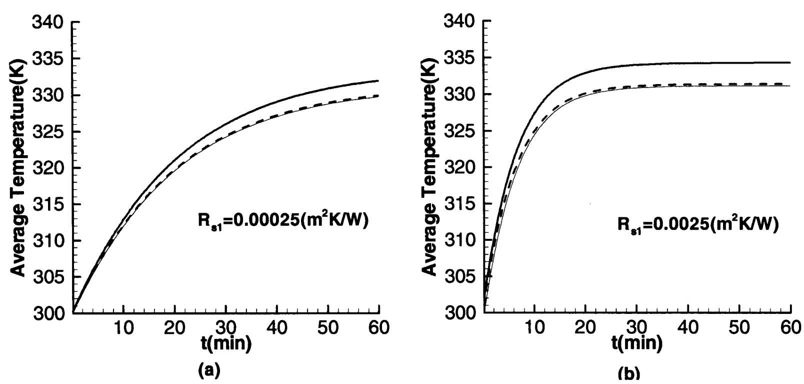

FIG. 3

The evolution of temperature for I_0=1000W/m^2 and K=20m^{-1}, where the thin solid curve, dashed curve and the coarse solid curve are compatible to the average temperature $\overline{T}_1(x=0)$, the average PV cell temperature $\overline{T}_2(x\in(0,1))$, and the average rear temperature $\overline{T}_3(x=1)$, respectively.

(a) $R_{s1}=R_{s2}$=0.00025m^{2}K/W; (b) $R_{s1}=R_{s2}$=0.0025m^{2}K/W.

Figure 4 shows a stereograph of temperature field (a) in glass cover, and (b) in rear paper for five instants: t=200,400,600,800,1000s when $R_{s1}=R_{s2}$=0.0025m^{2}K/W. It is the dependence of H_{c1} and h_{c3} on the vertical direction leads to the vertical variation of temperature.

The Effects of Thermal Storage

Thermal storage appears when the left hand side of the governing equations are retained. The effect of thermal storage leads to an alleviation of temperature increase, see Figure 2 and Figure 3. As soon as the effects of thermal storage in ignored, what obtained is the thermal equilibrium temperature. Evidently, this treatment is not appropriate to the heat transfer in a PV panel. However, it may be reasonable for long period operation of PV systems.

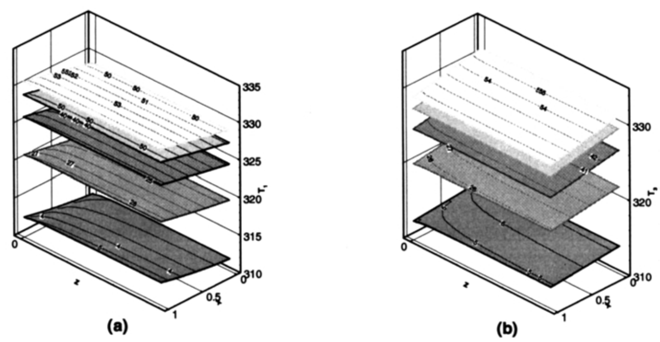

FIG. 4
The stereo-view of temperature field for five instants t=200, 400, 600, 800, 1000s in case of I_0=1000W/m^2 and K=20m^{-1} when thermal contact resistance is R_{s1}= R_{s2}=0.0025m^{2}K/Wwhere the curves labeled by 1,2,...,59,60 are for the temperature contours for values 311.5, 312.+ΔT, ..., 332.5, with ΔT= (332.5-311.5)/59 K.
(a) $T_1(x,z)$ in the cover, (b) $T_3(x,z)$ in the rear paper.

The Effects of Indoor Normal Irradiations

A large value of indoor normal irradiation results in higher steady state temperatures, this can be seen from Table 3. Clearly, the steady temperature also depends the surface thermal resistance, which can be expressed by the inverse of the sum of radiative and convective heat transfer coefficients for the frontal and rear surfaces of the PV panel. For indoor cases, an increase of 200W/m^2 leads the PV temperature grow 6 ~7 K.

The Effects of Extinction Coefficient

It appears a slight effect of K on the temperature field in a PV panel as shown in Table 4. The increase of temperature in glass cover appears is less than 0.3 K.

TABLE 2
The Effects of Thermal Contact Resistances

R_{s1}(m2K/W)	$T_1(x=0)$	$T_1(x=1)$	$T_2(x\in(0,1)$	$T_3(x=0)$	$T_3(x=1)$
		For R_{s1}=0.00025m^{2}K/W			
0.00025	329.6710	331.7732	331.8917	331.7818	329.8788
0.0025	331.1300	333.3430	333.4654	332.5854	330.6256
0.025	333.7588	3336.2170	336.3543	328.9416	327.2380
		For R_{s1}=0.0025m^{2}K/W			
0.00025	330.4811	332.6403	333.5971	333.4825	331.4627
0.0025	331.1139	333.3175	334.2913	333.3869	331.3714
0.025	333.7481	336.1959	337.2913	329.6637	327.9113
		For R_{s1}=0.025m^{2}K/W			
0.00025	327.6086	329.5109	337.5914	337.4605	335.1589
0.0025	328.1664	330.1114	338.3640	337.3308	335.0364
0.025	330.9277	333.1163	342.5580	333.6786	331.6474

TABLE 3
The Effects of Irradiation I_0 and Extinction Coefficient K
for R_{s1}= R_{s2}=0.0025m^{2}K/W

I_0(W/m^2)	$T_1(x=0)$	$T_1(x=1)$	$T_2(x\in(0,1)$	$T_3(x=0)$	$T_3(x=1)$
		Part (*a*)			
200	308.1718	308.6705	308.8851	308.7154	308.2947
400	314.4095	315.3460	315.7608	315.4073	314.5873
600	320.1338	321.5069	322.1237	321.5880	320.3722
800	325.6859	327.4950	328.3117	327.5915	325.9758
1000	331.1139	333.3175	334.2913	333.3869	331.3714
		Part (*b*)			
K(m^{-1})	$T_1(x=0)$	$T_1(x=1)$	$T_2(x\in(0,1)$	$T_3(x=0)$	$T_3(x=1)$
0.00025	330.8814	333.1691	334.2316	333.3291	331.3175
0.0025	331.1139	333.3175	334.2913	333.3869	331.3714
0.025	331.2618	333.3853	334.2762	333.3727	331.3589

Conclusions

Numerical analysis of heat transfer in a PV panel has been presented by using a mathematical model derived from energy conservation. The model has included the effects of thermal storage, optical energy absorption, and thermal contact resistances. The fractional scheme, together with first order backward time difference as well as the finite volume integration was used in the numerical study. It was found that thermal contact resistance is the key feature to determine the field of temperature in the PV panel. The time required for unsteady heating is shortened when thermal contact resistance increases. For detail thermal analysis, it is necessary to consider the thermal storage effects in the heat transfer problem unless long term PV system operation is involved. However, the optical energy absorption term in the glass cover shows just a very slight effect on the temperature distribution in the PV panel.

Finally, we emphasize that this numerical analysis is suppressed to indoor cases in which merely normal optical irradiation is supposed. The subsequent work in progress will concern with outdoor cases, the corresponding analysis and comparison with experiments will be presented in our companion paper.

Acknowledgement

This work was supported from the State Key laboratory of Fire Science.

Nomenclature

Cp	specific capacity under constant pressure
h_r	radiative heat transfer coefficient
h_c	convective heat transfer coefficient
I_0	indoor solar irradiation
K	extinction coefficient
Nu	Nusselt Number
Q	heat flux
Ra	Rayleigh Number
R_s	thermal contact resistance
T	temperature
W	Width of PV panel
δ	thickness
ρ	density
$(a\ \tau)$	transmittance-absorptance product
ΔT	contact temperature difference

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Received November 28, 2001